

A Comparative Study for Fuzzy Ranking Methods in Determining Economic Life of Equipment

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Abstract Replacing the equipment at its economic life reducing the direct cost of construction projects, which leads to increasing the probability of winning bids. This problem involves capital cost, scrap value, maintenance cost, and rate of interest. The main aim of this paper is to study the effect of fuzzy ranking method based on centroid indices in determining replacement time (economic life) of equipment. Normal trapezoidal fuzzy numbers were used for their simplicity and for the comparison of different ranking methods. The time value of money was considered. Nine ranking methods were considered. Two examples were conducted to demonstrate the effect of fuzzy ranking method on economic life of equipment. The main contribution of this research is that five methods give the same results for the economic life for fuzzy replacement problem which are consistent with human intuition. These methods are: Cheng [1], Chu and Taso [2], Wang et al. [3], Dat et al. [4] and Gani and Mohamed [5]. On the other hand, Chen and Chen [6] and Allahviranloo and Saneifard [7] ranking methods give another replacement time which is also consistent with human intuition. Yager [8] and Chen and Chen [9] ranking methods give different results for replacement time which are not consistent with human intuition.

Keywords Fuzzy numbers, Fuzzy ranking method, Time value of money, Economic life, Fuzzy replacement problem

1. Introduction

Replacement Problem (RP) is one of the practical areas in economic decision analysis for our real world system. It is used in engineering economics to determine an optimal decision for maintenance and replacement purposes. Park [10] reported that replacement time or economic life of asset is the remaining useful life that results in the minimum annual equivalent cost. However, the minimum annual equivalent cost for equipments is an important issue for reducing direct cost of construction projects, which results in increasing the probability of winning bids.

When any production facility is new, it works with full operating efficiency. With passage of time and due to usage, it may become old, some of its components wear out, and the operating efficiency of the facility may gradually decrease. To regain the efficiency, maintenance is to be needed. When first maintenance is attended, its performance is slightly reduced. In the second maintenance, it is more reduced than previous one. Like this the facility deteriorates, and finally the operating efficiency reduces to some desired level of performance. Thus, it is not economical to use the facility for further production, as the maintenance cost will be very high,

and the unit production cost also increases. Therefore, the replacement of the facility is due at this stage (Biswas and Pramanik [11]).

In our real world system, there are elements of uncertainty in the process or its parameters, which may lack precise definition or precise measurement especially when the system involves human judgment. When developing a model of a system of uncertainty, the decision maker can either ignore the uncertainty or try to deal with uncertainty. When, ignoring the uncertainty, the decision maker obtains the results in a deterministic model of the process with precise values of all parameters. To deal with uncertainty, a decision maker uses specific paradigms such as interval analysis, probability theory, fuzzy set theory, possibility theory, or evidence theory. The choice of paradigm depends on the nature of the uncertainty (Biswas and Pramanik [11]).

In their work, Biswas and Pramanik [11] considered a realistic replacing problem where capital cost, scrap value and maintenance cost or running cost of equipment are imprecise in nature and represented by positive triangular fuzzy number (TFNs) or trapezoidal fuzzy numbers (TrFNs). They assumed that the replacement of equipment deteriorates with time and the fuzzy maintenance cost goes on increasing with usage or age. Then, they found out optimum time of replacing the item considering the value of money decreases with fuzzy rate of interest that is known as its discounted factor or depreciation ratio. With this discounted factor, they determined the weighted average

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fuzzy cost, then found the minimum fuzzy average cost. Finally, Comparison of fuzzy average costs was done by a Yager's ranking method [8].

In this paper, the authors study a realistic view that capital cost, scrap value, maintenance or running cost of equipment are all of trapezoidal fuzzy numbers (TrFNs). They consider the value of money with time in the replacement of equipment that deteriorates with time. The previous definition of economic life given by Park [10] is adopted for determining the replacement year. The main objective of this paper is to compare different fuzzy ranking methods in determining replacement time (economic life) which seek for the minimum equivalent annual cost. Fuzzy ranking methods have been used to transform the fuzzy numbers to crisp version so that any conventional method can be applied to solve the problem. In the current research, for simplicity and for the comparison of different ranking methods, normal fuzzy numbers are only considered.

2. Literature Review

Many researchers have developed different RP with different criterion. Several RP models are available in the literature. Bellman [12] and Bellman and Dreyfus [13] developed the replacement problem as a dynamic programming (DP). They formulated a discounted DP version of the economic life of equipment and determined analytically the optimal age to replace the equipment. Alchian [14] considered replacement problem, when operating cost of equipment involves linear function of time. Dreyfus and Law [15] discussed RP with exponentially bounded operating costs. In order to introduce a more realistic view, they studied RP where decision can be made stochastically. Ohnishi [16] studied optimal repair and replacement problem under average cost criterion as a semi Markov decision. Wagner [17] formulated a replacement problem as a network and solved for the shortest path that corresponds to the minimum outlay. Oakford et al [18] generalized the Wagner's dynamic programming model that allows for multiple challengers and time varying parameters. Lohmann [19] again generalized Wagner's dynamic model in stochastic concept. Dimitrakos and Kyriakidis [20] considered a system that deteriorates with age and may experience a failure at any time. They developed an algorithm based on the embedding technique and then generate a sequence of improving control limit policy. Mahdavi and Mahdavi [21] discussed an optimization of age replacement policy using reliability based heuristic model. Zhao et al. [22] formulated three kinds of replacement models combined with additive and independent damages. Nezhad et al [23] discussed one stage two-machine replacement strategy based on the Bayesian inference method. Recently, Zvipore et al. [24] developed a comprehensive dynamic programming – based optimization solution methodology to solve equipment replacement

optimization problem on the replacement of conveyer belts at a Gold Mining Company in Zimbabwe. Sabu et al. [25] discussed the value and importance of replacement in production atmosphere. They focused on striking a balance between the cost and the competitive environment. Also, they discussed the possibilities of equipment replacement to ensure delivering the normal performance of equipment.

