ENHANCEMENT OF PUNCHING SHEAR RESISTANCE OF LIGHT AND NORMAL-WEIGHT REINFORCED CONCRETE FLAT SLABS USING SHEAR BAND REINFORCEMENT

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ملخص البحث

تعد البلاطات اللاكمرية واحدة من النظم الهيكلية الأكثر استخداما في المباني و يعتبر من أبرز عيوب هذا النظام الانهيار المفاجئ للخرسانة في المنطقة المحيطة بالعمود نتيجة القص الثاقب . توجد طريقتان لتحسين مقاومة القص الثاقب 1) زيادة سمك البلاطة جزئيا حول العمود أو استخدام تيجان الأعمدة 2) استخدام تسليح داخلي للقص حيث يعتبر أكثر تطورا و اقتصاداً من الطرق السابقة كما أنه يحافظ علي استمرارية البلاطة اثناء البناء. يستخدم هذا البحث برنامج ANSYS (14.5) و ذلك لدراسة مقاومة القص الثاقب للبلاطات اللاكمرية من الخرسانة المسلحة الخفيفة و العادية الوزن للأعمدة الداخلية و الطرفية و الركنية باستخدام نظام شرائط القص و هو عباره عن شرائط رقيقة مثنية يتم تركيبها بسهولة في الموقع كما سيتم المقارنة للحصول علي أفضل مقاس و توزيع لها حول العمود.

Abstract

Flat slabs are solid concrete slabs of uniform thickness that transfer the load directly to the columns without the presence of beams. The major and critical problem of this system is its sudden brittle failure which called punching shear failure. There are mainly two ways to increase the punching shear strength of concrete slabs, first, increasing slab thickness in the vicinity of the column by providing a drop panel or column capital, second providing shear reinforcement. Shear reinforcement is more sophisticated from both the structural and economical point of view, it maintains the flat slab construction uninterrupted.

A total of fifty finite element models analyzed using ANSYS (14.5)[3] to investigate punching shear resistance of interior, edge and corner slab-column [11] of light-weight (LWC) [2] and normal-weight (NWC) [17] reinforced concrete connection using shear band system. Two types of concrete grade used. First normal strength concrete with characteristic strength 30 MPa and the second is high strength concrete with characteristic strength 70 MPa [13].

Slab layout dimensions of 1700 mm x 1700 mm, 1700 mm x 95 mm and 95 mm x 95 mm for interior, edge and corner slab-column connection respectively. Thickness of all specimens is 160 mm. Steel reinforcement ration of all slabs is constant. An elongated this steel strips [6] of 40 mm width and 1.5 mm thickness undulated into the slab in different ways to investigate punching shear resistance [9].

The results such as crack pattern, failure modes, loads deflection curves, stiffness, ductility index and absorbed energy for the models were studied in this program.

Based on the results obtained from finite element analysis, it was observed that the arrangement of shear bands around the column provides enhancement ratio in the failure load capacity and punching shear resistance.

Keywords: Punching, Shear-Band, Crack pattern, Ductility Index and Absorbed Energy.

Introduction

There are many methods of enhancing punching shear resistance of flat slabs by providing shear reinforcement which placed in the slab around the column. The importance of shear reinforcement that can easily be installed will be emphasized. Shear-band system differs from all other existing systems. It's made of steel strips of high ductility. The strip can be bent to variety shapes which undulated into the slab from top surface after all flexural reinforcement placed with minimum loss of cover. The advantages of using shear-band system can be concluded in the following:

- Maximum effective depth.
- Prevents brittle punching shear failure and greatly improves the ductility of flat slabs.
- Increases the punching shear capacity of flat slabs without increasing the slab flexural capacity.
- Cost effective and it can allow for the design of thinner slabs.
- Easily fabricated, light weight, simple and efficient placement of reinforcement in addition to it is easy to store and transport.

Program Study

The analyzed flat slab specimens as shown in Table (1) were carried out on two types of concrete weight, normal weight (NW) and light weight (LW) concrete, characteristic strength of concrete used is normal strength with grade of 30 MPa (C30) [9] and high strength with grade 70 MPa (C70) [13].

Three location of column-slab connection, first, interior column-slab connection (I) has dimensions 1700 mm x 1700 mm and thickness 160 mm, second, edge column-slab connection (E) has dimensions 1700 mm x 950 mm and same thickness 160 mm, the third is corner column-slab connection (C) has dimensions 950 mm x 950 mm and also same thickness 160 mm. All slabs reinforced using bottom mesh of 16 mm bar diameter each 130 mm and top mesh of 10 mm bar diameter each 130 mm.

Shear-band system consists of steel strips with 40 mm with and 0.5 mm thickness installed around the column in two shapes, one is orthogonal and the other is box shape. Number of shear band strips installed as different to cover the expected cone are of punching around the column to reach three times the thickness of slab.

Refer to following tables (1) to (4), figures (1) to (6) indicate the dimensions and reinforcement of two specimens from each column-slab connection, one with orthogonal shear band system and the other with box shape shear band system.

The notation of speciemens number in the previous table expressed as the first is the locatoion of column, interior (I), edge (E) and corner (C), the second is the serial of specimens number S1 to S5, the third is the type of concrete weight, Normal weight (NW) and Light weight (LW), while the last is the grade of concrete, (C30) for normal strength concrte while (C70) for high strength concrete. S1 is the reference specimens in all groups whih is without shear band system.

Table (1): Specimen Details for Verification Specimens (Interior Column)

Specimen No.	Slab Dimensions $L_1 \times L_2 \times t_s$	Concrete Strength (MPa)	No of Shear Bands Strips	Layout Distribution
S1	1700 x 1700 x 160	35	-----	-----
S2	1700 x 1700 x 160	40	8	Orthogonal
S3	1700 x 1700 x 160	35	8	Orthogonal
S4	1700 x 1700 x 160	35	12	Box shaped
S5	1700 x 1700 x 160	40	16	Orthogonal

