

Prediction of Punching Shear Strength of HSC Interior Slab-Column Connections

Hussein M. Elsanadedy*, Yousef A. Al-Salloum**, and Saleh H. Alsayed***

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Abstract

Flat plate systems are widely used in reinforced concrete structures. Using of high-strength concrete has been common recently. In the current international codes of practice for concrete structures, the design methods for assessment of punching shear capacity are based on experimental data of flat plates with Normal-Strength Concrete (NSC). The aim of this research is to come up with new formula for punching shear resistance, consistent with data of flat plates made from High-Strength Concrete (HSC). Test results of 61 HSC interior slab-column connection specimens were collected from the literature. The available test results were not only compared with current code provisions but with equations proposed by other researchers as well. A new formula for predicting punching shear strength of HSC interior slab-column connections is proposed. An innovative design equation is also suggested.

Keywords: *punching shear strength, interior column connections, flat plates, high strength concrete*

1. Introduction

The use of high-strength concrete (HSC) for structural components, is becoming exceedingly common in the Kingdom of Saudi Arabia as well as the rest of the world. A number of structures including bridges are being built with High-Strength Concrete (HSC)/High Performance Concretes (HPC) with strengths exceeding 60 MPa, due to the number of advantages offered by such concretes (Subramanian, 2005).

Flat plate slab system does not have beams, column capitals, or drop panels, which make them both attractive and cost-effective at the same time. Moreover, the elimination of beams reduces the overall height of the floor in a multi-storey building thereby creating additional floor space. Another advantage of a flat plate slab system is the flexibility in partition location. As a result of these desirable features, the use of this slab system has become common in structures like multi-storey buildings, car parks, etc.

Flat plate slabs exhibit higher stress at the column connection and are most likely to fail due to a sudden and brittle punching shear rather than flexural failure, especially when a high reinforcement ratio is used. Such a failure generally occurs due to transfer of vertical shearing force and bending moment between the slab and the column. The vertical shearing force is mainly caused as a result of gravity loads, while the unbalanced bending moment is a result of non-uniform gravity loads or any

lateral loads due to wind or earthquake forces. As a result of this load concentration and the unbalanced moments, the punching shear failure of the slab occurs at a load well below the flexural capacity of the slab thereby resulting in concrete crushing along the periphery of the columns, before the steel reinforcement reaches the yield strain. Fig. 1 shows the typical punching failure of a slab near the vicinity of a column. The observed angle of failure surface was found to vary between 26° and 36° for normal strength concrete and 32° and 38° for HSC (Marzouk and Hussein, 1992).

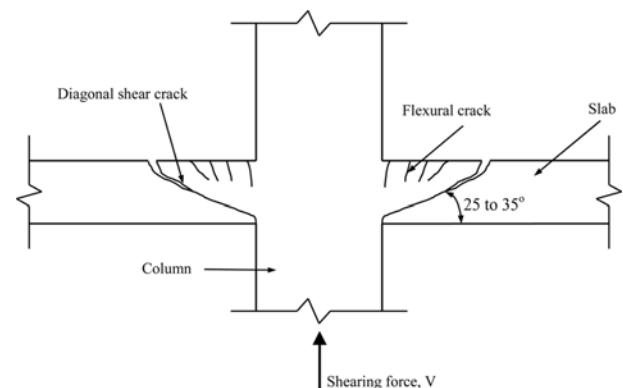


Fig. 1. Flexural and Shear Cracks in the Flat Slab Near the Vicinity of Column

*Assistant Professor, Specialty Units for Safety & Preservation of Structures, Dept. of Civil Engineering, King Saud University, Riyadh 11421, Saudi Arabia (Corresponding Author, E-mail: elsanadedy@yahoo.com)

**Professor, Specialty Units for Safety & Preservation of Structures, Dept. of Civil Engineering, King Saud University, Riyadh 11421, Saudi Arabia (E-mail: ysalloum@gmail.com)

***Professor, Specialty Units for Safety & Preservation of Structures, Dept. of Civil Engineering, King Saud University, Riyadh 11421, Saudi Arabia (E-mail: shalsayed@gmail.com)



Fig. 2. Punching Shear Failure at Interior Slab-Column Connections - Piper's Row Car Park, Wolverhampton, UK, 1997 (Wood, 2003a, 2003b)

Errors in predicting the punching shear capacity have been known to cause catastrophic failures resulting in huge loss of life and property. One such failure is the collapse of the six-year old, five-storey Sampoong Department store (originally designed as an office block and later converted to department store with reckless structural modifications) in Seoul, Korea in 1995. This collapse under service conditions killed 498 people (Gardner *et al.*, 2002). Another example is the partial collapse of the Pipers Row Car Park, Wolverhampton, West Midlands, UK, on 20th March 1997. The collapse of the deteriorating, 120tonne section of the top floor slab, measuring 15×15 m, occurred at night under dead load only. Fortunately, the car park was empty, so there were no injuries, but the closure and subsequent demolition of the whole 400-space car park caused substantial disruption. As seen in Fig. 2, punching shear failure at one column led to a progressive collapse as similar failures followed at seven adjacent columns (Wood, 2003a, 2003b).

In the case of slab-column connection, the use of HSC improves the punching shear resistance allowing higher forces to be transferred through the slab-column connection. However, this increase in the shear resistance is offset by the increased brittleness of the system. In fact, HSC members exhibit in some instances different failure mechanisms, compared with the normal-strength concrete members, and simple extrapolation of models and equations meant for normal strength to HSC may lead to unsafe designs (Subramanian, 2003).

The behavior of normal-strength concrete slab-column connection for flat plates has been sufficiently investigated under gravity

loading as well as lateral loading. However, only a few studies have been carried out on the behavior of HSC slab-column connection under gravity loading.

Smadi and Yasin (2008) studied the behavior of high strength fibrous concrete slab-column connections under gravity and lateral loads. For the study, ten slab-column connections were tested under combinations of gravity and lateral loads and the variables selected were, strength of concrete, volumetric ratio of steel fibers, type of steel fibers, and moment to shear ratio. Both Normal-Strength Concrete (NSC) and High-Strength Concrete (HSC) slabs were tested as per the testing regimen. Based on the results of the experimental study, it was found that, the addition of steel fibers does significantly enhance the performance of the tested slab. It improved the shear strength, increased the ductility due to deflection and rotation, yielded greater stiffness and smaller cracks widths. Further improvement was also obtained when larger aspect ratio of steel fibers was used. For specimens constructed with HSC, the ultimate shear strength increased by 7-21%, compared with specimens constructed with NSC. The displacement and rotation ductility ratios for HSC specimens were larger than those for NSC by 11-64% for displacement ductility and 106-123% for rotation ductility. Their corresponding energy absorptions due to deflection and rotation were also larger by 48-150% and 93-246%, respectively. Incorporating steel fibers with high-strength concrete improved the overall deformation characteristics of the tested specimens and resulted in less sudden and more gradual failure mode.

In another study, Ngo (2001) compiled the experimental results from 4 research studies on high-strength concrete flat slabs, which were used to review the existing recommendations in design codes for punching shear failure of normal strength slabs. Design codes referred in this study are AS3600 (1994) and CEB-FIP MC 90 (1993). Comparison of test results with code predictions revealed that the CEB-FIP formula is un-conservative for HSC flat slabs as it overestimated the punching shear strength for 34.5% of the data points. However, the AS3600 formula was un-conservative for only 10.3% of the data points.