The theory of Von Neumann and Morgenstern [26] provides the tools necessary to determine the optimal decision when the probabilities are specified for outcomes. To deal with uncertainty, which is different from probability theory, Zadeh [27] developed the concept of fuzzy set theory. This theory has been developed and applied to numerous areas such as control, decision-making, engineering, medicine, investment and finance. The uses of non-probabilistic uncertainty and especially fuzzy sets have caught much attention in the area of economic analysis (Buckley [28]; Buckley [29]; Choobineh, and Behrens [30]). Ward [31] studied discounted fuzzy cash flow problem. Uncertainty occurs in replacement and maintenance decisions in various ways. To solve the fuzzy replacement problem (FRP), Dong and Shash [32] proposed a method to bypass the problem involving interval and fuzzy arithmetic. Hearnes [33] formulated fuzzy versions of the economic life of an asset model and the finite single asset replacement problem. Chiu and Park [34] used fuzzy numbers in cash flow analysis and provide a good survey of the major methods for ranking mutually exclusive fuzzy projects. Biswas and Pramanik [35] developed a method of finding the optimal replacement time of equipment for fuzzy replacement problem with trapezoidal fuzzy numbers and triangular fuzzy numbers using Yager's ranking method [8]. They don't consider time value of money.

3. Research Scope and Methodology

In the current research, a comparative study for different fuzzy ranking methods in determining economic life of equipment which seek for the minimum equivalent annual cost will be conducted. A standard methodology will be adopted. As an initial step to meet the objectives, previous research papers that deal with the optimal replacement time of equipment for fuzzy replacement problem were reviewed in the previous section. The next step is to describe the normal trapezoidal fuzzy number. Fuzzy ranking methods based on centroid indices are then presented. Methodology adopted for the determination of replacement time for equipment are then explained. The next step is to apply these steps to an example project to show how to determine the economic life for each ranking method. A second example is then highlighted to draw a conclusion. Analysis of the results for the two examples helps indicate the effect of fuzzy ranking method based on centroid indices on economic life for fuzzy replacement problem. Based on the results, conclusions are drawn in the last section.

4. Generalized and Normal Trapezoidal Fuzzy Number

Kaufmann and Gupta, 1988, cited by kumar et al., [36] reported that a generalized fuzzy number, A (a,b,c,d;w) = is said to be a generalized trapezoidal fuzzy number if its membership function is given by:

$$\mu_A(x) = \begin{cases} \frac{w(x-a)}{(b-a)}, & a \leq x \leq b \\ w, & b \leq x \leq c \\ \frac{w(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

Thorani et al. [37] reported that, it is customary to write a trapezoidal fuzzy number as A (a,b,c,d;w). If w = 1, then A (a,b,c,d;1) is a normalized fuzzy number, otherwise A is said to be a generalized or non-normal fuzzy number.

5. Fuzzy Ranking Methods

Ranking fuzzy numbers is usually used in decision-making, data analysis, artificial intelligence, economic systems and operation research. In a fuzzy environment, ranking is a very important decision making procedure. In order to rank fuzzy numbers, one fuzzy number needs to be evaluated and compared to the others, but this may not be easy. Since fuzzy numbers are represented by possibility distributions, they can overlap with each other and, thus, it is difficult to determine clearly whether one fuzzy number is larger or smaller than another (Kwang and Lee, [38]). In recent years, many methods have been proposed for ranking different types of fuzzy numbers and can be classified into four major classes; preference relation, fuzzy mean and spread, fuzzy scoring and linguistic expression, but each method appears to have advantages as well as disadvantages (Chen and Hwang, [39]). One of the most commonly used methods under the class of fuzzy scoring is the centroid point method. Therefore, in this paper the centroid point methods in ranking fuzzy numbers are only applied. The famous ranking methods based on centroid point in almost previous researches are applied here: Yager [8], Cheng [1], Chu and Taso [2], Chen and Chen [6], Wang et al. [3], and Chen and Chen [9]. In addition to three new methods, these are: Dat et al. [4], Allahviranloo and Saneifard [7], and Gani and Mohamed [5].

Ever since Yager [8] presented the centroid concept in the ranking approach, numerous ranking techniques using the centroid concept have been proposed and investigated. Yager [8] was the first researcher to propose a centroid-index ranking method to calculate the value of fuzzy number A as

$$x_A^* = \frac{\int_0^l w(x)f_A(x)dx}{\int_0^l f_A(x)dx} \tag{1}$$

Where w(x) is a weighting function measuring the importance of the value x, and f_A denotes the membership function of the fuzzy number A. When w(x) = x, the value x_A^* becomes the geometric Center of Gravity (COG) with

$$x_A^* = \frac{\int_0^l xf_A(x)dx}{\int_0^l f_A(x)dx}$$

The larger the value is of x_A^*, the

better the ranking of A. However, Yager [8] made no assumption on the normality and on the convexity of the fuzzy number.