Table (2): Specimen Details for Interior Slab-Column Connections

Specimen No.	Slab Dimensions L_1 x L_2 x t_s	Concrete Strength (MPa)	No of Shear Bands Strips	Layout Distribution
I-S1-NW-C30	1700 x 1700 x 160	30	-----	-----
I-S2-NW-C30	1700 x 1700 x 160	30	8	Orthogonal
I-S3-NW-C30	1700 x 1700 x 160	30	8	Orthogonal
I-S4-NW-C30	1700 x 1700 x 160	30	12	Box shaped
I-S5-NW-C30	1700 x 1700 x 160	30	16	Orthogonal
I-S1-LW-C30	1700 x 1700 x 160	30	-----	-----
I-S2-LW-C30	1700 x 1700 x 160	30	8	Orthogonal
I-S3-LW-C30	1700 x 1700 x 160	30	8	Orthogonal
I-S4-LW-C30	1700 x 1700 x 160	30	12	Box shaped
I-S5-LW-C30	1700 x 1700 x 160	30	16	Orthogonal
I-S1-NW-C70	1700 x 1700 x 160	70	-----	-----
I-S2-NW-C70	1700 x 1700 x 160	70	8	Orthogonal
I-S3-NW-C70	1700 x 1700 x 160	70	8	Orthogonal
I-S4-NW-C70	1700 x 1700 x 160	70	12	Box shaped
I-S5-NW-C70	1700 x 1700 x 160	70	16	Orthogonal

Table (3): Specimen Details for Edge Slab-Column Connections

Specimen No.	Slab Dimensions L_1 x L_2 x t_s	Concrete Strength (MPa)	No of Shear Bands Strips	Layout Distribution
E-S1-NW-C30	1700 x 950 x 160	30	-----	-----
E-S2-NW-C30	1700 x 950 x 160	30	4	Orthogonal
E-S3-NW-C30	1700 x 950 x 160	30	4	Orthogonal
E-S4-NW-C30	1700 x 950 x 160	30	6	Box shaped
E-S5-NW-C30	1700 x 950 x 160	30	6	Orthogonal
E-S1-LW-C30	1700 x 950 x 160	30	-----	-----
E-S2-LW-C30	1700 x 950 x 160	30	4	Orthogonal
E-S3-LW-C30	1700 x 950 x 160	30	4	Orthogonal
E-S4-LW-C30	1700 x 950 x 160	30	6	Box shaped
E-S5-LW-C30	1700 x 950 x 160	30	6	Orthogonal
E-S1-NW-C70	1700 x 950 x 160	70	-----	-----
E-S2-NW-C70	1700 x 950 x 160	70	4	Orthogonal
E-S3-NW-C70	1700 x 950 x 160	70	4	Orthogonal
E-S4-NW-C70	1700 x 950 x 160	70	6	Box shaped
E-S5-NW-C70	1700 x 950 x 160	70	6	Orthogonal

Table (4): Specimen Details for Corner Slab-Column Connections

Specimen No.	Slab Dimensions L_1 x L_2 x t_s	Concrete Strength (MPa)	No of Shear Bands Strips	Layout Distribution
C-S1-NW-C30	950 x 950 x 160	30	-----	-----
C-S2-NW-C30	950 x 950 x 160	30	4	Orthogonal
C-S3-NW-C30	950 x 950 x 160	30	4	Orthogonal
C-S4-NW-C30	950 x 950 x 160	30	6	Box shaped
C-S5-NW-C30	950 x 950 x 160	30	6	Orthogonal
C-S1-LW-C30	950 x 950 x 160	30	-----	-----
C-S2-LW-C30	950 x 950 x 160	30	4	Orthogonal
C-S3-LW-C30	950 x 950 x 160	30	4	Orthogonal
C-S4-LW-C30	950 x 950 x 160	30	6	Box shaped
C-S5-LW-C30	950 x 950 x 160	30	6	Orthogonal
C-S1-NW-C70	950 x 950 x 160	70	-----	-----
C-S2-NW-C70	950 x 950 x 160	70	4	Orthogonal
C-S3-NW-C70	950 x 950 x 160	70	4	Orthogonal
C-S4-NW-C70	950 x 950 x 160	70	6	Box shaped
C-S5-NW-C70	950 x 950 x 160	70	6	Orthogonal

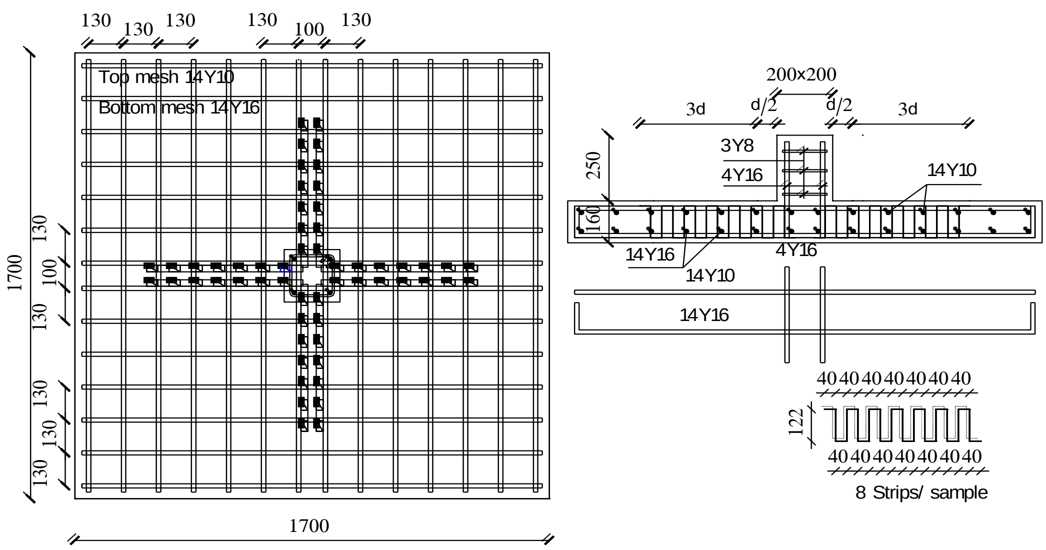

Figure (1): Details of Interior Slab-Column Model (Orthogonal Shear Band)