In a study, Hallgren and Kinnunen (1996) tested 10 circular HSC slabs, with and without shear reinforcement, supported on concrete column stubs. The parameters studied involved varying concrete strengths from 85 to 108 MPa and varying the two way main flexural slab reinforcement ratios from 0.003 to 0.012. The slabs were identical in shape and size with Normal-Strength Concrete (NSC) slabs tested previously. The tests showed that a significant increase of the punching shear strength can be gained by using HSC. The tests also indicated a more efficient use of the flexural reinforcement in HSC slabs than in NSC slabs. Comparisons between ultimate loads observed in the tests and corresponding punching loads calculated according to different current design methods showed that modifications of most of the design methods are required in order to be equally valid for HSC slabs and for NSC slabs. Some of the slab tests were simulated numerically by using the non-linear finite element method. The concrete behavior was modeled with a smeared crack approach

based on non-linear fracture mechanics. The finite element analysis showed that the brittleness of HSC reduces the rate of increase of punching strength with increasing concrete strength. The analysis also indicated the mechanism of punching shear failure. Based on findings obtained from the tests and from the finite element analysis, a modified mechanical model of punching was proposed. The failure criterion of the proposed model depends on the brittleness of the concrete and the depth of the tangential compression zone of the slab. The proposed model gave good predictions of the ultimate loads of the HSC slabs in the present investigation, as well as of HSC and NSC slabs tested by others. Based on the comparison with these tests, the model is equally valid for HSC slabs and for NSC slabs. The model also accurately predicts the size effect on the punching shear strength of the slabs.

Marzouk and Hussein (1992) tested 17 square HSC slab specimens to investigate the deformation and strength characteristics of punching shear failure of high-strength concrete slabs. The tested specimens had different slab depths and reinforcing ratios varying between 0.49 and 2.33 percent. Test results revealed that high-strength concrete slabs exhibit a more brittle failure than normal-strength concrete. Experimental results indicated that as the level of reinforcement is increased, the punching strength of the slabs is also increased. It was found that using the cubic root of the concrete compressive strength to predict the punching resistance of the concrete slabs generally yields better results than the square root expression used in North American codes.

In a study by Ghannoum (1998), the behavior of interior slab-column connections in flat plates was investigated. Six two-way slab-column specimens, which were designed to fail in punching shear, were tested. The parameters investigated were the use of high-strength concrete and the concentration of the slab flexural reinforcement in the immediate column region. The effects of these parameters on the punching shear capacity, negative moment cracking and stiffness of the two-way slab specimens were studied. The test results obtained from this experimental program were compared with the punching shear predictions of: the Canadian CSA A23.3-94 Standards (Canadian Standards Association, 1994), the ACI 318-95 Code (ACI Committee 318, 1995), the BS 8110-85 Standards (British Standards Institution, 1985) and the CEB-FIP MC 90 Code (1993). The beneficial effects of the use of high-strength concrete and of the concentration of flexural reinforcement in the immediate column vicinity were demonstrated. It was also concluded that the punching shear strength of slab-column connections is a function of the flexural reinforcement ratio, and the shear design of flat plates according to the current Canadian and American codes could be unconservative under certain conditions. It was also recommended to include the effect of flexural reinforcement ratio in the punching shear expressions of both the CSA Standards and the ACI Code.

Metwally *et al.* (2008) carried out experimental and analytical programs to investigate the punching shear behavior and strength of both NSC and HSC flat slabs. Investigated variables were

tension steel reinforcement ratio, concrete compressive strength and slab thickness. To examine the validity of the ECP-203 (Egyptian Code Committee 203, 2007), ACI 318-08 (ACI Committee 318, 2008), and BS 8110-97 (British Standards Institution, 1997) provisions on punching shear, the results of 55 reinforced concrete flat slabs were analyzed and compared with the ultimate punching shear strengths predicted using these codes. This study revealed that investigated variables have significant effects on increasing punching shear capacity. It was also found out that the BS 8110 formula gives more accurate estimation of punching shear capacity for both NSC and HSC flat slabs than the ACI 318-08 and the ECP-203 codes.

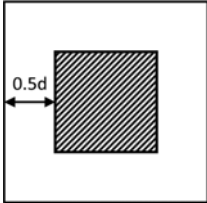
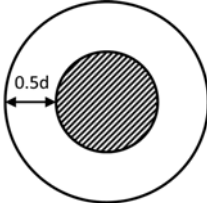
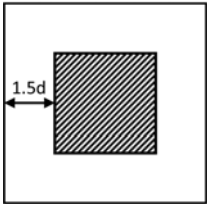
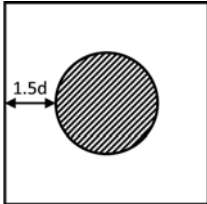
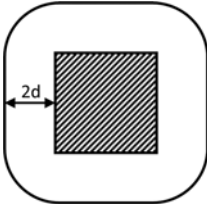
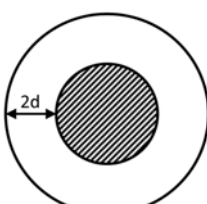
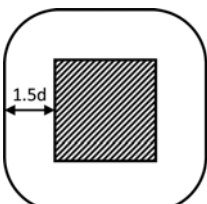
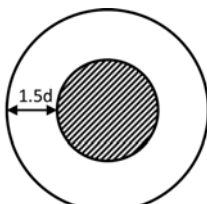
In the current codes of practice for concrete structures, the design methods for assessment of punching shear capacity are based on experimental data of flat plate specimens made from low and normal-strength concrete with compressive strength between 15 and 35 MPa. High-strength concrete above 40 MPa has been widely used in the construction industry. Since HSC not only has higher modulus of elasticity but also exhibits different failure mechanism from normal strength concrete, it is unrealistic to directly extrapolate the models and equations based on normal-strength concrete to HSC members. In addition, the influence of reinforcement ratio and size effect on punching shear capacity is ignored in some codes such as the ACI 318-08 (ACI Committee 318, 2008); however, it is recognized in some other codes. Although extensive research has been done on the punching shear strength of flat plates, to date there is still no general applicable rational theory. Additionally, in spite of the wide use of HSC, only a few research projects have been conducted on the punching shear resistance of HSC flat plates. Hence, it is necessary to re-examine the applicability of the punching shear design methods to HSC flat plates using available published data.

The objective of this study was to come up with new formula for punching shear resistance, not only consistent with data of flat plates made from High-Strength Concrete (HSC) but also takes into account the effect of all significant parameters (such as reinforcement ratio, column size and slab depth). This study is only limited to interior flat plate-column connections.

2. Major Code Provisions

There are significant variations in the approaches used to assess shear resistance of RC slab-column connections in the current major codes. Generally, all design codes adopt the simple “shear on certain critical perimeter” approach and involve only the most important parameters. The critical section for checking punching shear is usually situated a distance between 0.5 to 2.0 times the effective depth (d) from the edge of the loaded area [$0.5d$ for ACI 318-08 (ACI Committee 318, 2008), $1.5d$ in BS 8110-97 (British Standards Institution, 1997) and DIN 1045-1 (2001), and $2d$ in EC2-2003 (Eurocode 2, 2003)]. The other important difference amongst codes is in the way they represent the effect of concrete compressive strength (f'_c) on punching