Cheng [1] used a centroid-based distance approach to rank fuzzy numbers. For a trapezoidal fuzzy number A = (a,b,c,d;w), the distance index can be defined as:

$$R(A) = \sqrt{x_A^{-2} + y_A^{-2}} \tag{2}$$

$$x_A^- = \frac{\int_a^b xf_A^L dx + \int_b^c x dx + \int_c^d xf_A^R dx}{\int_a^b f_A^L dx + \int_b^c dx + \int_c^d f_A^R dx} \tag{3}$$

$$y_A^- = w \cdot \frac{\int_0^l yg_A^L dx + \int_0^l yg_A^R dy}{\int_0^l g_A^L dx + \int_0^l g_A^R dy} \tag{4}$$

f_A^R and f_A^L are the respective right and left membership functions of A, and g_A^R and g_A^L are the inverse of f_A^R and f_A^L respectively. The larger the value of R(A), the better the ranking will be of A. For trapezoidal fuzzy numbers equations 3 and 4 becomes as given in Eq.(s) 5 and 6.

$$x_A^- = \frac{w(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(c^2 - b^2)}{3w(d - c + b - a) + 6(c - b)} \tag{5}$$

$$y_A^- = \frac{w}{3} \left[1 + \frac{(b+c) - (a+d)(1-w)}{(b+c-a-d) + 2(a+d)w} \right] \tag{6}$$

Cheng [1] further proposed a coefficient of variation (CV) index that improves the concept of ranking fuzzy numbers, using fuzzy mean and fuzzy spread. Chu and Tsao [2] found that the distance approach and CV index proposed by Cheng [1] still had some shortcomings. Hence, to overcome the problems, Chu and Tsao [2] proposed a new ranking index function (Eq.7)

$$S_A = x_A^- \times y_A^- \tag{7}$$

Where: x_A^- is as defined in Cheng [1] and y_A^- as given in Eq. 8. The larger the value is of S_A, the better the

ranking will be of A .

$$y_A^- = \frac{\int_0^w yg_A^L dy + \int_0^w yg_A^R dy}{\int_0^w g_A^L dy + \int_0^w g_A^R dy} \tag{8}$$

For $w=1$ Eq.8 becomes as Eq.4 Accordingly, for normal trapezoidal fuzzy numbers Eq. 5 and 6 given by Cheng [1] are applied for Chu and Taso [2] for calculating x_A^- and y_A^- .

Dat et al. [4] declared that the drawback of Chu and Tsao [2] ranking method is that if $\bar{x} = 0$, then the value of $S_A = \bar{x}y$ is a constant zero. In other words, the fuzzy numbers with centroids $(0,y_1)$ and $(0,y_2)$, ($y_1 \neq y_2$) are considered the same. This is unreasonable.

Chen and Chen [6] proposed an approach for ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations to overcome the drawbacks of Cheng's [1] and Yager's [8] approaches. The ranking value for a generalized trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4 : w)$ (Rank (A)) is given in Eq. 9. The larger the value of Rank (A), the better the ranking of A .

$$Rank(A) = x_A + (w - y_A)^S (y_A + 0.5)^{1-w} \tag{9}$$

Where: y_A , x_A and S are as given in Eq.(s) 10-12. a as given in Eq.13.

$$y_A = \frac{w}{6} + \frac{(a_3 - a_2 + 2)}{(a_4 - a_1)} \tag{10}$$

$$x_A = \frac{y_A(a_3 + a_2) + (a_4 + a_1)(w_A - y_A)}{2w_A} \tag{11}$$

$$S = \sqrt{\frac{\sum_{i=1}^4 (a_i - \bar{a})^2}{3}} \tag{12}$$

$$\bar{a} = \frac{(a_1 + a_2 + a_3 + a_4)}{4} \tag{13}$$

In a study conducted by Wang et al. [3], the centroid formulae proposed by Cheng [1] and Chu and Tsao [2] are shown to be incorrect. Therefore, to avoid any more misapplication, Wang et al. [3] presented the correct centroid formulas as:

$$x_A = \frac{\frac{b}{a} \int x f_A^L dx + \frac{c}{b} \int x w dx + \frac{d}{c} \int x f_A^R dx}{\int f_A^L dx + \int w dx + \int f_A^R dx} \tag{14}$$

$$y_A = \frac{\int_0^w y [g_A^R(y) - g_A^L(y)] dy}{\int_0^w [g_A^R(y) - g_A^L(y)] dy} \tag{15}$$

In a comparative study by Ramli and Mohamad [38] for ranking fuzzy numbers of centroid methods, they reported that, the correct formula proposed by Wang et al. [3] is only limited to trapezoidal fuzzy numbers with invertible membership functions. For a trapezoidal fuzzy number $\bar{A} = (a,b,c,d;w)$, the value of \bar{x}_0 and \bar{y}_0 are as given in Eq.(s) 16 and 17.

$$\bar{x}_0 = \frac{1}{3} \left[a + b + c + d - \frac{dc - ab}{(d + c) - (a + b)} \right] \tag{16}$$

$$\bar{y}_0 = \frac{w}{3} \left[1 + \frac{c - b}{(d + c) - (a + b)} \right] \tag{17}$$

The ranking function associated with \bar{A} is given in Eq.18. The larger the value of $R(\bar{A})$, the better the ranking of A .

$$R(\bar{A}) = \sqrt{x_0^{-2} + y_0^{-2}} \tag{18}$$

Chen and Chen [9] found that the approach proposed by Chen and Chen [6] still have shortcomings. Thus, Chen and Chen [9] proposed an approach for ranking generalized fuzzy numbers with different heights and different spreads. The score value of each standardized generalized fuzzy number $A_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4} : w_{Ai})$ is defined as in $Score(A_i)$. The larger the value of $Score(A_i)$ the better the ranking of A_i .

$$Score(A_i) = (x_{Ai} \times w_{Ai}) / (1 + S_{Ai}) \tag{19}$$

S_{Ai} and x_{Ai} are as given in Eq.(s) 20 and 21, respectively.

