Effect of Fuzzy Ranking Method Based on Centroid Indices on Economic Life for Fuzzy Replacement Problem

A. M. El-Kholy
Assistant Professor, Civil Engineering Dept., Faculty of Engineering, Beni-Suef Univ., Beni-Suef , Egypt. E-mail: amrelkholy_2012@ yahoo.com

Ahmed M. Abdelalim
Assistant Professor, Civil Engineering Dept., Mataria Faculty of Engineering, Helwan University, Egypt. E-mail:Dr.ahmed.abdelalim@gmail.com

Abstract
This paper presents an algorithm for the determination of replacement time (economic life) of equipment considering the effect of fuzzy ranking method based on centroid indices. Replacing the equipment at its economic life results in reducing the direct cost of construction projects. This leads to increase the probability of winning bids. This problem involves capital cost, scrap value or salvage value, maintenance cost or operating cost, and rate of interest. Here the imprecise values are assumed as positive normal trapezoidal fuzzy numbers. The time value of money is considered. Nine ranking methods are considered. Two examples are conducted to demonstrate the algorithm's performance and its contributions. The results of the two examples revealed that: five methods gives the same results for the economic life or replacement time for fuzzy replacement problem. These methods are: Cheng (1998), Chu and Taso (2002), Wang et al. (2006), Dat et al. (2012) and Gani and Mohamed (2013). On the other hand, Chen and Chen (2003) and Viranloo and Saneifard (2012) ranking methods give the same replacement time. Yager (1981) and Chen and Chen (2009) ranking methods give different results for replacement time.

Keywords: Replacement; Fuzzy Ranking Method; Time Value of Money; Economic Life; Fuzzy Replacement Problem

Introduction
Replacement Problem (RP) is one of the practical areas in economic decision analysis for our real world system. It is used in engineering economics to determine an optimal decision for maintenance and replacement purposes. Park (2007) reported that replacement time or economic life of asset is the remaining useful life that results in the minimum annual equivalent cost. However, the minimum annual equivalent cost for equipments is an important issue for reducing direct cost of construction projects, which results in increasing the probability of winning bids.

When any production facility is new, it works with full operating efficiency. With passage of time and due to usage, it may become old, some of its components wear out, and the operating efficiency of the facility may gradually decreases. To regain the efficiency, maintenance is to be needed. When first maintenance is attended, its performance is slightly reduced. In the second maintenance, it is more reduced than previous one. Like this the facility deteriorates, and finally the operating efficiency reduces to some desired level of performance. Thus, it is not economical to use the facility for further production, as the maintenance cost will be very high, and the unit production cost also increases. Therefore, the replacement of the facility is due at this stage (Biswas and Pramanik (2011b)).

Many researchers have developed different RP with different criterion. Several RP models are available in the literature. Bellman (1955) and Bellman and Dreyfus (1962) developed the replacement problem as a dynamic programming (DP). They formulated a discounted DP version of the economic life of equipment and determined analytically the optimal age to replace the equipment. Alchian (1952) considered replacement problem, when operating cost of equipment involves linear function of time. Dreyfus and Law (1977) discussed RP with exponentially bounded operating costs. In order to introduce a more realistic view, they studied RP where decision can be made stochastically. Ohnishi (1997) studied optimal repair and replacement problem under average cost criterion as a semi Markov decision. Wagner (1975) formulated a replacement problem as a network and solved for the shortest path that corresponds to the minimum outlay. Oak ford et al (1984) generalized the Wagner's dynamic programming model that allows for multiple challengers and time varying parameters. Lohmann (1986) again generalized Wagner's dynamic model in stochastic concept. Dimitrakos and Kyriakidis (2007) considered a system that deteriorates with age and may experience a failure at any time. They developed an algorithm based on the embedding technique and then generate a sequence of improving control limit policy. Mahdavi and Mahdavi (2009) discussed an optimization of age replacement policy using reliability based heuristic model. Zhao et al. (2010) formulated three kinds of replacement models combined with additive and independent damages. Nezhad et al (2007) discussed one stage two-machine replacement strategy based on the Bayesian inference method.

Biswas and Pramanik (2011b) gave that in our real world system, there are elements of uncertainty in the process or its parameters, which may lack precise definition or precise measurement especially when the system involves human judgment. When developing a model of a system of uncertainty, the decision maker can either ignore the uncertainty or try to deal with uncertainty. When, ignoring the uncertainty, the decision maker obtains the results in a deterministic model of the process with precise values of all parameters. To deal with uncertainty, a decision maker uses specific paradigms such as interval analysis, probability theory, fuzzy set theory, possibility theory, or evidence theory. The choice of paradigm depends on the nature of the uncertainty.