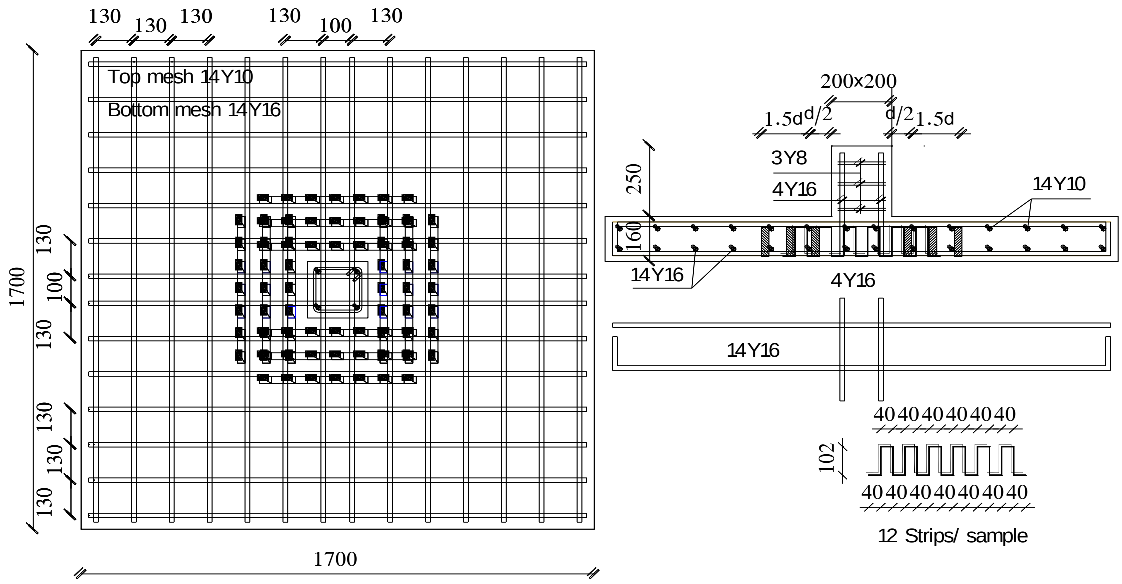

Figure (2): Details of Interior Slab-Column Model (Box Shapes Shear Band)

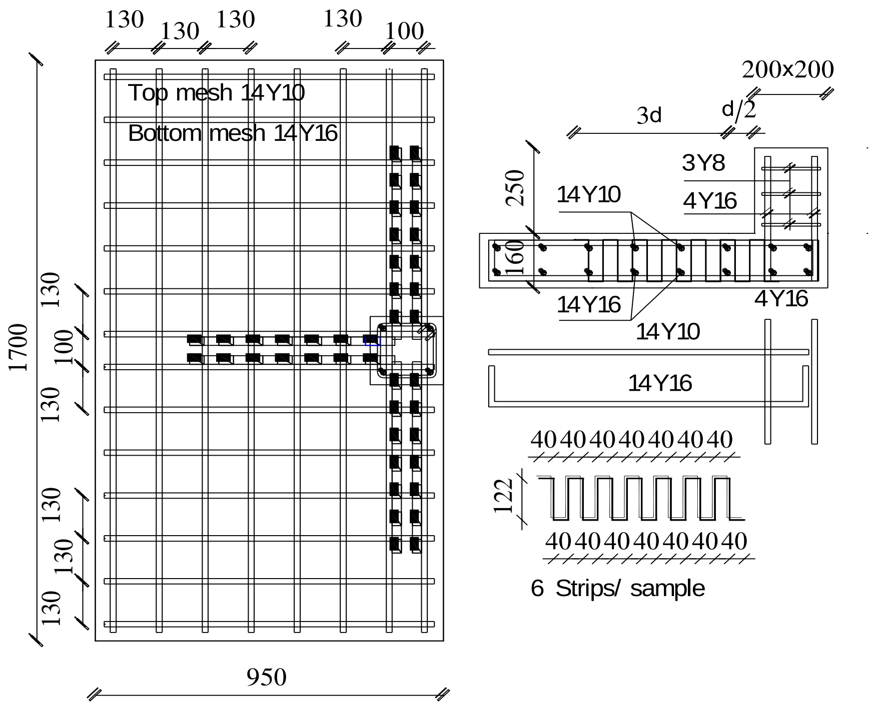

Figure (3): Details of Edge Slab-Column Model (Orthogonal Shear Band)

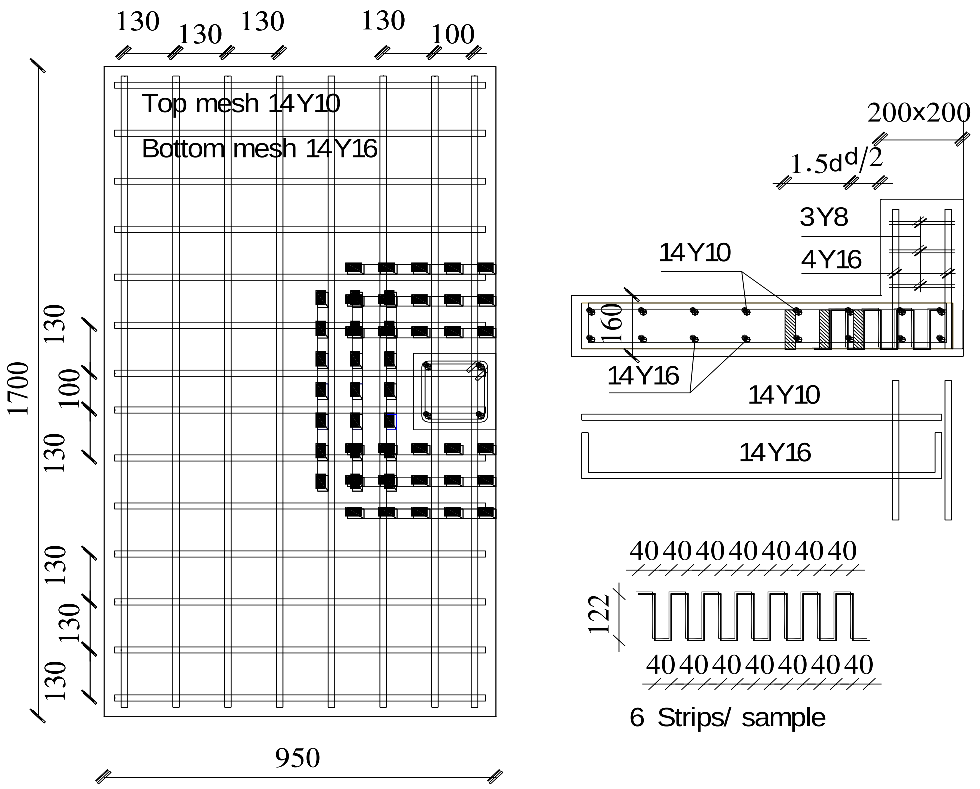

Figure (4): Details of Edge Slab-Column Model (Box Shapes Shear Band)