$$S_{Ai} = \sqrt{\frac{\sum_{j=1}^4 (a_{ij} - x_{Ai})^2}{3}} \tag{20}$$

$$x_{Ai} = \frac{(a_{i1} + a_{i2} + a_{i3} + a_{i4})}{4} \tag{21}$$

In their work, Dat et al. [4] proposed a new ranking method as follows: Suppose A_1, A_2, \dots, A_n are fuzzy numbers. First, the centroid point of all fuzzy numbers are calculated using Wang et al. [3]. Thus Eq. (s) 16 and 17 is used first for calculating \bar{x}_A and \bar{y}_A , then the distance between the

centroid point,

$$A_i = (x_{Ai}, y_{Ai}), i=1,2,\dots,n \text{ and the minimum point}$$

$G = (x_{min}, y_{min})$, is proposed as in Eq.22.

$$D(A_i, G) = \sqrt{(x_{Ai} - x_{min})^2 + (y_{Ai} - \frac{w}{3} y_{min})^2} \quad (22)$$

thus, if A_i, A_j are two fuzzy numbers, then their ranking order is defined as follows: (1) $A_i < A_j$, if $D(A_i, G) < D(A_j, G)$; (2) $A_i > A_j$, if $D(A_i, G) > D(A_j, G)$, and (3) $A_i = A_j$, if $D(A_i, G) = D(A_j, G)$.

Another ranking method was proposed by Allahviranloo and Saneifard [7]. They assumed that if there are n fuzzy numbers A_1, A_2, \dots, A_n . The proposed method for ranking fuzzy numbers A_1, A_2, \dots, A_n is presented as follows:

Step 1: Use formulas 14 and 15 given by Wang et al. [3] to calculate the centroid point (x_{A_j}, y_{A_j}) of each fuzzy number

A_j , where $1 \leq j \leq n$. For trapezoidal fuzzy number Eq.(s) 16 and 17 are equivalent to Eq.(s) 14 and 15, respectively.

Step 2: Calculate the maximum crisp value τ_{max} of all fuzzy numbers A_j , where $1 \leq j \leq n$.

Step 3: use the point (x_{A_j}, y_{A_j}) to calculate the ranking

value $Dist(A_j)$ of fuzzy numbers A_j , where $1 \leq j \leq n$, as in Eq. 23.

$$\begin{aligned} Dist(A_j) &= \sqrt{(x_{A_j} - \tau_{max})^2 + (y_{A_j} - 0)^2} \\ &= \sqrt{(x_{A_j} - \tau_{max})^2 + (y_{A_j})^2} \end{aligned} \quad (23)$$

$Dist(A_1) < Dist(A_2)$ if and only if $A_1 > A_2$; (2) $Dist(A_1) > Dist(A_2)$ if and only if $A_1 < A_2$ and (3) $Dist(A_1) = Dist(A_2)$ if and only if $A_1 = A_2$.

Recently, Gani and Mohamed [5] gave that, the centroid point of a trapezoid is considered to be the balancing point of the trapezoid. Divide the trapezoid into three plane figures. These three plane figures are two triangles and a rectangle. Consider a generalized trapezoidal fuzzy number $A = (a,b,c,d:w)$. The centroid of the first triangle is $G_1 = ((a+2b)/3, w/3)$, the centroid of the rectangle is $G_2 = ((b+c)/2, w/2)$ and the centroid of the second triangle is $G_3 = ((2c+d)/3, w/3)$. Thus, they concluded

that the centroid $G_A(x_0, y_0)$ of the triangle with vertices G_1, G_2 , and G_3 of the generalized trapezoidal fuzzy number

$$A=(a,b,c,d:w) \text{ as } G_A(x_0, y_0) = \left(\frac{2a+7b+7c+2d}{18}, \left(\frac{7w}{18} \right) \right).$$

The ranking function of the generalized trapezoidal fuzzy number $A=(a,b,c,d:w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as in Eq. 24.

$$R(A) = \bar{x}_0 \cdot \bar{y}_0 = \left(\frac{2a+7b+7c+2d}{18} \right) \left(\frac{7w}{18} \right) \quad (24)$$

The ranking procedure of two generalized trapezoidal fuzzy numbers $A=(a_1,b_1,c_1,d_1:w_1)$ and $B=(a_2,b_2,c_2,d_2:w_2)$, are as follows:

Step 1: Find $R(A)$ and $R(B)$. (i) If $R(A) > R(B)$, then

$A > B$; (ii) If $R(A) <$, then $A < B$; (iii) If $R(A) = R(B)$, then go to step 2.

Step 2: Find mode (A) and mode (B) using Eq. 25. (i) If mode $(A) >$ mode (B) , then $A > B$; (ii) If mode $(A) <$ mode (B) , then $A < B$; (iii) If mode $(A) =$ mode (B) , then go to step 3.

Step 3: Find average spread (AS) for A and B using Eq. 26. (i) If $AS(A) >$ $AS(B)$, then $A > B$; (ii) If $AS(A) <$ $AS(B)$, then $A < B$; (iii) If $AS(A) =$ $AS(B)$, then go to step 4.

Step 4: Examine w_1 and w_2 . (i) If $w_1 >$ w_2 , then $A > B$; (ii) If $w_1 <$ w_2 , then $A < B$; (iii) If $w_1 =$ w_2 , then $A = B$

$$mode = \frac{1}{2} x \int_0^w (b+c) dx = \frac{w}{2} (b+c) \quad (25)$$

$$\begin{aligned} AS &= \frac{1}{2} \left[\int_0^w (b-a) dx + \int_0^w (d-c) dx \right] \\ &= \frac{1}{2} [w(b-a) + w(d-c)] \end{aligned} \quad (26)$$

6. Methodology Adopted for the Determination of Replacement Time

Biswas and Pramanik [35] reported that replacement policy can be classified into the following categories. Case (1): When the equipment deteriorates with time and the value of money does not change with time or changes with time. Case (2): When the units fail completely all of a sudden.