The theory of Von Neumann and Morhenstern (1944) provides the tools necessary to determine the optimal decision when the probabilities are specified for outcomes. To deal with uncertainty, which is different from probability theory, Zadeh (1965) developed the concept of fuzzy set theory. This theory has been developed and applied to numerous areas such as control, decision-making, engineering, medicine, investment and finance. The uses of non-probabilistic uncertainty and especially fuzzy sets have caught much attention in the area of economic analysis (Buckley 1987; Buckley 1992; Choobineh, and Behrens 1992). Ward (1985) studied discounted fuzzy cash flow problem. Uncertainty occurs in replacement and maintenance decisions in various ways. To solve the fuzzy replacement problem (FRP), Dong and Shash (1987) proposed a method to bypass the problem involving interval and fuzzy arithmetic. Hearnes (1995) formulated fuzzy versions of the economic life of an asset model and the finite single asset replacement problem. Chiu and Park (1994) used fuzzy numbers in cash flow analysis and provide a good survey of the major methods for ranking mutually exclusive fuzzy projects. Biswas and Pramanik (2011a) developed a method of finding the optimal replacement time of equipment for fuzzy replacement problem with trapezoidal fuzzy numbers and triangular fuzzy numbers using Yager's ranking method. They don't consider time value of money. In their work, Biswas and Pramanik (2011b) considered a realistic replacing problem where capital cost, scrap value and maintenance cost or running cost of equipment are imprecise in nature and represented by positive triangular fuzzy number (TFNs) or trapezoidal fuzzy numbers (TrFNs). They assumed that the replacement of equipment deteriorates with time and the fuzzy maintenance cost goes on increasing with usage or age. Then, they found out optimum time of replacing the item considering the value of money decreases with fuzzy rate of interest that is known as its discounted factor or depreciation ratio. With this discounted factor, they determined the weighted average fuzzy cost, then found the minimum fuzzy average cost. Finally, Comparison of fuzzy average costs was done by a Yager's ranking method (1981).

In this paper, the author study a realistic view that capital cost, scrap value, maintenance or running cost of equipment are all of trapezoidal fuzzy numbers (TrFNs). He considers the value of money with time in the replacement of equipment that deteriorates with time. The previous definition of economic life given by Park (2007) is adopted for determining the replacement year. The objective is to propose an algorithm for the determination of replacement time which seek for the minimum equivalent annual cost using different ranking methods to find the effect of fuzzy ranking method on economic life. Fuzzy ranking methods have been used to transform the fuzzy numbers to crisp version so that any conventional method can be applied to solve the problem In the current research, for simplicity and for the comparison of different ranking methods, normal fuzzy numbers are only considered. The rest of this paper is organized as follows. The second section is devoted to present the preliminaries of fuzzy sets. The third section explains fuzzy ranking methods The fourth section demonstrates the proposed algorithm for the determination of replacement time. An illustrative example is then presented to demonstrate the performance of the proposed algorithm considering different ranking methods. Another example is highlighted to draw a conclusion. Analysis of the examples helps indicate the effect of fuzzy ranking method based on centroid indices on economic life for fuzzy replacement problem. Conclusions are drawn in the last section.

Preliminaries of Fuzzy Sets

Lotfi Zadeh (1965) first introduced Fuzzy set as a mathematical way for representing impreciseness.

Definition 1: Fuzzy set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined by $\tilde{A}=\{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, where $\mu_{\tilde{A}}(x)$: $X \to [0,1]$ is called the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership to which $x \in \tilde{A}$.

Definition 2: A fuzzy set $\tilde{A}$ on R is convex if for any $x_1, x_2 \in X$, the membership function of $\tilde{A}$ satisfies the inequality $\mu_{\tilde{A}}\{\lambda x_1+(1-\lambda) x_2\} \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$; $0 \leq \lambda \leq 1$.where min denotes the minimum operator.

Definition 3: Normal Fuzzy Set: A fuzzy set $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that there exists at least one x in X such that $\mu_{\tilde{A}}(x)$**=1.**

Definition 4:

Trapezoidal fuzzy number: A trapezoidal fuzzy number $\tilde{a}$ is denoted by (a_1 , a_2 , a_3 , a_4) where a_1 , a_2 , a_3 , a_4 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given by:

$$\mu_{\tilde{a}}(x)=\begin{cases} 0, x \leq a_1, \\ \dfrac{x-a_1}{a_2-a_1}, a_2 \leq x \leq a_1, \\ 1, a_2 \leq x \leq a_3, \\ \dfrac{a_4-x}{a_4-a_3}, a_3 \leq x \leq a_4, \\ 0, x \geq a_4 \end{cases}$$

$\mu_{\tilde{a}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{a}}(x)$ is a continuous mapping from R to closed interval [0,1].
2. $\mu_{\tilde{a}}(x) = 0$ for every $X \in [-\infty, a_1]$.
3. $\mu_{\tilde{a}}(x)$ is strictly increasing and continuous on $[a_1, a_2]$.

4. $\mu_{\tilde{a}}$ (x)=1 for every x $\in$ [a_2, a_3].

5. $\mu_{\tilde{a}}$ (x) is strictly decreasing and continuous on [a_3, a_4].

6. $\mu_{\tilde{a}}$ (x) = 0 for every x $\in$ [a_4, ∞].

Definition 5: The α -cut set of a fuzzy set $\tilde{A}$ is a crisp set defined by

$$\tilde{A}_\alpha = \{x \in X \,/\, \mu_{\tilde{A}}(x) \geq \alpha\}.$$

Fuzzy Ranking Methods

Ranking fuzzy numbers is usually used in decision-making, data analysis, artificial intelligence, economic systems and operation research. In a fuzzy environment, ranking is a very important decision making procedure. In order to rank fuzzy numbers, one fuzzy number needs to be evaluated and compared to the others, but this may not be easy. Since fuzzy numbers are represented by possibility distributions, they can overlap with each other and, thus, it is difficult to determine clearly whether one fuzzy number is larger or smaller than another (Kwang and Lee, 1999). In recent years, many methods have been proposed for ranking different types of fuzzy numbers and can be classified into four major classes; preference relation, fuzzy mean and spread, fuzzy scoring and linguistic expression, but each method appears to have advantages as well as disadvantages (Chen and Hwang, 1992). One of the most commonly used methods under the class of fuzzy scoring is the centroid point method. Therefore, in this paper the centroid point methods in ranking fuzzy numbers are only applied.