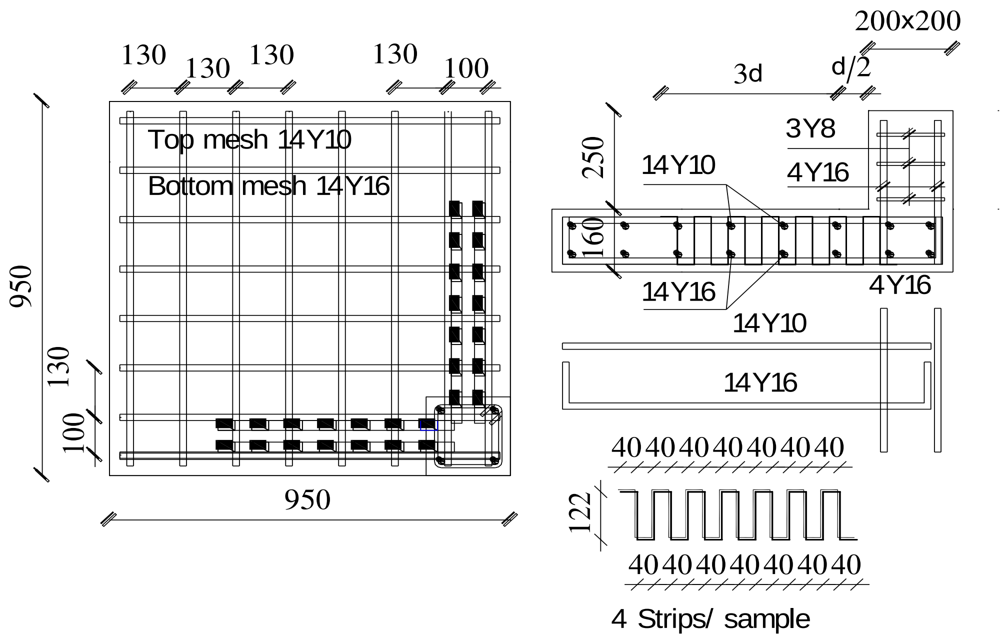

Figure (5): Details of Corner Slab-Column Model (Orthogonal Shear Band)

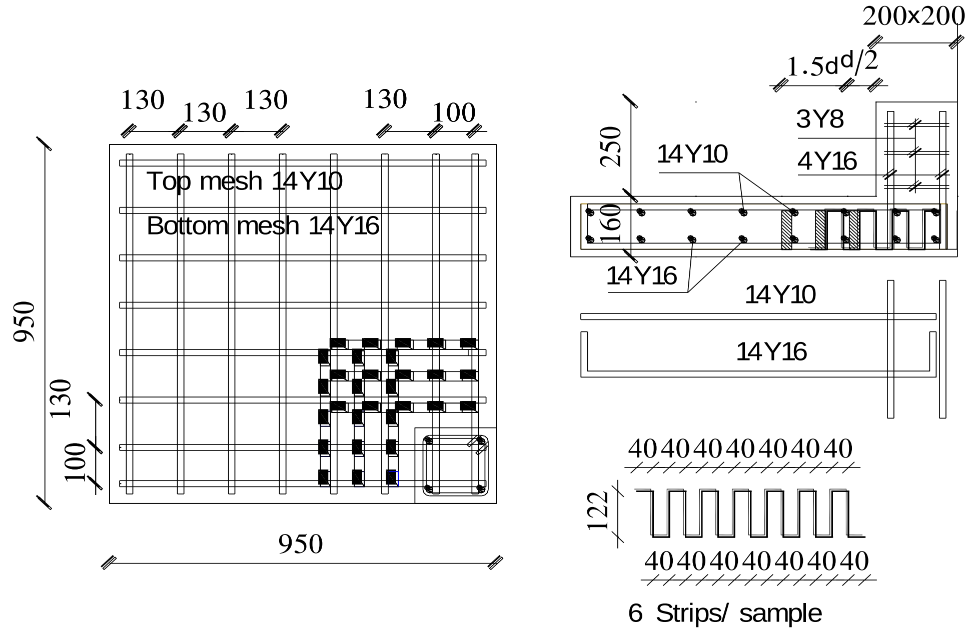

Figure (6): Details of Corner Slab-Column Model (Box Shape Shear Band)

Numerical Analysis

A nonlinear three dimensional brick element **(SOLID-65** in ANSYS) Element is used to model the concrete (NWC and LWC) with or without reinforcing bars (rebar). The element is capable of plastic deformation, cracking in three orthogonal directions at each integration point in tension and crushing in compression, and creep. The cracking is modeled through an adjustment of the material properties that is done by changing the element stiffness matrices. If the concrete at an integration point fails in uniaxial, biaxial, or tri-axial compression, the concrete is assumed crushed at that point. Crushing is defined as the complete deterioration of the structural integrity of the concrete.

The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. Up to three different rebar specifications may be defined.

A discrete axial element (**LINK180** in ANSYS) Element is used to model the longitudinal steel reinforcement; the element is a uniaxial tension-compression element with three degrees of freedom at each node: translations in the nodal x, y, and z directions..

The steel for F.E. models is assumed to be an elastic-perfectly plastic material.

The Slab-column, Steel plate, and supports were modeled as volumes. The combined volumes of the slab-column, steel plate and support created in ANSYS for the NWC and LWC.

Theoretical Results

In the following, the results and the behavior of all slab-column specimens is discussed listed in Table (5) to Table (8). Tables indicate the values of cracking deflection and load and deflection and load at failure stage respectively for all specimens relative to control specimen. Also the ductility index and absorbed energy calculated and listed in the same tables. Finally the mode of failure for each specimens determined according to the final cracks shapes before failure.

Load Deflection Relationship

Referring to Table (5) to Table (8); it can be noticed that the measured deflection for slab-column connection have almost the same profile and The load-deflection curves for all slabs (same type) have almost the same shapes. Where, Load–deflection behavior of concrete structures typically includes three stages. Stage I manifests the linear behavior of un-cracked elastic section. Stage II implies initiation of concrete cracking and Stage III relies relatively on the yielding of steel reinforcements and the crushing of concrete. In nonlinear iterative algorithms, ANSYS (14.5) utilizes Newton–Raphson method [00] for the incremental load analysis.
Fig. (7) to Fig. (16) show the load-deflection plots from the finite element analyses for all specimens at the last converged load step.

Mode of Failure

Failure load and final deflection of all slabs listed in Table (5) to Table (8), all specimens were designed to exhibit punching shear failure mode and already failed in a typical punching shear mode.

Ductility Index and Absorbed Energy

Also the ductility index and absorbed energy calculated for each specimen relative to control specimen and listed in Table (5) to Table (8).