In this paper the case when the value of money changes with time is considered. The purpose is to determine the optimum replacement time of equipment whose running or maintenance cost increases with time and the value of money changes during the period considering different fuzzy ranking methods.

- Let
- C Capital cost of equipment.
- S Scrap value of equipment.
- n number of years that equipment would be in use.
- F_t maintenance cost for year t.
- t year number, it may be 1, or 2, or 3,..., or n
- i rate of interest per year.
- PV Present Value
- FV Future value
- NPV Net Present Value
- AEA Annual Equivalent Amount

Austin et al. [41] reported that the present value PV is given by Eq. 27, whereas Eq. 28 shows annual equivalent amount. Kuhlemeyer et al. [42] gave that, present value (PV) is the current value of a future amount of money, or a series of payments, evaluated at a given interest rate. On the other hand, future value (FV) is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate. In Eq. 27, the factor multiplied by (FV) called single payment present worth factor. He added that, net present value measures the surplus in an investment project at time 0. In other words, net present value (NPV) is the summation of capital cost of equipment, maintenance cost, and subtracting scrap value (S) of equipment at time 0. On the other hand, annual equivalent amount (AEA) approach is used when companies want to determine whether or not they should replace an existing asset with a new asset (the case here). It can also be used if decided between two new assets [43]. It could be defined as a series of equal payments that are either paid to you or paid from you such as monthly rent payments, car payments [44]. In Eq. 28 the factor multiplied by (PV) called capital recovery factor.

$$PV = \frac{FV}{(1+i)^t} \quad (27)$$

$$AEA = PV \cdot \frac{i(1+i)^n}{(1+i)^n - 1} \quad (28)$$

If the equipment is used for n-years, then the present value of cost incurred during this period is given by:

$$PV(F_t) = \frac{F_t}{(1+i)^t} \quad (29)$$

$$\sum_1^t PV(F_t) = \sum_1^t \frac{F_t}{(1+i)^t} \quad (30)$$

$$PV(S_t) = \frac{S_t}{(1+i)^t} \quad (31)$$

$$NPV_t = C + \sum_1^t PV(F_t) - PV(S_t) \quad (32)$$

$$AEA_t = NPV_t \cdot \frac{i(i+1)^t}{(1+i)^t - 1} \quad (33)$$

So the replacement policy can be described as follows:

Replace the equipment at the end of t-th year, if annual equivalent amount of cost corresponding to this year is the least annual cost. To determine the replacement year for the

equipment we proceed as follows:

Step 1: Choose defuzzification method for ranking capital cost, maintenance cost, and scrap or salvage value.

Step 2: Apply equations 29, 30, and 31 to determine $PV(F_t)$, $\sum_1^t PV(F_t)$ and $PV(S_t)$ at each year.

Step 3: Calculate NPV_t at each year using Eq. 32. Then, determine AEA_t at each year by applying Eq. 33. Replace the equipment at least AEA_t .

The following section presents two examples which give a comparison for the effect of fuzzy ranking methods based on centroid indices on economic life of fuzzy replacement problem.

7. Application Examples

In this section two examples are given to show how the methodology adopted step by step for comparing the above methods of ranking for estimating economic life of fuzzy replacement problem. Normal trapezoidal fuzzy numbers are only considered. This is because trapezoidal fuzzy numbers pose several advantages over triangular fuzzy numbers as they are a more generalized form (Bansal, [45]). Also, the Arithmetic behavior of trapezoidal fuzzy numbers is better than triangular fuzzy number and it is more suitable for engineering and scientific problems. On the other hand, normal trapezoidal fuzzy numbers are applied for their simplicity and for the comparison of different ranking methods. Accordingly, normal trapezoidal fuzzy numbers are used in these examples. Analysis of the results for the two examples helps indicate the effect of fuzzy ranking method based on centroid indices on economic life for fuzzy replacement problem.

7.1. First Example

The data of this example was obtained from Biswas and Pramanik [35]. They did not consider the time value of money, thus they gave one value of salvage value and neglect interest rate. Thus, the authors assumed different values for salvage according to the year. Also, interest rate = 10% per year is assumed.

The example data are as follows: a construction company used a certain type of loader whose fuzzy cost (C) in rupees is (61000, 61300, 61700, 62000). Also, they reported that the running cost (maintenance cost (M.C)) in rupees are found from experience (see Table 1, column 2). Assumed different salvage values according to the year are also shown in Table 1, column 3).

Solution:

(1) To solve this problem, we take RS 1000 =1 unit.

Step 1: Choosing ranking method

1. For Yager’s [8] ranking method, to get the defuzzified value for example for maintenance cost in year 1, $F_1 = (1.200, 1.350, 1.400, 1.450)$ the following memberships function and their corresponding indices are as follows:

Table 1. Fuzzy maintenance cost and salvage value

Year(n)	Maintenance fuzzy cost (F_t)	Salvage value (S_t)
1	[1200, 1350, 1400, 1450]	[50000,50300,50700,51000]
2	[2500, 2600, 2750, 2900]	[33000,33300,33700,34000]
3	[3500, 3700, 3850,4000]	[22000,22300,22700,23000]
4	[4500, 4650, 4800, 5000]	[15000,15300,15700,16000]
5	[6000, 6500, 6700, 6800]	[10000,10300,10700,11000]
6	[8000, 8200, 8450, 8800]	[7000,7300,7700,8000]
7	[10500, 11000, 12500, 14000]	[5000,5300,5700,6000]
8	[16000, 17000, 18500, 20000]	[3000,3300,3700,4000]

$$\mu_{f_i} = \begin{cases} 0, x \leq 1.2 \\ \frac{x-1.2}{1.35-1.2}, 1.2 \leq x \leq 1.35 \\ 1, 1.35 \leq x \leq 1.4 \\ \frac{1.45-x}{0.05}, 1.4 \leq x \leq 1.45 \\ 0, x \geq 1.45 \end{cases}$$