Ever since Yager (1981) presented the centroid concept in the ranking approach, numerous ranking techniques using the centroid concept have been proposed and investigated. Yager (1981) was the first researcher to propose a centroid- index ranking method to calculate the value of fuzzy number A as

$$x_A^* = \frac{\int_0^1 w(x) f_A(x)dx}{\int_0^1 f_A(x)dx} \tag{1}$$

Where $w(x)$ is a weighting function measuring the importance of the value x, and f_A denotes the membership function of the fuzzy number A . When $w(x) = x$, the value x_A^* becomes the geometric Center of Gravity (COG) with $x_A^* = \frac{\int_0^1 x f_A(x)dx}{\int_0^1 f_A(x)dx}$ .The larger the value is of x_A^*, the better the ranking of A. However, Yager (1981) made no assumption on the normality and on the convexity of the fuzzy number.

Cheng (1998) used a centroid-based distance approach to rank fuzzy numbers. For a trapezoidal fuzzy number $A = (a,b,c,d;\varpi)$, the distance index can be defined as:

$$R(A) = \sqrt{\bar{x}_A^{\,2} + \bar{y}_A^{\,2}} \tag{2}$$

$$\bar{x}_A = \frac{\int_a^b x f_A^L dx + \int_b^c x dx + \int_c^d x f_A^R dx}{\int_a^b f_A^L dx + \int_b^c dx + \int_c^d f_A^R dx} \tag{3}$$

$$y_A^- = w.\frac{\int_0^1 yg_A^L dx + \int_0^1 yg_A^R dy}{\int_0^1 g_A^L dx + \int_0^1 g_A^R dy} \quad (4)$$

f_A^R and f_A^L are the respective right and left membership functions of A, and g_A^R and g_A^L are the inverse of f_A^R and f_A^L respectively. The larger the value of $R(A)$, the better the ranking will be of A. For trapezoidal fuzzy numbers equations 3 and 4 becomes as given in Eq.(s) 5 and 6.

$$x_A^- = \frac{w(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(c^2 - b^2)}{3w(d - c + b - a) + 6(c - b)} \quad (5)$$

$$y_A^- = \frac{w}{3}\left[1 + \frac{(b + c) - (a + d)(1 - w)}{(b + c - a - d) + 2(a + d)w}\right] \quad (6)$$

Cheng (1998) further proposed a coefficient of variation (CV) index that improves the concept of ranking fuzzy numbers, using fuzzy mean and fuzzy spread. Chu and Tsao (2002) found that the distance approach and CV index proposed by Cheng (1998) still had some shortcomings. Hence, to overcome the problems, Chu and Tsao (2002) proposed a new ranking index function (Eq.7)

$$S_A = x_A^- \times y_A^- \quad (7)$$

Where: x_A^- is as defined in Cheng (1998) and y_A^- as given in Eq. 8. The larger the value is of S_A, the better the ranking will be of *A*.

$$y_A^- = \frac{\int_0^w yg_A^L dy + \int_0^w yg_A^R dy}{\int_0^w g_A^L dy + \int_0^w g_A^R dy} \quad (8)$$

For w=1 Eq.8 becomes as Eq.4 Accordingly, for normal trapezoidal fuzzy numbers Eq. 5 and 6 given by Cheng (1998) are applied for Chu and Taso (2002) for calculating x_A^- and y_A^-.

Dat et al. (2012) declared that the drawback of Chu and Tsao (2002) ranking method is that if $\bar{x} = 0$, then the value of $S_A = \bar{x}\bar{y}$ is a constant zero. In other words, the fuzzy numbers with centroids (0,y_1) and (0,y_2), ($y_1 \neq y_2$) are considered the same. This is unreasonable.

Chen and Chen (2003) proposed an approach for ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations to overcome the drawbacks of Cheng's (1998) and Yager's (1981) approaches. The ranking value for a generalized trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4 : w)$ (Rank (*A*)) is given in Eq. 9.The larger the value of Rank (*A*), the better the ranking of *A*.

$$Rank(A) = x_A + (w - y_A)^S (y_A + 0.5)^{1-w} \quad (9)$$

Where: y_A, x_A and S are as given in Eq.(s) 10-12. $\bar{a}$ as given in Eq.13.

$$y_A = \frac{w}{6} + (\frac{a_3 - a_2}{a_4 - a_1} + 2) \quad (10)$$

$$x_A = \frac{y_A(a_3 + a_2) + (a_4 + a_1)(w_A - y_A)}{2w_A} \quad (11)$$

$$S = \sqrt{\frac{\sum_{i=1}^{4}(a_i - \bar{a})^2}{3}} \tag{12}$$

$$\bar{a} = \frac{(a_1 + a_2 + a_3 + a_4)}{4} \tag{13}$$

In a study conducted by Wang et al. (2006), the centroid formulae proposed by Cheng (1998) and Chu and Tsao (2002) are shown to be incorrect. Therefore, to avoid any more misapplication, Wang et al. (2006) presented the correct centroid formulas as:

$$\bar{x}_A = \frac{\int_a^b x f_A^L dx + \int_b^c xwdx + \int_c^d x f_A^R dx}{\int_a^b f_A^L dx + \int_b^c wdx + \int_c^d f_A^R dx} \tag{14}$$

$$\bar{y}_A = \frac{\int_0^w y\left[g_A^R(y) - g_A^L(y)\right]dy}{\int_0^w \left[g_A^R(y) - g_A^L(y)\right]dy} \tag{15}$$

In a comparative study by Ramli and Mohamad (2009) for ranking fuzzy numbers of centroid methods, they reported that, the correct formula proposed by Wang et al. (2006) is only limited to trapezoidal fuzzy numbers with invertible membership functions. For a trapezoidal fuzzy number $\bar{A}$ = (a,b,c,d; w), the value of $\bar{x}_0$ and $\bar{y}_0$ are as given in Eq.(s) 16 and 17.