Table 1. Comparison of Major Code Provisions

Code	Critical perimeter		Nominal shear capacity (Units: N and mm)
	Square columns	Circular columns	
ACI 318-08	 $b_o = 4(c + d)$	 $b_o = \pi(c + d)$	$V_c = \frac{1}{3} \sqrt{f'_c} b_o d$ f'_c = specified cylinder strength (MPa) The upper limit of $f'_c = 35$ MPa has been ignored b_o = critical shear perimeter d = average effective depth c = column width or diameter
BS 8110-97	 $b_o = 4(c + 3d)$	 $b_o = 4(c + 3d)$	$V_c = 0.79 \left(\frac{400}{d} \right)^{1/4} (100\rho)^{1/3} b_o d$ The value of V_c shall be multiplied by $\sqrt[3]{f_{cu}/25}$ for $f_{cu} > 25$ MPa The value of $\sqrt[3]{400/d}$ shall not be less than 1.0 The upper limit of $f_{cu} = 40$ MPa has been ignored f_{cu} = characteristic cube strength (MPa) ρ = average steel ratio within 1.5d of column face = $0.5(\rho_x + \rho_y)$
EC2-2003	 $b_o = 4(c + \pi d)$	 $b_o = \pi(c + 4d)$	$V_c = \frac{0.18}{\gamma} \left[1 + \sqrt{\frac{200}{d}} \right] (100\rho f_{ck})^{1/3} b_o d$ f_{ck} = characteristic cylinder strength (MPa) ρ = flexural steel ratio = $\sqrt{\rho_x \rho_y}$ γ_c = partial safety factor = 1.5 (taken as 1.0) $1 + \sqrt{200/d}$ = size factor ≤ 2.0
DIN 1045-1	 $b_o = 4c + 3\pi d$	 $b_o = \pi(c + 3d)$	$V_c = 0.14 \left[1 + \sqrt{\frac{200}{d}} \right] (100\rho f_{ck})^{1/3} b_o d$ f_{ck} = characteristic cylinder strength (MPa) ρ = steel ratio = $0.5(\rho_x + \rho_y) < 0.4 f_{cd}/f_y < 0.02$ $f_{cd} = 0.85 f_{ck}$ f_y = yield strength of flexural steel (MPa) $1 + \sqrt{200/d}$ = size factor ≤ 2.0

shear capacity. Generally, these codes expressed this effect in terms of $(f'_c)^n$, where (n) varies from $(1/2)$ in the ACI code to $(1/3)$ in the European codes. Punching shear provisions of current major codes are illustrated in Table 1.

3. Available Data Collection

In this research, forty five square slab specimens with square columns in addition to sixteen circular slab specimens with circular columns have been collected from the literature. Square slab specimens have been tested by Ghannoum (1998), Marzouk and Hussein (1992), Tomaszewicz (1993), Gardner (1990), Elstner and Hognestad (1956), Marzouk and Jiang (1997), Marzouk *et al.* (1998), Adetifa and Polak (2005), Abdel Hafez

(2005) and Osman *et al.* (2000). Circular slab specimens have been tested by Ramdane (1996), Hallgren and Kinnunen (1996) and Metwally *et al.* (2008). Slab specimens selected in this study were constructed without transverse shear reinforcement where the punching shear load was only resisted by concrete contribution. Only slabs that failed in punching shear mode were chosen for this study. Details of the selected specimens are listed in Tables 2 and 3 for square and circular slabs, respectively. The intent is to predict the punching shear strength considering the main influential variables, namely: 1) concrete compressive strength, 2) flexural reinforcement ratio, 3) effective slab depth, and 4) column geometry. Past investigations showed that these declared four variables have the most significant effect on flat plates without shear reinforcement and supported on circular or square

Table 2. Details of Square Specimens

Slab source	Slab width (mm)	f'_c (MPa)	f_y (MPa)	Column width (mm)	Slab depth (mm)	Average effective depth (mm)	Steel ratio (ρ) %	Ultimate load (kN)
Ghannoum (1998)								
S2-U	2300	57.1	450	225	150	110	0.96	363
S2-B	2300	57.1	450	225	150	110	1.92	447
S3-U	2300	67.1	450	225	150	110	0.96	443
S1-U	2300	67.1	450	225	150	110	1.92	485
Marzouk and Hussein (1992)								
NS1	1500	42	490	150	120	95	1.47	320
HS1	1500	67	490	150	120	95	0.491	178
HS7	1500	74	490	150	120	95	1.193	356
HS3	1500	69	490	150	120	95	1.473	356
HS4	1500	66	490	150	120	90	2.37	418
HS5	1500	68	490	150	150	120	0.64	365
HS6	1500	70	490	150	150	120	0.944	489
HS8	1500	69	490	150	150	120	1.111	436
HS9	1500	74	490	150	150	120	1.611	543
HS10	1500	80	490	150	150	120	2.333	645
HS12	1500	75	490	150	90	70	1.524	258
HS13	1500	68	490	150	90	70	2	267
HS14	1500	72	490	220	120	95	1.47	498
Tomaszewicz (1993)								
nd65-1-1	2500	64.3	500	200	320	275	1.42	2050
nd95-1-1	2500	83.7	500	200	320	275	1.42	2250
nd95-1-3	2500	89.9	500	200	320	275	2.43	2400
nd115-1-1	2500	112	500	200	320	275	1.42	2450
nd65-2-1	2200	70.2	500	150	240	200	1.66	1200
nd95-2-1	2200	88.2	500	150	240	200	1.66	1100
nd95-2-3	2200	89.5	500	150	240	200	2.49	1450
nd115-2-1	2200	119	500	150	240	200	1.66	1400
nd115-2-3	2200	108.1	500	150	240	200	2.49	1550
nd95-3-1	1100	85.1	500	100	120	88	1.72	330
Gardner (1990)								
25	610	66.8	450	180	150	122	0.66	306.6
27	470	66.8	450	135	120	81	1.47	243
28	470	66.8	450	135	120	86	0.45	147.7
Elstner and Hognestad (1956)								
B-2	1780	47.6	326	254	150	114	0.476	200
B-4	1780	47.7	330	254	150	114	1.01	334
B-9	1780	43.9	341	254	150	114	2	505
B-14	1780	50.5	325	254	150	114	3.02	578
Marzouk and Jiang (1997)								
HS 17	1500	67	490	250	150	120	1	511
Marzouk <i>et al.</i> (1998)								
H.H.Z.S.1.0	1500	67.2	460	250	150	119	1	512
Adetifa and Polak (2005)								
SB1	1200	44	455	150	120	89	1.2	253
Abdel Hafez (2005)								
HS-16	540	63.28	550	100	60	45	1.72	129.5
HS-17	540	63.28	550	100	60	45	2.81	158
HS-18	540	65.6	550	100	60	45	2.81	136
HS-19	540	65.6	550	100	60	45	4.24	160
Osman <i>et al.</i> (2000)								
HSLW 0.5 P	1900	76.1	490	250	150	120	0.5	303.7
HSLW 1.0 P	1900	73.4	490	250	150	115	1	473.5
HSLW 1.5 P	1900	75.5	490	250	150	115	1.5	538.5
HSLW 2.0 P	1900	74	490	250	150	115	2	613.4

Table 3. Details of Circular Specimens

Slab source	Slab diameter (mm)	f'_c (MPa)	f_y (MPa)	Column diameter (mm)	Slab depth (mm)	Average effective depth (mm)	Steel ratio (p) %	Ultimate load (kN)
Ramdane (1996)								
5	1372	54.4	650	150	125	98	0.58	190
12	1372	60.4	650	150	125	95	1.28	319
15	1372	68.4	650	150	125	95	1.28	276
16	1372	99.2	650	150	125	95	1.28	362
21	1372	41.92	650	150	125	98	1.28	286
22	1372	84.24	650	150	125	98	1.28	405
23	1372	56.4	650	150	125	100	0.87	341
24	1372	44.64	650	150	125	98	1.28	270
Hallgren and Kinnunen (1996)								
HSC0	2400	90.3	643	250	240	200	0.8	965
HSC 1	2400	91	627	250	240	200	0.8	1021
HSC2	2400	85.7	620	250	240	194	0.82	889
HSC4	2400	91.6	596	250	240	200	1.2	1041
HSC6	2400	108.8	633	250	240	201	0.6	960
HSC8	2400	95	634	250	240	198	0.8	944
HSC9	2400	84.1	631	250	240	202	0.33	565
Metwally <i>et al.</i> (2008)								
H15-2	1100	53.28	360	140	150	120	1.6	410

RC columns.