Therefore, α -cut of the fuzzy number (1.200, 1.350, 1.400, 1.450) is $(f_{1\alpha}^L, f_{1\alpha}^U) = (0.15\alpha + 1.2, 1.45 - 0.05\alpha)$

Therefore,

$$\begin{aligned} Y(f_1) &= Y(1.2, 1.35, 1.4, 1.45) = \int_0^1 0.5(f_{1\alpha}^L + f_{1\alpha}^U) d\alpha \\ &= \int_0^1 0.5(0.15\alpha + 1.2 + 1.45 - 0.05\alpha) d\alpha = \int_0^1 0.5(2.65 + 0.1\alpha) d\alpha \\ &= \int_0^1 (1.325 + 0.05\alpha) d\alpha = 1.325 + 0.025 = 1.350 \end{aligned}$$

Similarly, the values of F_t at $t = 2, 3, \dots, 8$ are calculated (see Table 2). Also, the defuzzified values of capital cost and salvage value for each year are calculated (see Table 2).

2. For Cheng’s [1] ranking method, the defuzzified value for F_1 is calculated as follows: apply Eq.5 and 6 to calculate $x_{F_1}^-$ and $y_{F_1}^-$, then determine R_{F_1} by Eq. 2. See Table 2 for all the defuzzified values.

$$\begin{aligned} x_{F_1}^- &= \frac{1.45^2 - 2 \times 1.4^2 + 2 \times 1.35^2 - 1.2^2 + 1.4 \times 1.45 - 1.2 \times 1.35 + 3(1.4^2 - 1.35^2)}{3(1.45 - 1.4 + 1.35 - 1.2) + 6(1.4 - 1.35)} = 1.3444 \\ y_{F_1}^- &= \frac{1}{3} \left[1 + \frac{1.35 + 1.4 - 0}{(1.35 + 1.4 - 1.2 - 1.45) + 2(1.2 + 1.45)} \right] = 0.5031 \end{aligned}$$

$$R_{F_i} = \sqrt{(1.3444)^2 + (0.5031)^2} = 1.4354$$

3. For Chu and Taso [2] ranking method, $x_{F_i}^-$ and $y_{F_i}^-$ are as given in Cheng (1998), then determine S_A by Eq.7. $S_A = 1.3444 \times 0.5031 = 0.6764$. See Table 2 for the all defuzzified values.

Table 2. Ranking index for capital, maintenance and salvage fuzzy costs (example 1)

Ranking method	Defuzzified values in different years									
	C	other costs	1	2	3	4	5	6	7	8
Yager (1981)	61.50	F	1.35	2.61	3.76	4.74	6.50	8.363	12.00	17.88
		S	50.50	33.50	22.50	15.50	10.50	7.500	5.500	3.500
Cheng (1998)	61.50	F	1.435	2.735	3.794	4.766	6.499	8.384	12.04	17.91
		S	50.50	33.50	22.50	15.50	10.50	7.502	5.502	3.502
Chu and Taso (2002)	30.75	F	0.676	1.3435	1.882	2.368	3.257	4.179	5.974	9.299
		S	25.25	16.750	11.25	7.750	5.250	3.7500	2.750	1.750
Chen and Chen (2003)	62.29	F	2.2952	3.606	4.661	5.6414	7.321	9.217	12.49	18.32
		S	51.299	34.299	23.299	16.299	11.299	8.299	6.299	4.299
Chen and Chen (2009)	42.72	F	1.218	2.287	3.100	3.904	4.794	6.218	4.649	6.500
		S	35.077	23.269	15.628	10.766	7.293	5.209	3.820	2.431
Wang et al. (2006)	61.50	F	1.399	2.723	3.783	4.758	6.492	8.379	12.04	17.89
		S	50.502	33.502	22.502	15.502	10.502	7.502	5.502	3.502
Dat et al. (2012)	60.16	F	0.259	1.377	2.432	3.407	5.143	7.030	10.69	16.55
		S	49.157	32.157	21.157	14.159	9.161	6.163	4.166	2.176
Viranloo and Saneifard (2012)	0.429	F	60.157	58.812	57.741	56.762	55.022	53.133	49.47	43.61
		S	11.008	28.003	39.003	46.002	51.002	54.002	56.00	58.00
Gani and Mohamed (2013)	23.92	F	0.530	1.042	1.466	1.839	2.549	3.244	4.613	6.924
		S	19.639	13.028	8.750	6.028	4.083	2.917	2.139	1.361

4. For Chen and Chen's [6] ranking method, the defuzzified value for F_i is calculated by applying Eq.(s)10, 13, 12, 11 and 9 to calculate y_{F_i} , a_{F_i} , S_{F_i} , x_{F_i} , and R_{F_i} , respectively. See Table 2 for the all defuzzified values.

$$y_{F_i} = \frac{1}{6} \left[\frac{1.4 - 1.35}{1.45 - 1.2} + 2 \right] = 0.36667$$

$$a_{F_i} = \frac{1.2 + 1.35 + 1.4 + 1.45}{4} = 1.35$$

$$S_{F_i} = \sqrt{\frac{(1.2 - 1.35)^2 + (1.35 - 1.35)^2 + (1.4 - 1.35)^2 + (1.45 - 1.35)^2}{3}} = 0.10803$$

$$x_{F_i} = \frac{0.36667(1.4 + 1.35) + (1.2 + 1.45)(1 - 0.36667)}{2} = 1.3433$$

$$R_{F_i} = 1.3433 + (1 - 0.36667)^{0.10803} (0.36667 + 0.5)^{1-1} = 2.29518$$

5. For Wang et.al. [3] ranking method, the defuzzified value for F_i is calculated by applying Eq.(s)16,17 and 18 to calculate X_{F_i} , Y_{F_i} , and R_{F_i} , respectively. See Table 2 for the all defuzzified values.