$$\bar{x}_0 = \frac{1}{3}\left[a + b + c + d - \frac{dc - ab}{(d + c) - (a + b)}\right] \tag{16}$$

$$\bar{y}_0 = \frac{w}{3}\left[1 + \frac{c - b}{(d + c) - (a + b)}\right] \tag{17}$$

The ranking function associated with $\bar{A}$ is given in Eq.18. The larger the value of $Rank(\bar{A})$, the better the ranking of A.

$$R(\bar{A}) = \sqrt{\bar{x}_0^{\,2} + \bar{y}_0^{\,2}} \tag{18}$$

Chen and Chen (2009) found that the approach proposed by Chen and Chen (2003) still have shortcomings. Thus, Chen and Chen (2009) proposed an approach for ranking generalized fuzzy numbers with different heights and different spreads. The score value of each standardized generalized fuzzy number $A_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4} : w_{Ai})$ is defined as in $Score(A_i)$.The larger the value of $Score(A_i)$ the better the ranking of A_i .

$$Score(A_i) = (x_{Ai} \times w_{Ai}) / (1 + S_{Ai}) \tag{19}$$

S_{Ai} and x_{Ai} are as given in Eq.(s) 20 and 21, respectively.

$$S_{Ai} = \sqrt{\frac{\sum_{j=1}^{4}(a_{ij} - x_{Ai})^2}{3}} \tag{20}$$

$$x_{Ai} = \frac{(a_{i1} + a_{i2} + a_{i3} + a_{i4})}{4} \quad (21)$$

In their work, Dat et al. (2012) proposed a new ranking method as follows: Suppose $A_1, A_2, ..., A_n$ are fuzzy numbers. First, the centroid point of all fuzzy numbers are calculated using Wang et al.(2006). Thus Eq. (s) 16 and 17 is used first for calculating $\bar{x}_A$ and $\bar{y}_A$, then the distance between the centroid point,

$A_i = (\bar{x}_{Ai}, \bar{y}_{Ai})$, i=1,2,…,n and the minimum point $G = (x_{min}, y_{min})$, is proposed as in Eq.22.

$$D(A_i, G) = \sqrt{(\bar{x}_{Ai} - x_{min})^2 + (\bar{y}_{Ai} - \frac{w}{3} y_{min})^2} \quad (22)$$

thus, if A_i, A_j are two fuzzy numbers, then their ranking order is defined as follows:(1) $A_i \prec A_j$, if $D(A_i, G) \prec D(A_j, G)$;(2) $A_i \succ A_j$, if $D(A_i, G) \succ D(A_j, G)$, and (3) $A_i \sim A_j$, if $D(A_i, G) = D(A_j, G)$.

Another ranking method was proposed by Allahviranloo and Saneifard (2012). They assumed that if there are n fuzzy numbers $A_1, A_2, ..., A_n$. The proposed method for ranking fuzzy numbers $A_1, A_2, ..., A_n$ is presented as follows:

Step 1: Use formulas 14 and 15 given by Wang et al. (2006) to calculate the centroid point ($\bar{x}_{A_j}$, $\bar{y}_{A_j}$) of each fuzzy number A_j, where $1 \le j \le n$. For trapezoidal fuzzy number Eq.(s) 16 and 17 are equivalent to Eq.(s) 14 and 15, respectively.

Step 2: Calculate the maximum crisp value τ_{max} of all fuzzy numbers A_j, where $1 \le j \le n$.

Step 3: use the point ($\bar{x}_{A_j}$, $\bar{y}_{A_j}$) to calculate the ranking value *Dist* (A_j) of fuzzy numbers A_j, where $1 \le j \le n$, as in Eq. 23.

$$Dist(A_j) = \sqrt{(\bar{x}_{A_j} - \tau_{max})^2 + (\bar{y}_{A_j} - 0)^2} = \sqrt{(\bar{x}_{A_j} - \tau_{max})^2 + (\bar{y}_{A_j})^2} \quad (23)$$

Dist (A_1) $\prec$ *Dist* (A_2) if and only if $A_1 \succ A_2$; (2) *Dist* (A_1) $\succ$ *Dist* (A_2) if and only if $A_1 \prec A_2$ and (3) Dist (A_1) = Dist (A_2) if and only if $A_1 \sim A_2$.

Recently, Gani and Mohamed (2013) gave that, the centroid point of a trapezoid is considered to be the balancing point of the trapezoid. Divide the trapezoid into three plane figures. These three plane figures are two triangles and a rectangle. Consider a generalized trapezoidal fuzzy number A=(a,b,c,d:w). The centroid of the first triangle is $G_1 = ((a + 2b)/3, w/3)$, the centroid of the rectangle is $G_2 = ((b + c)/2, w/2)$ and the centroid of the second triangle is $G_3 = ((2c + d)/3, w/3)$. Thus, they concluded that the centroid $G_A(\bar{x}_0, \bar{y}_0)$ of the triangle with vertices G1, G2, and G3 of the generalized trapezoidal fuzzy number *A=(a,b,c,d:w)* as $G_A(\bar{x}_0, \bar{y}_0) = \left(\frac{2a + 7b + 7c + 2d}{18}\right), \left(\frac{7w}{18}\right)$. The ranking function of the generalized trapezoidal fuzzy number *A=(a,b,c,d:w)* which maps the set of all fuzzy numbers to a set of real numbers is defined as in Eq. 24.

$$R(A) = \bar{x}_0 . \bar{y}_0 = \left(\frac{2a + 7b + 7c + 2d}{18}\right)\left(\frac{7w}{18}\right) \quad (24)$$

The ranking procedure of two generalized trapezoidal fuzzy numbers $A=(a_1,b_1,c_1,d_1:w_1)$ and $B=(a_2,b_2,c_2,d_2:w_2)$, are as follows:

Step 1: Find $R(A)$ and $R(B)$. (i) If $R(A) \succ R(B)$, then $A \succ B$; (ii) If $R(A) \prec R(B)$, then $A \prec B$; (iii) If $R(A)=R(B)$, then go to step 2.