Table (5): FEM Results of Verification Specimens:

Group	Specimen No.	Cracking Stage		Failure Stage		Ductility Index (μ_d) %	Absorbed Energy (KN.mm)
		P_{cr} (KN)	Δ_{cr} (mm)	P_f (KN)	Δ_f (mm)		
Group (1)	**S1**	113	0.353	395	3.59	9.17	862.86
	S2	115	0.274	544	5.24	18.12	1707.26
	S3	113	0.352	517	5.15	13.63	1609.24
	S4	113	0.293	536	4.91	15.76	1532.73
	S5	115	0.273	461	4.01	13.69	1094.04

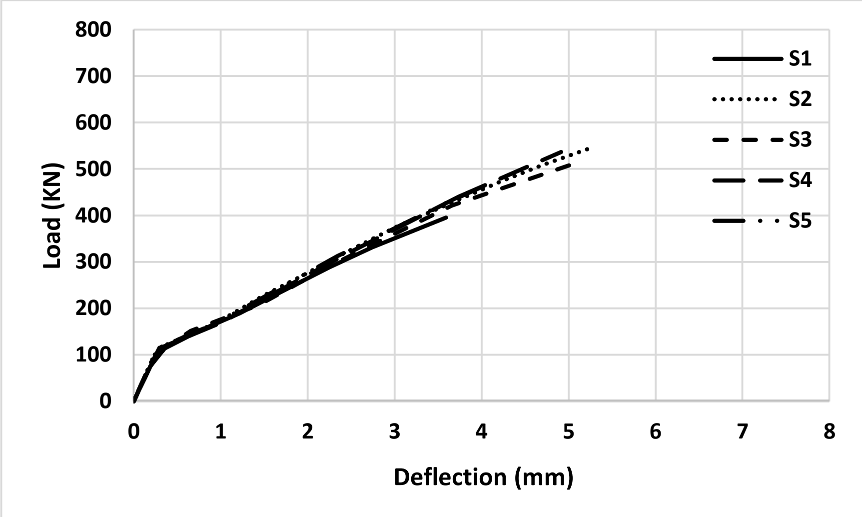

Figure (7): Load-Deflection Curves for Group (1)

Table (6): F.E. Results of Interior Slab-Column Connection

Group	Specimen No.	Cracking Stage		Failure Stage		Ductility Index (μ_d) %	Absorbed Energy (KN.mm)
		P_{cr} (KN)	Δ_{cr} (mm)	P_f (KN)	Δ_f (mm)		
Group (I-1)	**I-S1-NW-C30**	113	0.404	377	3.93	8.73	937.00
	I-S2-NW-C30	113	0.401	476	5.18	11.92	1354.02
	I-S3-NW-C30	113	0.403	459	4.67	10.59	1277.90
	I-S4-NW-C30	113	0.399	455	4.29	9.75	1135.33
	I-S5-NW-C30	113	0.395	470	4.24	9.73	1195.63
Group (I-2)	**I-S1-LW-C30**	75.8	0.346	363	4.82	12.93	1083.88
	I-S2-LW-C30	75.8	0.341	465	6.11	16.92	1662.29
	I-S3-LW-C30	75.8	0.343	443	5.96	16.38	1564.28
	I-S4-LW-C30	75.8	0.338	447	5.37	14.89	1383.81
	I-S5-LW-C30	75.8	0.340	459	5.76	15.94	1526.07
Group (I-3)	**I-S1-NW-C70**	170	0.433	497	3.39	6.83	999.29
	I-S2-NW-C70	170	0.405	703	5.64	12.93	2362.84
	I-S3-NW-C70	170	0.407	688	5.38	12.22	2181.92
	I-S4-NW-C70	170	0.397	678	5.02	11.64	1990.44
	I-S5-NW-C70	170	0.413	689	5.35	11.95	2162.64

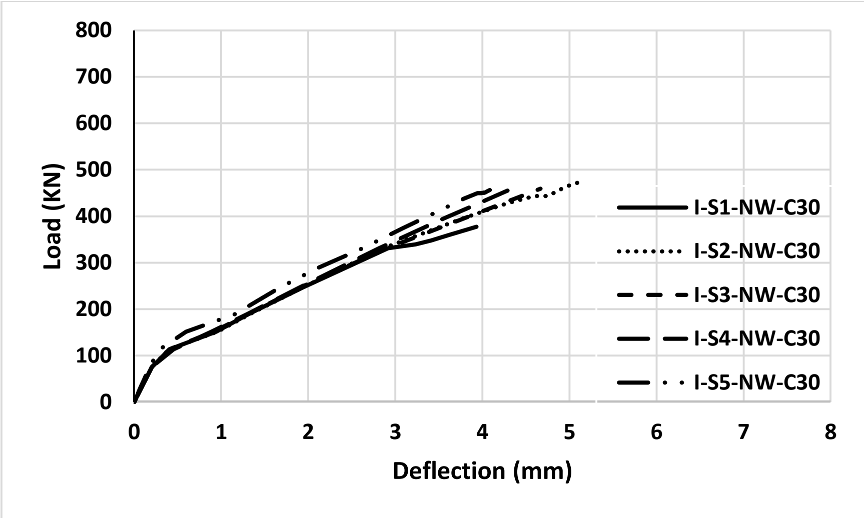

Figure (8): Load-Deflection Curves for Group (I-1)

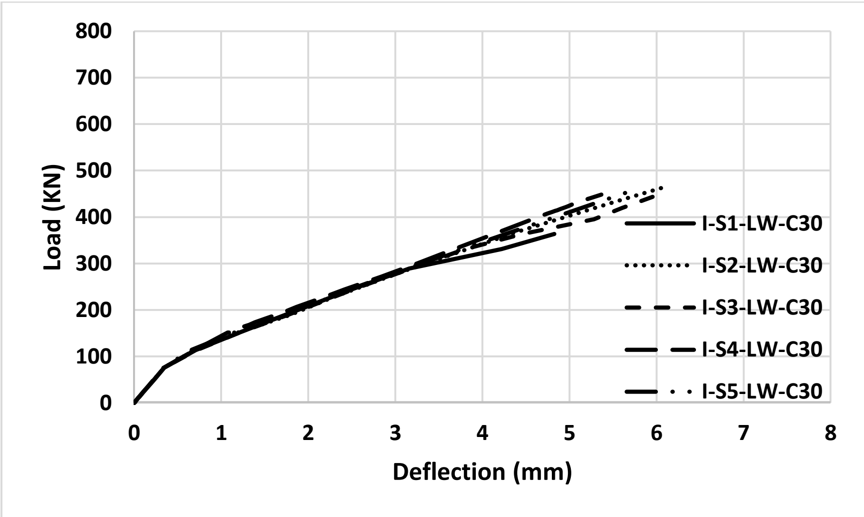

Figure (9): Load-Deflection Curves for Group (I-2)