$$x_{F_i}^- = \frac{1}{3} \left[1.2 + 1.35 + 1.4 + 1.45 - \frac{1.4 \times 1.45 - 1.2 \times 1.35}{(1.45 + 1.4) - (1.2 + 1.35)} \right] = 1.3443$$

$$\bar{y}_{F_1} = \frac{1}{3} \left[1 + \frac{1.4 - 1.35}{0.3} \right] = 0.3888$$

$$R_{F_1} = \sqrt{(1.3443)^2 + (0.3888)^2} = 1.3992$$

6. For Chen and Chen's [9] ranking method, the defuzzified value for F_1 is calculated by applying Eq.13 or 21 to calculate a_{F_1} , Eq.12, or 20 to calculate S_{F_1} . As previously calculated $a_{F_1} = 1.35$ and $S_{F_1} = 0.10803$. $Score_{F_1}$ is calculated by using Eq. 19. See Table 2 for the all defuzzified values.

$$Score_{F_1} = \frac{1.35 \times 1}{1 + 0.10803} = 1.2184$$

7. For Dat et al. [4] ranking method, the defuzzified value for F_1 is calculated by applying Eq.(s)16,17 to calculate \bar{x}_{F_1} , \bar{y}_{F_1} . As given above in Wang et al. (2006) $\bar{x}_{F_1} = 1.3443$ and $\bar{y}_{F_1} = 0.3888$. It must be noted that these values correspond to x_{min} and y_{min} (see Table 1). Thus, $x_{min} = 1.3443$ and $y_{min} = 0.3888$. $D(F_1, G)$ is calculated using Eq. 22. See Table 2 for the all defuzzified values.

$$D(F_1, G) = \sqrt{(1.3443 - 1.3443)^2 + (0.3888 - 0.3888 / 3)^2} = 0.2592$$

8. For Allahviranloo and Saneifard [7] ranking method, the defuzzified value for F_1 is calculated by applying Eq.(s)16,17 to calculate \bar{x}_{F_1} , \bar{y}_{F_1} . As given previously $\bar{x}_{F_1} = 1.3443$ and $\bar{y}_{F_1} = 0.3888$. τ_{max} is the maximum $X_{0, \Delta}$. This value is 60.5 correspond to fuzzy capital cost (61000, 61300, 61700, 62000). Applying Eq. 23, $Dist(F_1)$ is calculated as given below. See Table 2 for the all defuzzified values.

$$Dist(F_1) = \sqrt{(1.3443 - 61.5)^2 + (0.3888 - 0)^2} = 60.1569$$

9. For Gani and Mohammed [5] ranking method, the defuzzified value for F_1 is calculated by applying Eq. 24 to calculate $R(F_1)$. See Table 2 for all the defuzzified values.

$$R(F_1) = \left(\frac{2 \times 1.2 + 7 \times 1.35 + 7 \times 1.4 + 2 \times 1.45}{18} \right) \left(\frac{7}{18} \right) = 1.36388 \times \frac{7}{18} = 0.530$$

Step 2: Applying Eq.(s) 29, 30, and 31 and using Yager 's [8] ranking method for example

$$\text{gives } PV(F_1) = 1.227, \sum_1^t PV(F_1) = 1.227 \text{ and } PV(S_1) = 45.909 \text{ for the first year.}$$

Step 3: Apply Eq.(s) 32 and 33 to calculate, NPV_1 and AEA_1 . These values are 16.818 and 18.5, respectively. All other values of AEA_t are given in Table 3.

Step 2 and 3 are applied to all other ranking methods. Table 3 shows AEA_t for all ranking methods at different years. From this table it is clear that the minimum annual equivalent amount of cost depends on the ranking method. The year correspond to each minimum value is the year of replacement and this year represents the economic life of equipment.

Table 3 shows that eight ranking methods (Yager [8]; Cheng [1]; Chu and Taso [2]; Chen and Chen [6]; Wang et al. [3], Dat et al. [4], Allahviranloo and Saneifard [7], and Gani and Mohamed [5] give the same result that the loader should be replaced at the end of 7th year. According to Chen and Chen [9] ranking method, the loader should be replaced at the end of 8th.

Table 3. Replacement year (economic life) of equipment for different ranking methods (example 1)

Ranking method	Annual equivalent amount of cost (AEA_t) for different years							
	1	2	3	4	5	6	7	8
Yager (1981)	18.500	21.435	20.431	19.043	18.060	17.329	17.057*	17.351
Cheng (1998)	18.586	21.538	20.512	19.111	18.119	17.375	17.109*	17.399
Chu and Taso (2002)	9.251	10.736	10.228	9.531	9.039	8.668	8.533*	8.709
Chen and Chen (2003)	19.525	22.484	21.457	20.059	19.059	18.310	18.002*	18.256
Chen and Chen (2009)	13.131	15.261	14.598	13.678	12.968	12.458	11.836	11.524*
Wang et al. (2006)	18.549	21.514	20.491	19.093	18.108	17.359	17.096*	17.385
Dat et al. (2012)	18.374	20.141	19.084	17.669	16.669	15.919	15.651*	15.936
Viranloo and Saneifard (2012)	49.621	46.429	47.367	48.735	49.691	50.385	50.667*	50.354
Gani and Mohamed (2013)	7.199	8.352	7.957	7.414	7.032	6.744	6.635*	6.745

* the minimum annual equivalent amount of cost

7.2. Second Example

The data of this example was obtained from Biswas and Pramanik [11]. The fuzzy cost of a machine is US\$ (5900, 5950, 6050, 6100). The fuzzy running cost and the salvage value at the end of the year are given in Table 4. The author assumed the interest to be 10% per year. Find when the machine is to be replaced.