Step 2: Find mode (A) and mode (B) using Eq. 25. (i) If mode (A) $\succ$ mode (B), then $A \succ B$; (ii) If mode (A) $\prec$ mode (B), then $A \prec B$; (iii) If mode (A) = mode (B), then go to step 3.
Step 3: Find average spread (AS) for A and B using Eq. 26. (i) If $AS(A) \succ AS(B)$, then $A \succ B$; (ii) If $AS(A) \prec$ AS (B), then A $\prec$ B; (iii) If AS (A) = AS (B), then go to step 4.
Step 4: Examine w_1 and w_2. (i) If $w_1 \succ w_2$, then $A \succ B$; (ii) If $w_1 \prec w_2$, then $A \prec B$; (iii) If $w_1 = w_2$, then $A = B$

$$mode = \frac{1}{2} x \int_0^w (b+c)dx = \frac{w}{2}(b+c) \tag{25}$$

$$AS = \frac{1}{2}\left[\int_0^w (b-a)dx + \int_0^w (d-c)dx\right] = \frac{1}{2}\left[w(b-a)+w(d-c)\right] \tag{26}$$

Proposed Algorithm for the Determination of Replacement Time

Biswas and Pramanik (2011a) reported that replacement policy can be classified into the following categories. Case (1): When the equipment deteriorates with time and the value of money does not change with time or changes with time. Case (2): When the units fail completely all of a sudden.
In this paper, the case when the value of money changes with time is considered. The purpose is to determine the optimum replacement time of equipment whose running or maintenance cost increases with time and the value of money changes during the period considering different fuzzy ranking methods.
Let

C Capital cost of equipment.
S Scrap value of equipment.
n Number of years that equipment would be in use.
F_t Maintenance cost for year t.
t Year number, it may be 1, or 2, or 3,…, or n
i Rate of interest per year.
PV Present Value
FV Future value
NPV Net Present Value
AEA Annual Equivalent Amount

Austin et al. (1996) reported that the present value PV is given by Eq. 27, whereas Eq. 28 shows annual equivalent amount

$$PV = \frac{FV}{(1+i)^t} \tag{27}$$

$$AEA = PV . \frac{i(1+i)^n}{(1+i)^n - 1} \tag{28}$$

If the equipment is used for n-years, then the present value of cost incurred during this period is given by:

$$PV(F_t) = \frac{F_t}{(1+i)^t} \tag{29}$$

$$\sum_{1}^{t} PV(F_t) = \sum_{1}^{t} \frac{F_t}{(1+i)^t} \quad (30)$$

$$PV(S_t) = \frac{S_t}{(1+i)^t} \quad (31)$$

$$NPV_t = C + \sum_{1}^{t} PV(F_t) - PV(S_t) \quad (32)$$

$$AEA_t = NPV_t * \frac{i(i+1)^t}{(1+i)^t - 1} \quad (33)$$

So the replacement policy can be described as follows:
Replace the equipment at the end of t-th year, if annual equivalent amount of cost corresponding to this year is the least annual cost.
To determine the replacement year for the equipment we proceed as follows:
Step 1: Choose defuzzification method for ranking capital cost, maintenance cost, and scrap or salvage value.

Step 2: Apply equations 29, 30, and 31 to determine $PV(F_t)$, $\sum_{1}^{t} PV(F_t)$ and $PV(S_t)$ at each year.

Step 3: Calculate NPV_t at each year using Eq. 32. Then, determine AEA_t at each year by applying Eq. 33. Replace the equipment at least AEA_t.

The following examples give a comparison for the effect of fuzzy ranking methods based on centroid indices on economic life of fuzzy replacement problem.

First Example

The data of this example was obtained from Biswas and Pramanik (2011a).They did not consider the time value of money, thus they gave one value of salvage value and neglect interest rate. Thus, the author assumed different values for salvage according to the year. Also, interest rate = 10% per year is assumed.
The example data are as follows: a construction company used a certain type of loader whose fuzzy cost (C) in rupees is (61000, 61300, 61700, 62000). Also, they reported that the running cost (maintenance cost (M.C)) in rupees are found from experience (see Table 1, column 2). Assumed different salvage values according to the year are also shown in Table 1, column 3).

Solution:
(1) To solve this problem, we take RS 1000 =1 unit.

Step 1: Choosing ranking method

1. For Yager's (1981) ranking method, to get the defuzzyfied value for example for maintenance cost in year 1, F_1 = (1.200, 1.350, 1.400, 1.450) the following memberships function and their corresponding indices are as follows:

$$\mu_{f_t} = \begin{cases} 0, x \le 1.2 \\ \dfrac{x-1.2}{1.35-1.2}, 1.2 \le x \le 1.35 \\ 1, 1.35 \le x \le 1.4 \\ \dfrac{1.45-x}{0.05}, 1.4 \le x \le 1.45 \\ 0, x \ge 0 \end{cases}$$

Therefore, α -cut of the fuzzy number (1.200, 1.350, 1.400, 1.450) is

$(f_{1\alpha}^L, f_{1\alpha}^U) = (0.15\alpha + 1.2, 1.45 - 0.05\alpha)$

Therefore, $Y(f_1) = Y(1.2, 1.35, 1.4, 1.45) = \int_0^1 0.5(f_{1\alpha}^L + f_{1\alpha}^U) d\alpha$

$$= \int_0^1 0.5(0.15\alpha + 1.2 + 1.45 - 0.05\alpha) d\alpha = \int_0^1 0.5(2.65 + 0.1\alpha) d\alpha$$

$$= \int_0^1 (1.325 + 0.05\alpha) d\alpha = 1.325 + 0.025 = 1.350$$

Similarly, the values of F_t at t = 2, 3, …, 8 are calculated (see Table 2). Also, the defuzzified values of capital cost and salvage value for each year are calculated (see Table 2).