4. Comparison of Code Predictions with Experimental Results

Comparison of the code provisions with experimental results is not straightforward because the code expressions were developed to be conservative. In this study, the code punching shear capacities were calculated using the reported mean cylinder compressive strength, in place of the specified strength in the ACI code and the characteristic strength in the BS 8110-97, EC2-2003, and DIN 1045-1 codes. All limitations on the magnitude of the concrete compressive strength are ignored in the comparisons given below.

For all codes formulas, the ratios of the tested-to-predicted punching shear capacity are calculated. Statistical indicators in terms of: mean (m), standard deviation (σ), coefficient of variation, and 5% percentile are given in Table 4. The predicted value of the punching shear strength of HSC flat plate specimens has been plotted against its observed value in Figs. 3 to 6 for the ACI 318-08, BS 8110-97, EC2-2003 and DIN 1045-1 codes, respectively. The correlation parameters for the predicted versus tested punching shear strength are listed in Table 5 for the

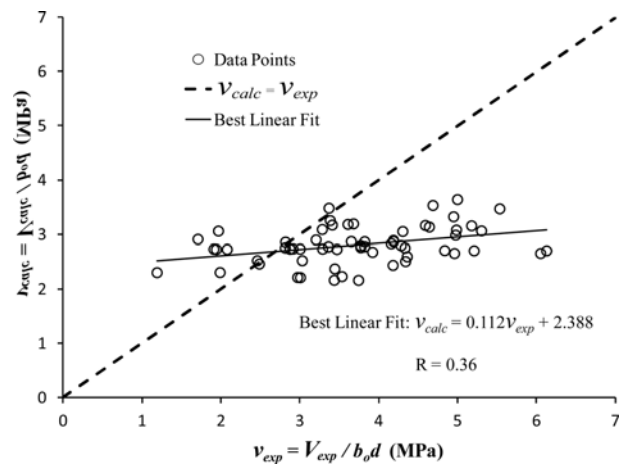


Fig. 3. Observed versus Predicted Punching Shear Strength for ACI 318-08 Code Provisions

different code provisions. It is indicated that among all codes, the BS 8110 formula gives the best fit to the experimental data as it has the highest coefficient of determination (R^2) of 0.90 as shown in Table 5. This is also evident from Table 4 as the BS 8110 formula has the least values of mean, standard deviation and coefficient of variation among all code provisions. However,

Table 4. Statistical Indicators for Different Code Provisions for Punching Shear of HSC Flat Plates

Statistical parameter for (V_{exp}/V_{calc})	ACI 318-08	BS 8110-97	EC2-2003	DIN 1045-1	Proposed design formula of Eq. (5)
Mean (m)	1.33	1.06	1.19	1.84	1.28
Standard deviation (σ)	0.37	0.15	0.25	0.37	0.13
Coefficient of variation	0.29	0.12	0.18	0.26	0.10
5% Percentile	0.70	0.81	0.88	1.36	1.09

Table 5. Correlation Indicators for Different Code Provisions for Punching Shear of HSC Flat Plates

Correlation parameter	ACI 318-08	BS 8110-97	EC2-2003	DIN 1045-1	Proposed design formula of Eq. (5)
Coefficient of correlation (CC)	0.36	0.95	0.77	0.80	0.95
Coefficient of determination (R^2)	0.13	0.90	0.60	0.64	0.90
Percent data with non-conservative prediction	16.4	32.8	19.7	0.0	0.0

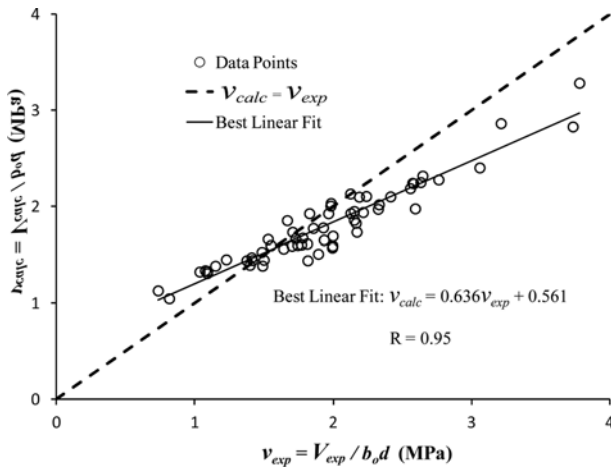


Fig. 4. Observed versus Predicted Punching Shear Strength for BS 8110-97 Code Provisions

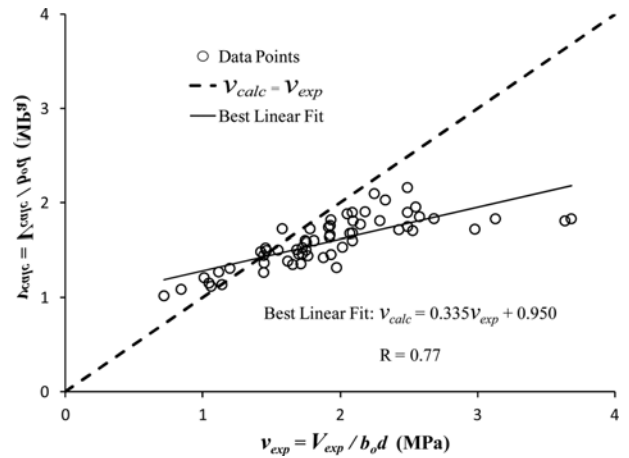


Fig. 5. Observed versus Predicted Punching Shear Strength for EC2-2003 Code Provisions

as seen in Fig. 4 and Table 5, the BS 8110 formula is un-conservative as 32.8% of the data points have their calculated punching shear capacity exceeding the experimental value. Yet, among all codes, the least accurate prediction was given by the ACI 318 code as it has the lowest coefficient of determination (R^2) of 0.13 and the highest coefficient of variation of 0.29. Moreover, the ACI 318 formula is un-conservative as it overestimated the punching shear strength for 16.4% of the data points as seen in Fig. 3 and Table 5. This may be due to the lack of reinforcement ratio and size effect terms in the ACI 318 code equations. It is also noted that the DIN 1045-1 code is the most conservative in estimating the punching shear strength as none of the data points had its predicted shear strength exceeding the experimental value. Yet, the DIN 1045-1 code has a poor correlation with fairly low R^2 of 0.64 and relatively high coefficient of variation of 0.26. As observed from Table 4, the shear resistance coefficients of the ACI 318-08, BS 8110-97, and EC2-2003 codes should be reduced by 30%, 19%, and 12%, respectively, to meet a 5% percentile value of unity.