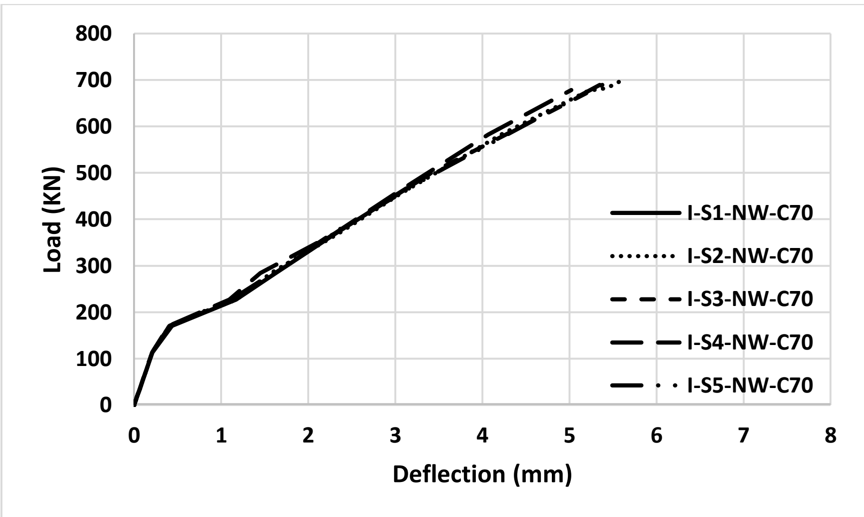

Figure (10): Load-Deflection Curves for Group (I-3)

Table (7): F.E. Results of Edge Slab-Column Connection:

Group	Specimen No.	Cracking Stage			Failure Stage			Ductility Index (μ_d) %	Absorbed Energy (KN.mm)
		P_{cr} (KN)	M_{cr} (KN. mm)	Δ_{cr} (mm)	P_f (KN)	M_f (KN. mm)	Δ_f (mm)		
Group (E-1)	E-S1-NW-C30	67.3	6.73	0.900	233	23.3	9.94	10.04	1519.40
	E-S2-NW-C30	67.3	6.73	0.804	314	31.4	16.6	19.65	3411.21
	E-S3-NW-C30	75.8	7.58	0.998	293	29.3	14.1	13.13	2736.86
	E-S4-NW-C30	75.8	7.58	0.997	300	30	14.5	13.54	2918.22
	E-S5-NW-C30	75.8	7.58	0.994	306	30.6	14.5	13.59	2888.82
Group (E-2)	E-S1-LW-C30	75.8	7.58	1.53	249	24.9	12.6	7.24	1972.20
	E-S2-LW-C30	75.8	7.58	1.52	299	29.9	18.9	11.43	3744.99
	E-S3-LW-C30	75.8	7.58	1.41	256	25.6	11.8	7.37	2027.56
	E-S4-LW-C30	75.8	7.58	1.40	265	26.5	12.9	8.21	2140.90
	E-S5-LW-C30	72.6	7.26	1.44	274	27.4	14.5	9.07	2484.05
Group (E-3)	E-S1-NW-C70	101	10.1	0.920	349	34.9	12.6	13.03	2899.15
	E-S2-NW-C70	114	11.4	0.623	409	40.9	15.6	24.04	4112.84
	E-S3-NW-C70	101	10.1	0.895	370	37	13.6	13.73	3301.23
	E-S4-NW-C70	114	11.4	0.898	395	39.5	12.9	11.06	3190.42
	E-S5-NW-C70	114	11.4	0.893	384	38.4	12.4	10.70	2948.77

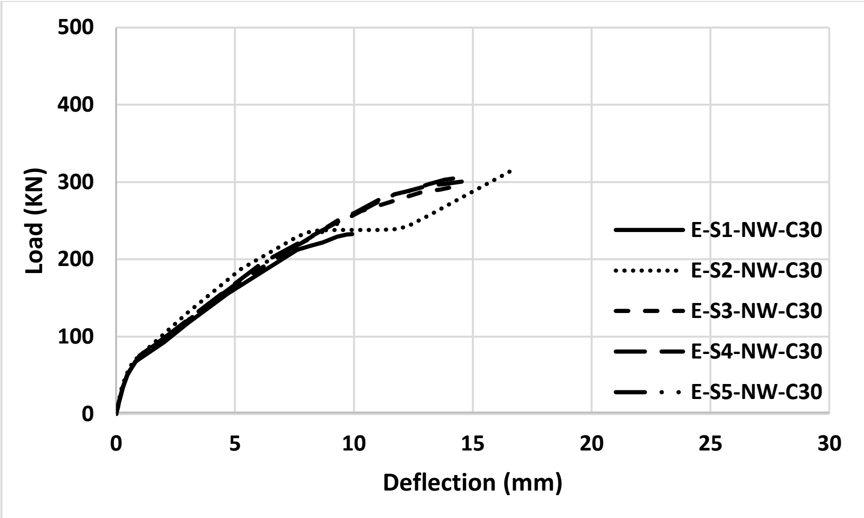

Figure (11): Load-Deflection Curves for Group (E-1)

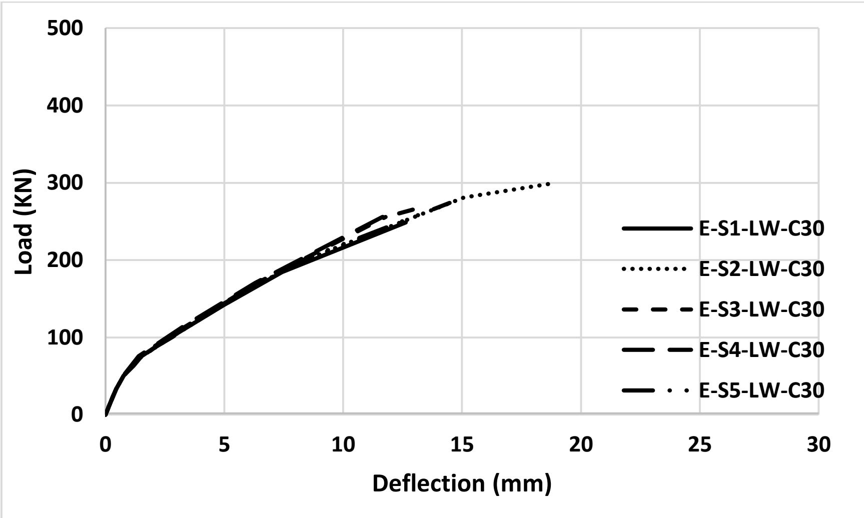

Figure (12): Load-Deflection Curves for Group (E-2)