2. For Cheng's (1998) ranking method, the defuzzified value for F_1 is calculated as follows: apply Eq.5 and 6 to calculate $x_{F_1}^-$ and $y_{F_1}^-$, then determine R_{F_1} by Eq. 2. See Table 2 for all the defuzzified values.

$$x_{F_1}^- = \frac{1.45^2 - 2\times 1.4^2 + 2\times 1.35^2 - 1.2^2 + 1.4\times 1.45 - 1.2\times 1.35 + 3(1.4^2 - 1.35^2)}{3(1.45 - 1.4 + 1.35 - 1.2) + 6(1.4 - 1.35)} = 1.3444$$

$$y_{F_1}^- = \frac{1}{3}\left[1 + \frac{1.35 + 1.4 - 0}{(1.35 + 1.4 - 1.2 - 1.45) + 2(1.2 + 1.45)}\right] = 0.5031$$

$$R_{F_1} = \sqrt{(1.3444)^2 + (0.5031)^2} = 1.4354$$

3. For Chu and Taso (2002) ranking method, $x_{F_1}^-$ and $y_{F_1}^-$ are as given in Cheng (1998), then determine S_A by Eq.7.

$S_A = 1.3444 \times 0.5031 = 0.6764$ .See Table 2 for the all defuzzified values.

4. For Chen and Chen's (2003) ranking method, the defuzzified value for F_1 is calculated by applying Eq.(s)10,13,12,11 and 9 to calculate $y_{F_1}, a_{F_1}, S_{F_1}, x_{F_1}$, and R_{F_1}, respectively. See Table 2 for the all defuzzified values.

$$y_{F_1} = \frac{1}{6}\left[\frac{1.4 - 1.35}{1.45 - 1.2} + 2\right] = 0.36667$$

$$a_{F_1} = \frac{1.2 + 1.35 + 1.4 + 1.45}{4} = 1.35$$

$$S_{F_1} = \sqrt{\frac{(1.2 - 1.35)^2 + (1.35 - 1.35)^2 + (1.4 - 1.35)^2 + (1.45 - 1.35)^2}{3}} = 0.10803$$

$$x_{F_1} = \frac{0.36667(1.4+1.35)+(1.2+1.45)(1-0.36667)}{2} = 1.3433$$

$$R_{F_1} = 1.3433+(1-0.36667)^{0.10803}(0.36667+0.5)^{1-1} = 2.29518$$

5. For Wang et.al (2006) ranking method, the defuzzified value for F_1 is calculated by applying Eq.(s)16,17 and 18 to calculate X_{F_1}, Y_{F_1}, and R_{F_1}, respectively. See Table 2 for the all defuzzified values.

$$\bar{x}_{F_1} = \frac{1}{3}\left[1.2+1.35+1.4+1.45-\frac{1.4\times1.45-1.2\times1.35}{(1.45+1.4)-(1.2+1.35)}\right] = 1.3443$$

$$\bar{y}_{F_1} = \frac{1}{3}\left[1+\frac{1.4-1.35}{0.3}\right] = 0.3888$$

$$R_{F_1} = \sqrt{(1.3443)^2+(0.3888)^2} = 1.3992$$

6. For Chen and Chen's (2009) ranking method, the defuzzified value for F_1 is calculated by applying Eq.13or 21 to calculate a_{F_1}, Eq.12, or 20 to calculate S_{F_1}.As previously calculated $a_{F_1} = 1.35$ and $S_{F_1} = 0.10803$. $Score_{F_1}$ is calculated by using Eq. 19. See Table 2 for the all defuzzified values.

$$Score_{F_1} = \frac{1.35\times1}{1+0.10803} = 1.2184$$

7. For Dat et al. (2012) ranking method, the defuzzified value for F_1 is calculated by applying Eq.(s)16,17 to calculate $\bar{x}_{F_1}$, $\bar{y}_{F_1}$. As given above in Wang et al.(2006) $\bar{x}_{F_1} = 1.3443$ and $\bar{y}_{F_1} = 0.3888$. It must be noted that these values correspond to x_{min} and y_{min} (see Table 1). Thus, $x_{min} = 1.3443$ and $y_{min} = 0.3888$. $D(F_1,G)$ is calculated using Eq. 22. See Table 2 for the all defuzzified values.

$$D(F_1,G) = \sqrt{(1.3443-1.3443)^2+(0.3888-0.3888/3)^2} = 0.2592$$

8. For Allahviranloo and Saneifard (2012) ranking method, the defuzzified value for F_1 is calculated by applying Eq.(s)16,17 to calculate $\bar{x}_{F_1}$, $\bar{y}_{F_1}$. As given previously $\bar{x}_{F_1} = 1.3443$ and $\bar{y}_{F_1} = 0.3888$. τmax is the maximum $X_{0_{AJ}}$. This value is 60.5 correspond to fuzzy capital cost (61000, 61300, 61700, 62000). Applying Eq. 23, $Dist(F_1)$ is calculated as given below. See Table 2 for the all defuzzified values.