5. Approaches Proposed by Other Researchers

Other researchers (Rankin and Long, 1987; Zhang, 2003; Gardner and Shao, 1996; Sherif and Dilger, 1996; El-Gamal and Benmokrane, 2004) proposed some formulas to predict the punching shear strength of interior column-HSC flat plate connections. Most of the proposed equations are similar in assuming the critical section at a distance of $d/2$ from the column face. Therefore, the critical perimeter (b_o) is equal to $4(c + d)$ for

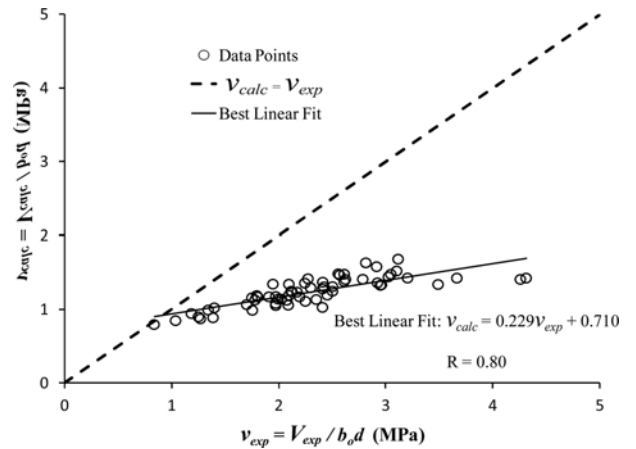


Fig. 6. Observed versus Predicted Punching Shear Strength for DIN 1045-1 Code Provisions

square columns and $\pi(c + d)$ for circular columns, which is similar to the ACI 318-08 provisions. However, the researchers are disparate in estimating the effect of concrete compressive strength; additionally, they are different in assuming the effect of flexural steel ratio. Rankin's modified equation (Rankin and Long, 1987; Zhang, 2003) for HSC flat plates gives concrete punching shear strength (V_c) as proportional to the cubic root of the concrete compressive strength and the fourth root of the reinforcement ratio. Gardner and Shao (1996) suggested another approach, which included the effect of yield strength of flexural reinforcement (f_y) in the proposed equation. Yet, they preferred the power of one-third to express the effect of both reinforcing

steel and concrete strength, which was the same for Sherif and Dilger (1996). El-Gamal and Benmokrane (2004) proposed an equation that was modified from the ACI 318 code provisions. They kept the same form of the ACI equation but with the introduction of two new parameters in order to take into account the effects of axial stiffness of the main steel reinforcement and the continuity of the RC slab. Their new formula could be also used for flat plates reinforced with FRP rods. Table 6 summarizes the equations for predicting the punching shear proposed by the investigators acknowledged above.

For all researchers' formulas, the ratios of the tested-to-predicted punching shear capacity are calculated. Statistical indicators in terms of: mean (m), standard deviation (σ), coefficient of variation, and 5% percentile are given in Table 7. The predicted value of the punching shear strength has been plotted against its observed value in Figs. 7 to 10 for Rankin's modified formula, Sherif and Dilger's formula, Gardner and Shao's equation and El-Gamal and Benmokrane's formula, respectively. The error estimate parameters for the predicted versus tested punching shear strength are listed in Table 8 for the different

Table 6. Formulas Proposed by Other Researchers for Concrete Punching Shear Capacity of HSC Flat Plates

Researcher	Concrete punching shear capacity (Units: N and mm)
Modified Rankin (1987; 2003)	$V_c = 0.78\sqrt[3]{f_c^2} \sqrt[3]{100\rho} \cdot b_o d$
Sherif and Dilger (1996)	$V_c = 0.7\sqrt[3]{f_c^2} \sqrt[3]{100\rho} \cdot b_o d$
Gardner and Shao (1996)	$V_c = 0.79\sqrt[3]{f_c^2} \sqrt[3]{\rho f_y} \sqrt{1 + \frac{200}{d} \sqrt{\frac{d}{b_o}}} \cdot b_o d$
El-Gamal and Benmokrane (2004)	$V_c = 0.33\sqrt{f_c} \left[0.5\sqrt{\rho E} \left(1 + \frac{8d}{b_o} \right) \right] \cdot b_o d$ where E = modulus of elasticity of the reinforcing material (MPa)

models. In terms of coefficient of variation and coefficient of determination (R^2), all researchers' formula, except for the

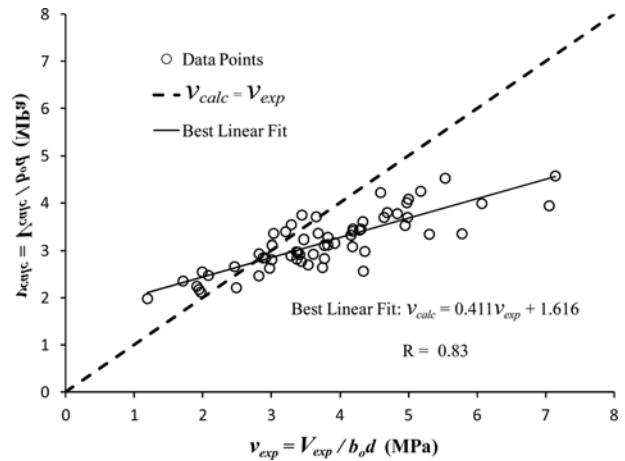


Fig. 8. Observed versus Predicted Punching Shear Strength for Formula of Sherif and Dilger

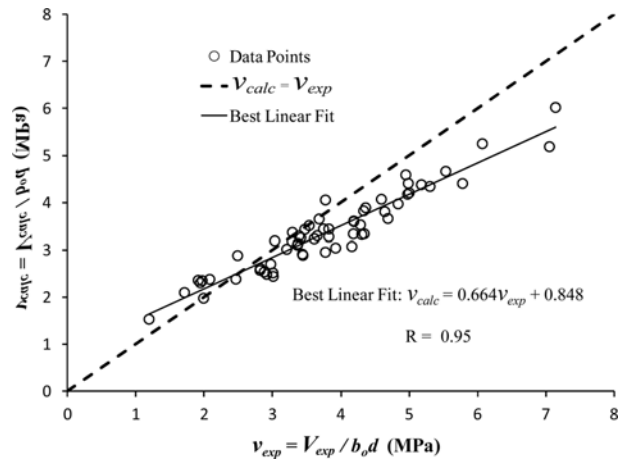


Fig. 9. Observed versus Predicted Punching Shear Strength for Formula of Gardner and Shao

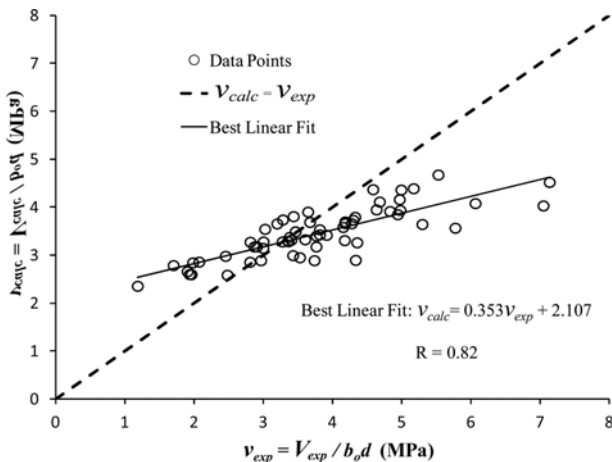


Fig. 7. Observed versus Predicted Punching Shear Strength for Rankin's Modified Formula

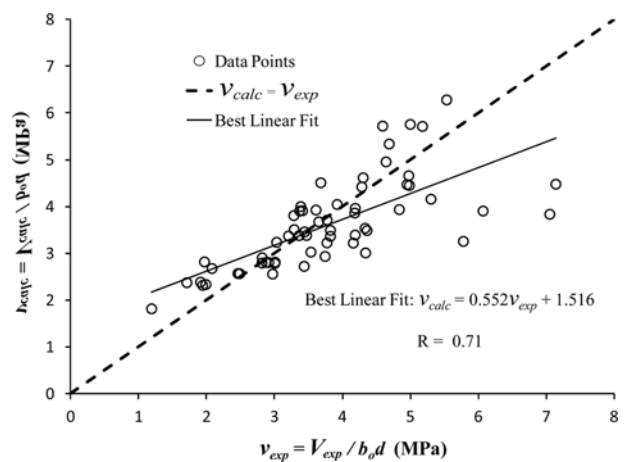


Fig. 10. Observed versus predicted punching shear strength for formula of El-Gamal and Benmokrane.