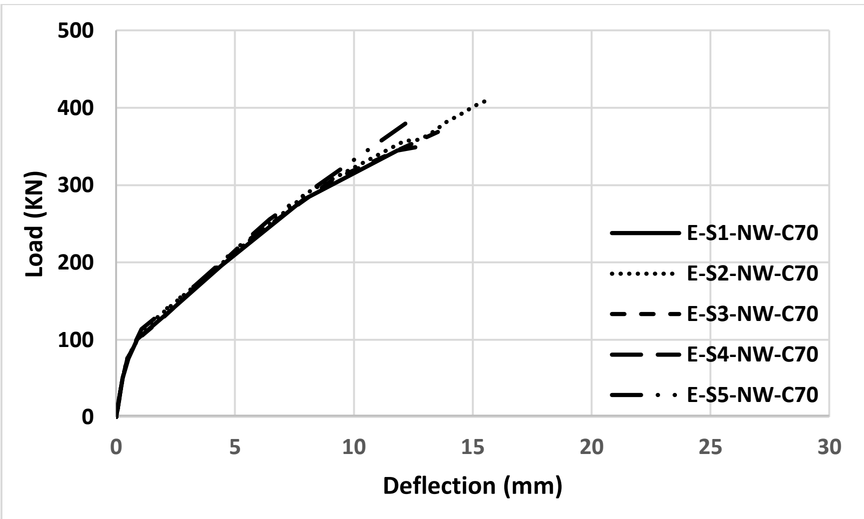

Figure (13): Load-Deflection Curves for Group (E-3)

Table (8): F.E. Results of Corner Slab-Column Connection

Group	Specimen No.	Cracking Stage			Failure Stage			Ductility Index (μ_d) %	Absorbed Energy (KN.mm)
		P_{cr} (KN)	M_{cr} (KN. mm) (X,Y)	Δ_{cr} (mm)	P_f (KN)	M_f (KN.mm) (X,Y)	Δ_f (mm)		
Group (C-1)	**C-S1-NW-C30**	44.9	4.49	2.08	82.3	8.23	8.55	3.63	474.74
	C-S2-NW-C30	44.9	4.49	1.47	130	13	19.1	11.99	1715.79
	C-S3-NW-C30	44.9	4.49	1.88	107	10.7	13.4	6.13	972.87
	C-S4-NW-C30	44.9	4.49	1.87	101	10.1	10.8	4.78	701.92
	C-S5-NW-C30	44.9	4.49	1.88	114	11.4	14.2	6.55	1066.72
Group (C-2)	**C-S1-LW-C30**	44.9	4.49	2.46	89.8	8.98	12.5	4.08	749.29
	C-S2-LW-C30	44.9	4.49	2.27	126	12.6	19.5	7.59	1591.19
	C-S3-LW-C30	44.9	4.49	2.45	114	11.4	17.5	6.14	1304.15
	C-S4-LW-C30	44.9	4.49	2.48	101	10.1	13.8	4.56	905.34
	C-S5-LW-C30	44.9	4.49	2.45	120	12	21	7.57	1707.91
Group (C-3)	**C-S1-NW-C70**	67.3	6.73	2.81	149	14.9	14.7	4.23	1450.10
	C-S2-NW-C70	67.3	6.73	1.52	210	21	26.4	16.37	3812.47
	C-S3-NW-C70	67.3	6.73	2.01	168	16.8	16.9	7.41	1892.36
	C-S4-NW-C70	67.3	6.73	2.03	160	16	15.8	6.78	1723.79
	C-S5-NW-C70	67.3	6.73	2.63	174	17.4	16.8	5.39	1882.82

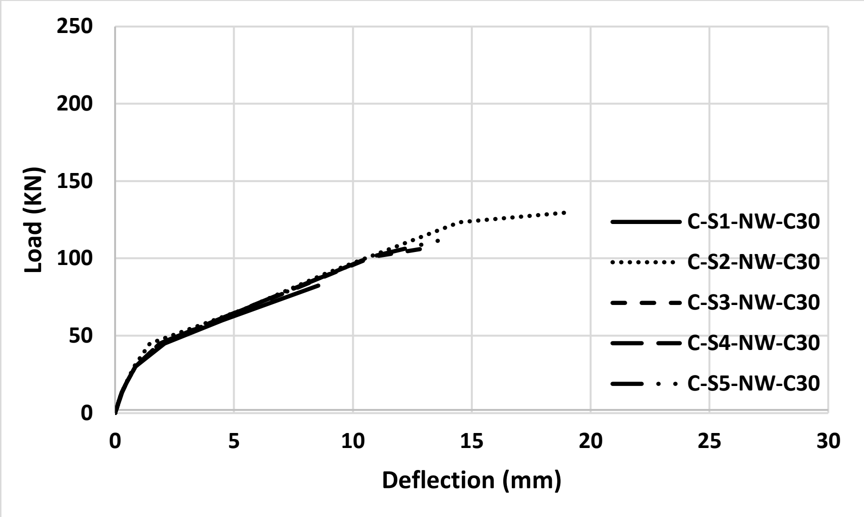

Figure (14): Load-Deflection Curves for Group (C-1)

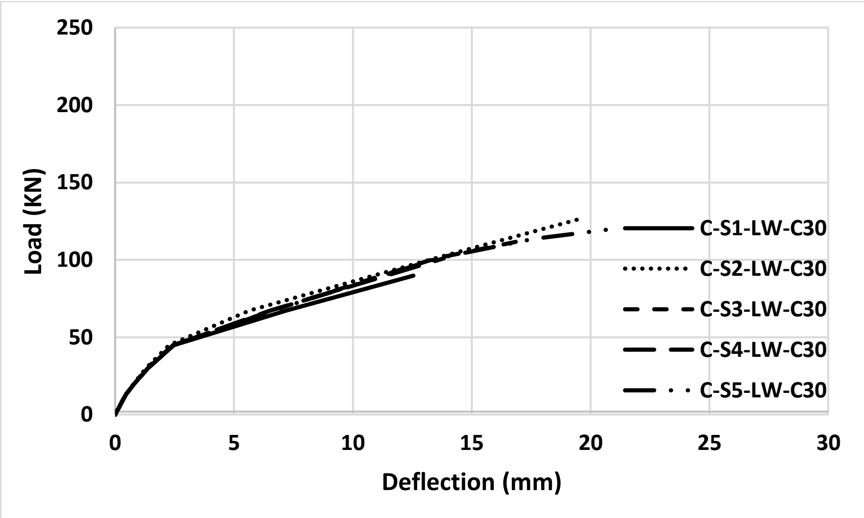

Figure (15): Load-Deflection Curves for Group (C-2)