$$Dist(F_1) = \sqrt{(1.3443-61.5)^2+(0.3888-0)^2} = 60.1569$$

9. For Gani and Mohammed (2013) ranking method, the defuzzified value for F_1 is calculated by applying Eq. 24 to calculate $R(F_1)$. See Table 2 for all the defuzzified values.

$$R(F_1) = \left(\frac{2\times1.2+7\times1.35+7\times1.4+2\times1.45}{18}\right)\left(\frac{7}{18}\right) = 1.36388\times\frac{7}{18} = 0.530$$

Step 2: Appling Eq.(s) 29, 30, and 31 and using Yager 's (1981) ranking method for example gives $PV(F_1) = 1.227$, $\sum_{1}^{t} PV(F_1) = 1.227$ and $PV(S_1) = 45.909$ for the first year.

Step 3: Apply Eq.(s) 32 and 33 to calculate, NPV_1 and AEA_1. These values are 16.818 and 18.5, respectively. All other values of AEA_t are given in Table 3.

Step 2 and 3 are applied to all other ranking methods. Table 3 shows AEA_t for all ranking methods at different years. From this table it is clear that the minimum annual equivalent amount of cost depends on the ranking method. The year correspond to each minimum value is the year of replacement and this year represents the economic life of equipment.

Table 3 shows that eight ranking methods (Yager 1981; Cheng (1998);Chu and Taso(2002); Chen and Chen (2003); Wang et al. (2006) , Dat et al. (2012), Viranloo and Saneifard (2012) , and Gani and Mohamed 2013) give the same result that the loader should be replaced at the end of 7 th year. According to Chen and Chen (2009) ranking method, the loader should be replaced at the end of 8 th.

Second Example

The data of this example was obtained from Biswas and Pramanik (2011b). The fuzzy cost of a machine is US$ (5900, 5950, 6050, 6100). The fuzzy running cost and the salvage value at the end of the year are given in Table 4. The author assumed the interest to be 10% per year. Find when the machine is to be replaced.

Solution: To solve this problem, we take $1000 =1 unit. The same steps applied in solving example 1 are applied here. Table 5 shows AEA_t for all ranking methods at different years. From this table it is clear that the minimum annual equivalent amount of cost depends on ranking method.. The year corresponds to each minimum value is the year of replacement.

Table 5 shows that ranking methods: Cheng (1998); Chu and Taso (2002); Chen and Chen (2009); Wang et al. (2006) , Dat et al. (2012), and Gani and Mohamed (2013) give the same result that the loader should be replaced at the end of 6 th year. Two ranking methods:Chen and Chen (2003) and Viranloo and Saneifard (2012)) give that the loader should be replaced at the end of 5 th year. On the other hand, Yager (1981) ranking method gives that the loader should be replaced at the end of 7 th year.

Remarks on the Results of the Two Examples

Depending on the results obtained from examples 1 and 2, it can be concluded that:

1. Five ranking methods give the same results for the economic life or replacement time for fuzzy replacement problem. These methods are: Cheng (1998),Chu and Taso (2002), Wang et al. (2006), Dat et al. (2012) and Gani and Mohamed (2013). On the other hand, Chen and Chen (2003), Viranloo and Saneifard (2012) ranking methods give the same replacement time.
2. Yager (1981) and Chen and Chen (2009) ranking methods give different results for replacement time.

Conclusions

In this paper, the fuzzy replacement problem is considered in the sense that the capital cost, scrap value, maintenance or running cost are all imprecise in nature represented by fuzzy numbers. It is more realistic and closer to our daily life situation. In modeling of replacement problem it is often observed that the parameters of the problem are not known precisely. This impreciseness is handled by using fuzzy numbers, as it is expected to express the situation more realistically. In this paper, to deal with this uncertainty, fuzzy replacement problem was presented when the capital cost, scrap value, maintenance or running cost are normal trapezoidal fuzzy numbers. An algorithm for the determination of replacement time was proposed.

Nine ranking methods are dealt with through the proposed algorithm. Two numerical examples have been provided to show the effect of ranking method on fuzzy replacement problem. The results revealed that of the nine ranking methods dealt with, five methods gives the same results for the economic life or replacement time for fuzzy replacement problem. These methods are: Cheng 1998, Chu and Taso (2002) Wang et al. 2006, Dat et al. 2012 and Gani and Mohamed 2013. On the other hand, Chen and Chen (2003) and Viranloo and Saneifard (2012) ranking methods give the same replacement time. Yager (1981) and Chen and Chen (2009) ranking methods give different results for replacement time. Replacing the equipment at it's economic life leads to reducing direct cost of construction projects, which intern leads to increasing the probability of winning bids.

In the future research, the author hopes that the proposed algorithm may be used to study fuzzy replacement problem for non normal fuzzy numbers, i.e. for different values of membership function. On the other hand, further research is required to use the defuzzification method (ranking method) for the annual equivalent amount instead applying defuzzification method firstly for maintenance cost, capital cost and scarp value.