Table 7. Statistical Indicators for Researchers' Formulas for Punching Shear of HSC Flat Plates

Statistical parameter for (V_{exp} / V_{calc})	Modified Rankin (1987; 2003)	Sherif and Dilger (1996)	Gardner and Shao (1996)	El-Gamal and Benmokrane (2004)	Proposed predictive formula of Eq. (4)
Mean (m)	1.1	1.2	1.1	1.1	1.0
Standard deviation (σ)	0.24	0.24	0.14	0.24	0.10
Coefficient of variation	0.18	0.18	0.11	0.17	0.08
5% Percentile	0.72	0.84	0.84	0.78	0.86

Table 8. Error Estimates for Researchers' Formulas for Punching Shear of HSC Flat Plates

Parameter for error estimate	Modified Rankin (1987; 2003)	Sherif and Dilger (1996)	Gardner and Shao (1996)	El-Gamal and Benmokrane (2004)	Proposed Predictive formula of Eq. (4)
Mean percent error (MPE)	-2.49	-11.25	-8.30	-0.65	0.24
Mean absolute deviation in percent (MAD)	18.13	19.00	13.47	15.27	8.13
Root mean square error (RMSE)	0.90	1.00	0.64	0.87	0.38
Coefficient of correlation (CC)	0.82	0.83	0.95	0.71	0.95
Coefficient of determination (R^2)	0.67	0.69	0.90	0.50	0.90
Percent data for error within 15%	54.1	39.3	54.1	57.4	88.5
Percentage error enveloping 80% data	31.45	26.35	19.4	22.45	13.05

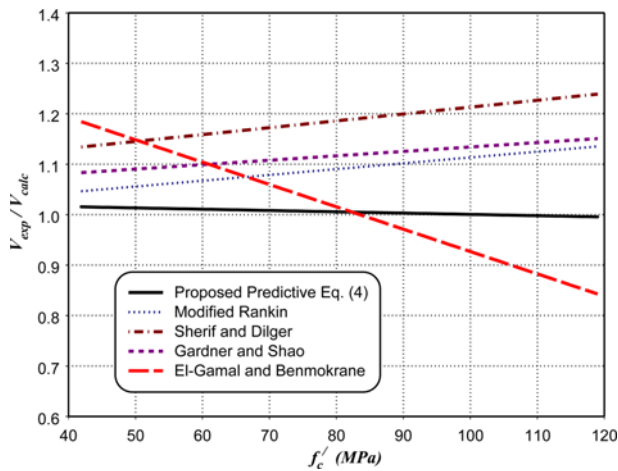


Fig. 11. Performance of Proposed Predictive and Researchers' Formulas with Respect to Concrete Strength

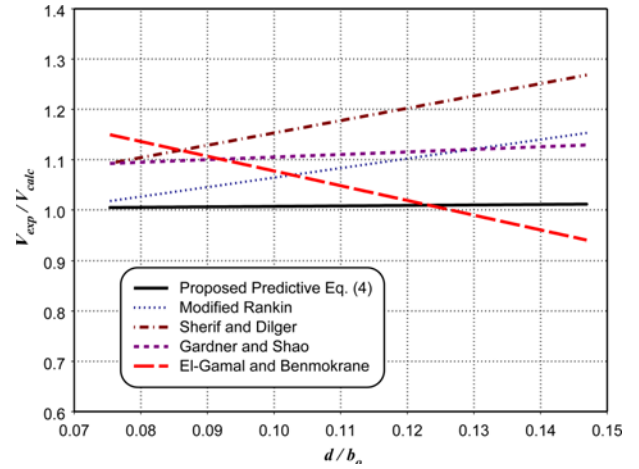


Fig. 13. Performance of Proposed Predictive and Researchers' Formulas with Respect to Average Effective Depth-to-control Perimeter Ratio

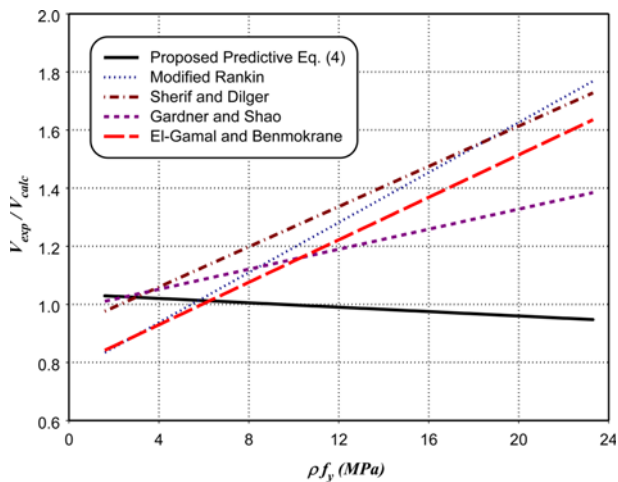


Fig. 12. Performance of Proposed Predictive and Researchers' Formulas with Respect to Reinforcement Parameter (ρf_y)

equation given by Gardner and Shao, give fairly poor correlation with the experimental results as seen in Tables 7 and 8. However, for all researchers' formulas, Table 8 shows that only 39.3 to 57.4% of the data has error less than 15%. It is also observed from Table 8 that for about 80% of the data, the percentage error is between 19.4 and 31.45% for all models.

The ratios of tested-to-calculated punching shear capacity are plotted against the main influencing parameters and only the trend lines are shown in Figs. 11 to 14 for all different investigators. From Fig. 11, it is clear that the equation suggested by El-Gamal and Benmokrane (2004) overestimates the punching shear capacity for concrete of strength more than 82 MPa. However, the formulas proposed by other investigators underestimate the punching shear capacity for all range of concrete strength. Fig. 14 shows that Rankin's modified equation (Rankin and Long, 1987; Zhang, 2003) overestimates the punching shear strength for slabs of average effective depth more than 235 mm; whereas,

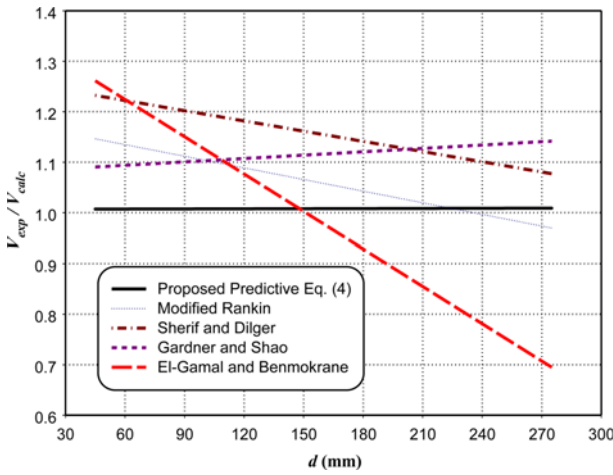


Fig. 14. Performance of Proposed Predictive and Researchers' Formulas with Respect to Average Effective Depth

the formula of El-Gamal and Benmokrane (2004) overestimates the punching shear capacity for specimens of average effective depth more than 150 mm. None of the investigators' formulas matches well the experimental data and as a result, a new proposed formula is highly needed.