Solution: To solve this problem, we take \$1000 = 1 unit. The same steps applied in solving example 1 are applied here. Table 5 shows AEA_t for all ranking methods at different years. From this table it is clear that the minimum annual equivalent amount of cost depends on ranking method. The year corresponds to each minimum value is the year of replacement.

Table 5 shows that ranking methods: Cheng [1]; Chu and Taso [2]; Chen and Chen [9]; Wang et al. [3], Dat et al. [4], and Gani and Mohamed [5] give the same result that the loader should be replaced at the end of 6th year. Two ranking methods: Chen and Chen [6] and Allviranloo and Saneifard [7] give that the loader should be replaced at the end of 5th year. On the other hand, Yager [8] ranking method gives that the loader should be replaced at the end of 7th year.

7.3. Analysis of the Results for the Two Examples

Dat et al. [4] reported that almost no approach can rank fuzzy numbers satisfactorily in all situations. In some situations, almost all methods cannot rank fuzzy numbers correctly with the same centroid point. However, he declared the strong discrimination power of his proposed ranking approach and its advantages through many comparative examples with other ranking methods. Also, he explained that his proposed ranking method is more consistent with human intuitions than existing methods. Taking this issue in consideration together with analysis of the results reveals that:

1. The ranking methods: Cheng [1], Chu and Taso [2], Wang et al. [3], Dat et al. [4] and Gani and Mohamed [5] give the same results for the economic life or replacement time for fuzzy replacement problem which are more consistent with human intuitions.
2. On the other hand, Chen and Chen [6], Allahviranloo and Saneifard [7] ranking methods give the same replacement time which are also consistent with human intuitions.
3. Yager [8] and Chen and Chen [9] ranking methods give different results for replacement time which are not consistent with human intuitions, since economic life always less than normal life.

Table 4. Fuzzy maintenance cost and salvage value (example 2)

Year(n)	Maintenance fuzzy cost (F_t)	Salvage value (S_t)
1	[1100, 1150, 1200, 1270]	[3800, 3900, 3950, 4000]
2	[1300, 1360, 1400, 1450]	[2600, 2650, 2700, 2760]
3	[1500, 1580, 1600, 1650]	[1900, 1950, 2000, 2060]
4	[1800, 1840, 1850, 1870]	[1450, 1470, 1480, 1500]
5	[2000, 2030, 2050, 2070]	[950, 980, 1000, 1050]
6	[2350, 2400, 2460, 2500]	[550, 570, 590, 600]
7	[2900, 2920, 2950, 3000]	[500, 530, 540, 550]

Table 5. Replacement year (economic life) of machine for different ranking methods (example 2)

Ranking method	Annual equivalent amount of cost (AEA_t) for different years						
	1	2	3	4	5	6	7
Yager (1981)	3.926	3.435	3.167	3.032	2.973	2.945	2.926*
Cheng (1998)	3.963	3.541	3.261	3.117	3.048	3.009*	3.041
Chu and Taso (2002)	1.933	1.728	1.591	1.522	1.482	1.473*	1.492
Chen and Chen (2003)	4.924	4.806	4.242	4.103	4.016*	4.019	4.050
Chen and Chen (2009)	3.543	3.168	2.934	2.814	2.782	2.778*	2.794
Wang et al. (2006)	3.938	3.514	3.233	3.091	3.025	3.010*	3.021
Dat et al. (2012)	3.346	2.922	2.639	2.497	2.428	2.398*	2.431
Viranloo and Saneifard (2012)	3.197	3.405	3.607	3.709	3.751*	3.739	3.725
Gani and Mohamed (2013)	1.500	1.344	1.238	1.184	1.160	1.153*	1.157

* the minimum annual equivalent amount of cost

8. Summary and Conclusions

In this paper, the fuzzy replacement problem is considered in the sense that the capital cost, scrap value, maintenance or running cost are all imprecise in nature represented by fuzzy numbers. It is more realistic and closer to our daily life situation. In modeling of replacement problem it is often observed that the parameters of the problem are not known precisely. This impreciseness is handled by using fuzzy numbers, as it is expected to express the situation more realistically. In this paper, to deal with this uncertainty, fuzzy replacement problem was presented when the capital cost, scrap value, maintenance or running cost are normal trapezoidal fuzzy numbers. Nine ranking methods have been compared in determining the replacement time (economic life) of equipment. Two numerical examples have been provided to show the effect of ranking method on fuzzy replacement problem. The results revealed that of the nine ranking methods dealt with, five methods give the same results for the economic life or replacement time for fuzzy replacement problem which are consistent with human intuitions. These methods are: Cheng [1], Chu and Taso [2], Wang et al. [3], Dat et al. [4] and Gani and Mohamed [5]. On the other hand, Chen and Chen [6] and Allahviranloo and Saneifard [7] ranking methods give the same replacement time which is also consistent with human intuitions. Yager [8] and Chen and Chen [9] ranking methods give different results for replacement time which are not consistent with human intuitions, since economic life always less than normal life. Replacing the equipment at its economic life leads to reducing direct cost of construction projects, which in turn leads to increasing the probability of winning bids. One limitation of the current research is that the results are true for normal trapezoidal fuzzy numbers.

In the future research, the authors hope that the steps adopted for the determination of replacement time of equipment in the current research are applied for non normal fuzzy numbers, i.e. for different values of membership

function. On the other hand, further research is required to use the defuzzification method (ranking method) for the annual equivalent amount instead applying defuzzification method firstly for maintenance cost, capital cost and scarp value.

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