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Table1. Fuzzy maintenance cost and salvage value

Year(n)	Maintenance fuzzy cost (F_t)	Salvage Value (S_t)

1	[1200, 1350, 1400, 1450]	[50000,50300,50700,51000]
2	[2500, 2600, 2750, 2900]	[33000,33300,33700,34000]
3	[3500, 3700, 3850,4000]	[22000,22300,22700,23000]
4	[4500, 4650, 4800, 5000]	[15000,15300,15700,16000]
5	[6000, 6500, 6700, 6800]	[10000,10300,10700,11000]
6	[8000, 8200, 8450, 8800]	[7000,7300,7700,8000]
7	[10500, 11000, 12500, 14000]	[5000,5300,5700,6000]
8	[16000, 17000, 18500, 20000]	[3000,3300,3700,4000]

Table 2. Ranking index for capital, maintenance and salvage fuzzy costs (example 1)

Ranking Method	Defuzzified values in different years									
	C	other costs	1	2	3	4	5	6	7	8
Yager (1981)	61.5	F S	1.3500 50.500	2.6125 33.500	3.7625 22.500	4.7375 15.500	6.5000 10.500	8.3625 7.5000	12.000 5.5000	17.875 3.5000
Cheng (1998)	61.502	F S	1.4354 50.502	2.7350 33.502	3.7940 22.502	4.7660 15.502	6.4990 10.502	8.3840 7.502	12.043 5.5020	17.908 3.5020
Chu and Taso (2002)	30.75	F S	0.6764 25.250	1.3425 16.750	1.8824 11.250	2.3679 7.7500	3.2566 5.2500	4.1786 3.7500	5.9744 2.7500	9.2993 1.7500
Chen and Chen (2003)	62.299	F S	2.2952 51.299	3.606 34.299	4.6614 23.299	5.6414 16.299	7.3209 11.299	9.2165 8.299	12.488 6.2988	18.315 4.299
Chen and Chen (2009)	42.718	F S	1.218 35.077	2.287 23.269	3.100 15.628	3.904 10.766	4.794 7.293	6.218 5.209	4.649 3.820	6.500 2.431
Wang et al. (2006)	61.502	F S	1.399 50.502	2.723 33.502	3.783 22.502	4.758 15.502	6.492 10.502	8.379 7.502	12.041 5.502	17.899 3.502
Dat et al. (2012)	60.156	F S	0.259 49.157	1.377 32.157	2.432 21.157	3.407 14.159	5.143 9.161	7.030 6.163	10.693 4.166	16.552 2.176
Viranloo and Saneifard (2012)	0.429	F S	60.157 11.008	58.812 28.003	57.741 39.003	56.762 46.002	55.022 51.002	53.133 54.002	49.469 56.002	43.608 58.002
Gani and Mohamed 2013	23.917	F S	0.530 19.639	1.042 13.028	1.466 8.750	1.839 6.028	2.549 4.083	3.244 2.917	4.613 2.139	6.924 1.361

Table 3. Replacement year (economic life) of equipment for different ranking methods (example1)

Ranking Method	Annual equivalent amount of cost (AEA_t) for different years							
	1	2	3	4	5	6	7	8
Yager (1981)	18.500	21.435	20.431	19.043	18.060	17.329	17.057*	17.351
Cheng (1998)	18.586	21.538	20.512	19.111	18.119	17.375	17.109*	17.399
Chu and Taso (2002)	9.251	10.736*	10.228	9.531	9.039	8.668	8.533	8.709
Chen and Chen (2003)	19.525	22.484	21.457	20.059	19.059	18.310	18.002*	18.256
Chen and Chen (2009)	13.131	15.261	14.598	13.678	12.968	12.458	11.836	11.524*
Wang et al. (2006)	18.549	21.514	20.491	19.093	18.108	17.359	17.096*	17.385
Dat et al. (2012)	18.374	20.141	19.084	17.669	16.669	15.919	15.651*	15.936
Viranloo and Saneifard (2012)	49.621	46.429	47.367	48.735	49.691	50.385	50.667*	50.354
Gani and Mohamed 2013	7.199	8.352	7.957	7.414	7.032	6.744	6.635*	6.745
* the minimum annual equivalent amount of cost								

Table 4. Fuzzy maintenance cost and salvage value (Biswas and Pramanik 2011b).

Year(n)	Maintenance fuzzy cost (F_t)	Salvage Value (S_t)
1	[1100,1150, 1200, 1270]	[3800,3900, 3950, 4000]
2	[1300, 1360, 1400,1450]	[2600, 2650, 2700, 2760]
3	[1500, 1580, 1600, 1650]	[1900, 1950, 2000, 2060]
4	[1800, 1840, 1850, 1870]	[1450,1470, 1480, 1500]
5	[2000, 2030, 2050, 2070]	[950, 980, 1000, 1050]
6	[2350, 2400, 2460, 2500]	[550, 570, 590, 600]
7	[2900, 2920, 2950, 3000]	[500, 530, 540, 550]

Table 5. Replacement year (economic life) of machine for different ranking methods (example 2)

Ranking Method	Annual equivalent amount of cost (AEA_t) for different years						
	1	2	3	4	5	6	7
Yager (1981)	3.926	3.435	3.167	3.032	2.973	2.945	2.926*
Cheng (1998)	3.963	3.541	3.261	3.117	3.048	3.009*	3.041
Chu and Taso (2002)	1.933	1.728	1.591	1.522	1.482	1.473*	1.492
Chen and Chen (2003)	4.924	4.806	4.242	4.103	4.016*	4.019	4.050
Chen and Chen (2009)	3.543	3.168	2.934	2.814	2.782	2.778*	2.794
Wang et al. (2006)	3.938	3.514	3.233	3.091	3.025	3.010*	3.021
Dat et al. (2012)	3.346	2.922	2.639	2.497	2.428	2.398*	2.431
Viranloo and Saneifard (2012)	3.197	3.405	3.607	3.709	3.751*	3.739	3.725
Gani and Mohamed 2013	1.500	1.344	1.238	1.184	1.160	1.153*	1.157
* the minimum annual equivalent amount of cost							