6. New Proposed Formula

A general-purpose statistical analysis package was utilized to find out the best-fit equation to predict the punching shear strength of HSC interior slab-column connections using the database given earlier in Tables 2 and 3. Previous investigations showed that the main variables affecting punching shear strength are (1) concrete compressive strength, (2) grade and ratio of flexural reinforcement, (3) effective slab depth, and (4) column size. The last two parameters are expressed by the (d/b_o) ratio. Accordingly, the concrete punching shear strength can be expressed as follows:

$$V_c = C1 \times (f'_c)^{C2} \times (\rho f_y)^{C3} \times \left(1 + C4 \times \frac{d}{b_o}\right) \times b_o d \quad (1)$$

In the above equation, $C1$, $C2$, $C3$ and $C4$ are constants to be determined from the regression analysis. After cycles of iterations, it was found that the best-fit equation is as follows:

$$V_c = 0.34 \sqrt[3]{f'_c} \sqrt{\rho f_y} \left(1 + \frac{d}{3b_o}\right) \cdot b_o d \quad (\text{N and mm}) \quad (2)$$

The above proposed equation has $R^2 = 0.80$ (considering all data points for square and circular specimens) and it was used to estimate concrete punching shear strength for the 61 specimens. The ratios of tested-to-predicted punching shear force are plotted against the average effective depth as displayed in Fig. 15. From the trend line displayed in Fig. 15, it is evident that as the average effective depth increases the ratio of tested-to-calculated punching shear strength decreases and the proposed equation overestimates the punching shear capacity when the average effective depth

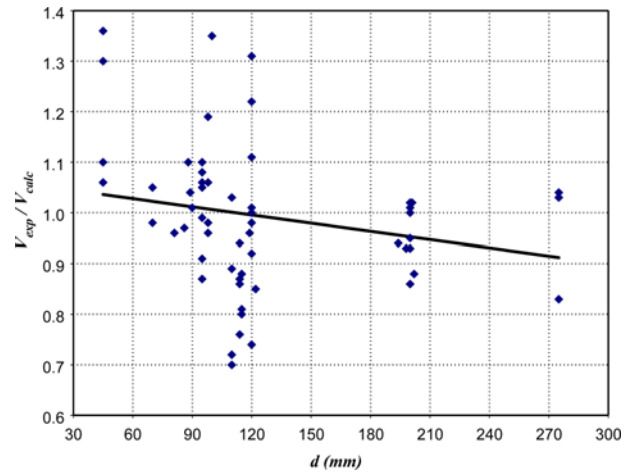


Fig. 15. Performance of Proposed Predictive Formula of Eq. (2) with Respect to Average Concrete Effective Depth

exceeds 120 mm. As a result, the size effect is pronounced and its term has to be incorporated in the proposed formula for predicting the punching shear strength of HSC flat plates as proposed by: the BS 8110-97 code (British Standards Institution, 1997), the EC2-2003 code (Eurocode 2, 2003), the DIN 1045-1 code (2001) and Gardner and Shao's formula (Gardner and Shao, 1996). Eq. (2) has to be revised in the following form:

$$V_c = C1 \times (f'_c)^{C2} \times (\rho f_y)^{C3} \times \left(1 + C4 \times \frac{d}{b_o}\right) \times \left(1 + \frac{C5}{d}\right)^{C6} \times b_o d \quad (3)$$

The terms $C1$ to $C6$ are constants to be calculated from the regression analysis. The best-fit equation for predicting the punching shear strength of HSC flat plates is as follows:

$$V_c = 0.127 \sqrt[3]{f'_c} \sqrt{\rho f_y} \left(1 + \frac{8d}{b_o}\right) \sqrt{\left(1 + \frac{125}{d}\right)} \cdot b_o d \quad (\text{N and mm}) \quad (4)$$

The above equation has $R^2 = 0.90$. It should be illustrated that in the proposed equation, the critical section was assumed to be at a distance of $d/2$ from the column face because this value has been used to define the critical section in the ACI code since the 1960's. Therefore, the critical perimeter (b_o) is equal to $4(c + d)$ for square columns and $\pi(c + d)$ for circular columns. In the proposed equation, it is noted that the term that has the ratio of the average effective depth to critical perimeter (d/b_o) is the same as that proposed by El-Gamal and Benmokrane (2004).

The above suggested formula of Eq. (4) was utilized to assess the concrete punching shear capacity for the 61 specimens and the ratios of the tested-to-predicted punching shear capacity are then calculated and plotted against the average effective depth as displayed in Fig. 16. From the trend line displayed in Fig. 16, it is evident that Eq. (4) gives the same ratio of tested-to-calculated punching shear strength for all range of average effective depth.

Statistical indicators of the tested-to-predicted punching shear capacity in terms of: mean (m), standard deviation (σ), coefficient of variation, and 5% percentile are given in Table 7 for Eq. (4).

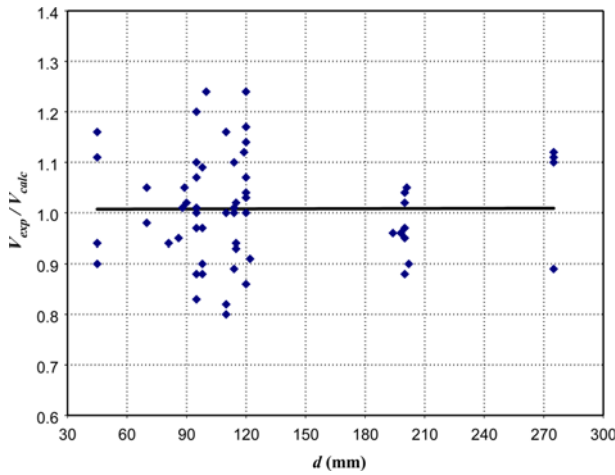


Fig. 16. Performance of Proposed Predictive Formula of Eq. (4) with Respect to Average Concrete Effective Depth

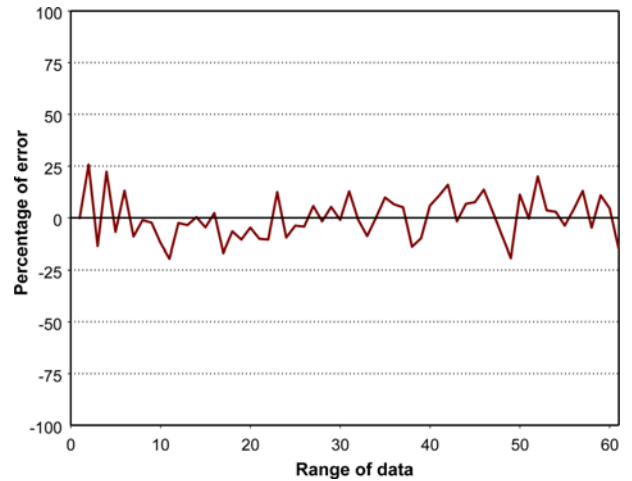


Fig. 18. Percentage Error in Prediction of Punching Shear Strength using Proposed Formula of Eq. (4) for Individual Data Points

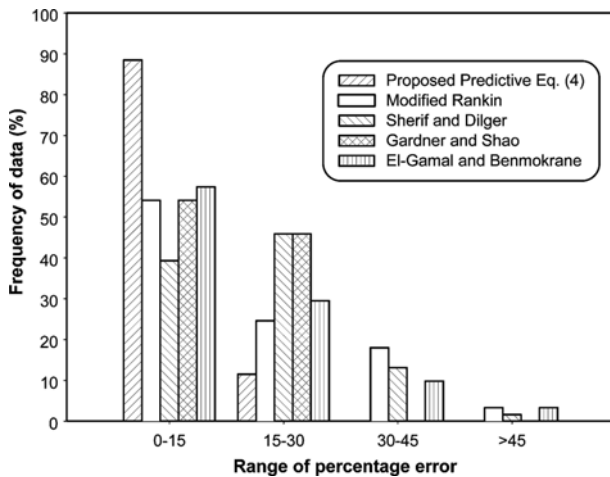


Fig. 17. Histogram of Percentage Error for Different Predictive Formulas of Punching Shear Strength