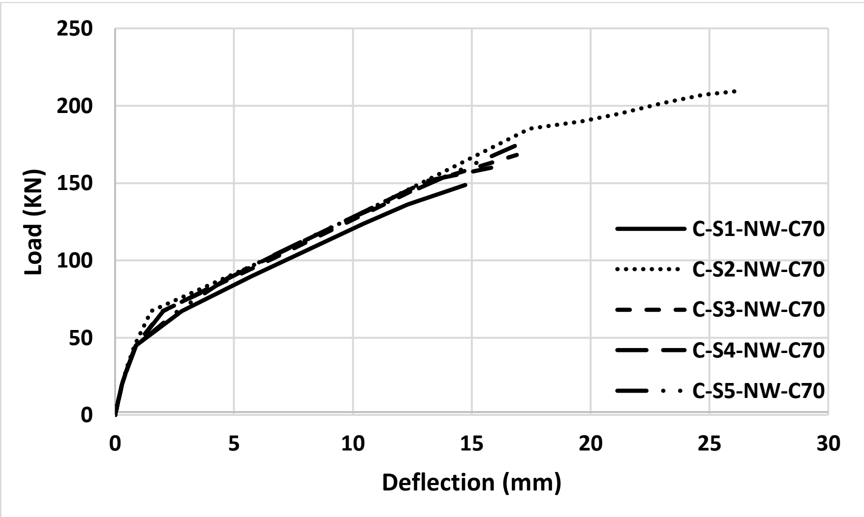

Figure (16): Load-Deflection Curves for Group (C-3)

Conclusions

Based on the results obtained from finite element analysis, the following can be concluded:

1. The control slabs from all groups which have left without shear reinforcement failed abruptly in punching shear.
2. Some of Specimens failed eventually in punching shear failure after reaching yielding strain and undergoing substantial deflection in more ductile mode. The failure reflects the huge potential of strip reinforcement in preventing failure in the shear reinforcement zone.
3. Shear bands distributed over the critical punching shear zone provide a very good economic solution regarding increasing the punching shear capacity, ductility and energy absorption. Their efficiency depends on their reinforcement installation method with flexural reinforcement, layout of distribution and concentration.
4. It was observed that the failure capacity load increase obviously when shear bands installed in woven way with the flexure steel mats than when hanging on tension flexure mat only. The knitting method provides additional anchorage to reinforcement with the compression zone; all characteristics improved strongly, load capacity, Absorbed Energy, measure of ductility. This installation method considered the most improved specimen of all specimens absolutely.
5. We can concluded that all specimens enhanced comparing with the control sample, it reflect significant increase in initial stiffness in addition to increasing in energy absorption and measure of ductility.
6. Concentration of shear bands (orthogonal distribution) by doubling quantity of shear bands over the punching zone area leads to efficient system, the slightly enhancement ratio in failure load capacity.
7. The arrangement of shear bands around the column circumference (Box layout distribution) it provides enhancement ratio in the failure load capacity comparing with
8. Comparison between NWC and LWC with same Concrete Grade (C30) shows that the failure load capacity and the measure of ductility of all specimens were enhanced by using NWC.
9. Comparison between NSC (C30) and HSC (C70) with same Concrete Weight shows that the failure load capacity and the measure of ductility of all specimens were enhanced by using HSC.

References

1. Heba Mohammed, (2014). “Enhancement of Punching Shear Resistance of Flat Slabs Using Shear-Band System”. M.Sc. Thesis, Faculty of Engineering, El-Mattaria, Helwan University.
2. Shatha S., ShaimaaT. and Mohammad Z. (2013) " Non- Linear FE Modeling of Two-Way Reinforced Concrete Slab of NSC, HSC and LWC under Concentrated Load" Journal of Engineering and Development, Vol. 17, No.2, 2013, ISSN 1813- 7822.
3. ANSYS, "ANSYS Help", Release 14.5, Copyright 2012.
4. Anthony J. Wolanski, B.S. (Flexural Behavior of Reinforced and Pre-stressed Concrete Beams Using Finite Element Analysis), M.Sc of Thesis, Marquette University (2004).
5. Megally, S. and A. Ghali. (2000)”Punching of Concrete Slabs due to Column Moment Transfer”. Journal of Structural Engineering. Feb.
6. Presentation on "Shear bands verification of a novel punching shear reinforcement system for flat slabs" by Professor Kypors pilakoutas, (2000). Center for cement and concrete – University of Sheffield.

7. Iravani Said and MacGregor G. James, (Sustained Load Strength and Short Term Load Behavior of High Strength Concrete), ACI Structural Journal. Vol 95. No 5 (1998).
8. Nilson, A. H., "Design of Concrete Structures," Twelfth Edition, the McGraw Hill Companies, Inc., 1997, 780 p.
9. American Concrete Institute Committee 363, State-of-the-Art Report on High-Strength Concrete, ACI 363R-97, American Concrete Institute, Detroit, MI, September 1997, 55 pp.
10. P. Desayi, BE, MTech, PhD, DSc and H. K. Seshadri s, (1997). “Punching shear strength of flat slab corner column connections” Journals of Proceedings of the ICE- Structures and Buildings, Volume 122, Issue 1
11. Neil Hammill and Amin Ghali "Punching Shear Resistance of Corner Slab-Column Connections" ACI Structural Journal , V. 91, No. 6,Nov-Dec.1994,PP.697-707
12. Regan P.E., Al-Hussaini, A. Ramdane, Xue H.Y., (1993)."Behavior of High Strength Concrete Slabs". Proceedings of International Conference, University of Dundee, Scotland, UK, September 7-9, Vol. 1, Cambridge, pp. 761-773.
13. Wafa, F.F. and Ashour, S.A. (1992), Mechanical Properties of High Strength of Fiber Reinforced Concrete (ACI Material Journal). Volume 89. No 5. PP. 449-455
14. El-Mezaini, N., and Citipitiouglu, E., “Finite Element Analysis of Prestressed and Reinforced Concrete Structures,” ASCE Journal of Structural Engineering, Vol. 97, No. 2, October, 1991, pp. 252-258.
15. Bazant, Z P and Cao, Z., (1987) "Size effect in punching shear failure of slabs ". ACI Structural Journal. Vol. 84, No. 1, pp. 44-52.
16. Rankin, G.I.B. and Long, A.E., (1987), "Punching Shear in Reinforced Concrete", Proceedings of the Institution of Civil Engineers, London, Part 1, vol. 82, no. 2, December, pp. 1165-1186.
17. P. T. WANG, S. P. SHAH, and A. E. NAAMAN, (1978). Stress-Strain Curves of Normal and Lightweight Concrete in Compression, ACI JOURNAL/ NOVEMBER 1978