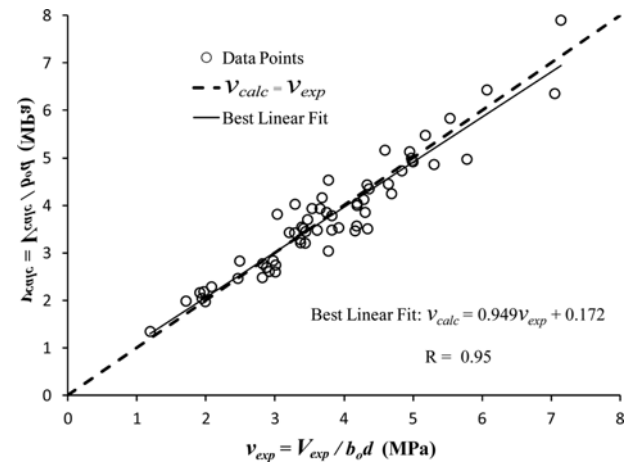


Fig. 19. Observed versus Predicted Punching Shear Strength for Proposed Formula of Eq. (4)

In addition, the error estimate parameters for Eq. (4) are listed in Table 8. With respect to all statistical indicators as well as error estimate parameters, it is clear from Tables 7 and 8 that the proposed predictive equation is the best to match the experimental data.

The histogram of error in the prediction of the punching shear strength of HSC flat plates for proposed Eq. (4) is plotted in Fig. 17. The error in the equations by other investigators is also plotted in Fig. 17. It is obvious from Fig. 17 that the error in calculating punching shear strength by proposed predictive Eq. (4) is less than 15% for about 90% of the data points. Yet, for formulas by other researchers, the error is less than 15% for less than 60% of the data points. The percentage error in the prediction of punching shear strength for different data sets is plotted in Fig. 18 for proposed predictive Eq. (4). It is shown that the percentage error for Eq. (4) is between -20 and 26% for all data points. The predicted value of the punching shear strength has been plotted against its observed value in Fig. 19 for the proposed formula of Eq. (4). It is evident from Fig. 19 that Eq.

(4) provides good correlation with the experimental data.

For the proposed predictive formula of Eq. (4), the ratio of measured-to-calculated punching shear capacity are plotted against the main influencing parameters and only the trend lines are shown in Figs. 11 to 14. It is evident that the proposed predictive equation is the best to match the experimental data with respect to all the independent parameters. However, similar to all other investigators and as demonstrated in Table 7, the proposed predictive equation does not satisfy the requirement of a 5% percentile by having a value less than one. Therefore, a proposed design equation is highly needed. The design knockdown factor for punching shear strength shall be based upon statistical lower bounds for (V_{exp}/V_{calc}) ratio in terms of the minimum of 5% percentile and $(m-2\sigma)$ values. From Table 7, it is apparent that the proposed predictive equation should be multiplied by about 80% in order to have a suitable design formula. Accordingly, the following formula is suggested as a code design equation for concrete punching shear strength of interior column-HSC flat plate connections without moment transfer.

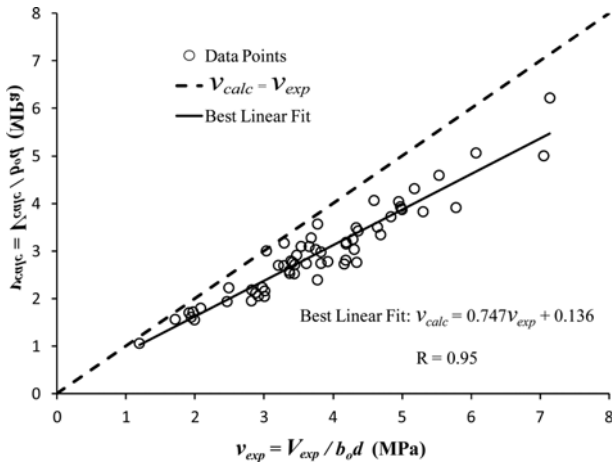


Fig. 20. Observed versus Predicted Punching Shear Strength for Proposed Design Formula of Eq. (5)

$$V_c = 0.1 \sqrt[3]{f'_c} \sqrt{\rho_f y} \left(1 + \frac{8d}{b_o} \right) \sqrt{\left(1 + \frac{125}{d} \right)} \cdot b_o d \quad (\text{N and mm}) \quad (5)$$

For the above proposed design formula, the predicted value of the punching shear strength has been plotted against its observed value as seen in Fig. 20. In addition, statistical and correlation parameters for the predicted versus tested punching shear strength are listed in Tables 4 and 5 for the proposed design formula of Eq. (5). It is obvious that among all design formulas, Eq. (5) is the most accurate by having coefficient of determination (R^2) of 0.90 and coefficient of variation of 0.10; yet, it is the most conservative in estimating the punching shear strength as none of the data points had its predicted shear strength exceeding the experimental value.

As mentioned previously, Eq. (5) is only valid for interior column-HSC flat plate connections without moment transfer. Yet, unbalanced moment transfer between flat slab and column occurs in practical applications due to uneven distribution of live load, uneven spacing of columns, or in some cases lateral loads such as wind and seismic actions. Tests conducted by Hanson and Hanson (1968) and Kamaraldin (1990) on NSC slab-column connections indicated that even though an increase in the moment transferred between the slab and column produces a reduction in the punching strength, the limiting concrete stress for eccentric loading is the same as that for concentric loading. Accordingly, the proposed equation for predicting punching stress limit of HSC slabs with concentric loading may be applied to eccentrically loaded slabs. In line with the ACI 318-08 code, the design of interior HSC flat plate connections with unbalanced moment transferred between the slab and the column may be governed by the following equation:

$$v_t = v_V + v_M \leq 0.1 \sqrt[3]{f'_c} \sqrt{\rho_f y} \left(1 + \frac{8d}{b_o} \right) \sqrt{\left(1 + \frac{125}{d} \right)} \quad (\text{N and mm}) \quad (6)$$

where v_t = total punching shear stress; v_V = punching stress due to vertical shear force and v_M = punching stress due to unbalanced

moment transferred by shear.

7. Conclusions

All current code provisions, except for the DIN 1045-1 code, are un-conservative in estimating the punching shear capacity of interior column-HSC flat plate connections. The shear resistance coefficients of the ACI 318-08, BS 8110-97, and EC2-2003 codes should be reduced by 30%, 19%, and 12%, respectively. Comprehensive regression analysis carried out in this study indicated that concrete punching shear strength of interior column-HSC flat plate connections is proportional to the cubic root of concrete compressive strength rather than the square root as used in normal strength concrete. Additionally, concrete punching shear capacity is proportional to the effective depth-to-critical perimeter ratio (d/b_o) and the square root of the reinforcement parameter ($\rho_f y$). These parameters are significant to include in current code design provisions such as the ACI 318-08. Moreover, as the concrete effective depth increases, the shear stress at punching failure decreases. This size effect is significant and is an important feature to include in the code design expressions. The innovative design equation proposed in this study is consistent with data of flat plates made from high-strength concrete with the assumption of the critical perimeter at a distance $0.5d$ from the column face as defined in the ACI 318-08 code. However, the proposed design equation is limited to HSC flat slabs with depth not more than 300 mm and concrete compressive strength below 120 MPa. For slabs exceeding these limits, experimental validation is required.

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