Water Science Magazine, National Water Research Center,
No. 12, October 1992, pp. 10-12

A METHOD FOR DETECTING NO-CHECK OBSERVATIONS IN GPS NETWORKS

By

Eng. Gomaa M. Dawod*

ABSTRACT

The harmful impact of no-check observations in GPS networks is discussed. The concept of redundancy number as an indication of no-check observations is examined. A new method is proposed to detect these observation defects. The suggested algorithm is based on analyzing the least-squares adjustment results and, therefore, it could be easily included in any least-squares adjustment procedure without additional computations.

1. INTRODUCTION

With the high precision of GPS relative positions, effects of no-check observations are critical and must be considered. A no-check observation is that observation which is not checked by other observations in the network. An example of no-check observations is a sideshot observed only once. No-check observations can be thought of as observation defects in an observation campaign, i.e. the number and geometry of the observations are inadequate to estimate uniquely the network parameters.

* Survey Research Institute, National Water Research Center,
308 Al-Ahram street, Talbia, Giza 12111,Egypt

The hazard of this type of observation defects lies in two features: (1) a blunder detection algorithm will fail to detect any gross error in a no-check observation; (2) the uncertainties of the distant point of a no-check observation (the point that is not connected to the network) cannot be judged correctly even though this point might have small standard deviations. Moreover, gross errors in no-check observations will adversely affect the estimated coordinates of the network stations. Consequently, this type of observation defect must be detected and proper actions should be taken to overcome the danger effects of no-check observations. Such a procedure becomes a must in high precision GPS networks as far as the network quality is concerned.

2. IMPLEMENTATION OF REDUNDANCY NUMBERS

The quality of an observation is described by two measures. One is precision (the standard deviation is set up as a measure of precision), and the other is reliability. A redundancy number is used to judge the reliability of the adjustment of individual observations. To define the redundancy number mathematically, the cofactor matrix of the residual, Q_v, can be written as:

$$Q_v = Q_Y - Q_{AY} \tag{1}$$

where Q_Y and Q_{AY} are the cofactor matrices of the observations and the adjusted observations respectively.

Equation (1) could be rewritten as:

$$Q_v = P^{-1} - A(A^T PA)^{-1} A^T \tag{2}$$

where A is the design matrix, and P is the weight matrix.

Multiplying Q_v by P yields the so-called redundancy matrix:

$$R = Q_v P = I - A(A^T PA)^{-1} A^T P \tag{3}$$

The matrix $[A(A^TPA)^{-1} A^T P]$ is an idempotent matrix (Pope 1976) i.e.,

$$[A(A^TPA)^{-1} A^TP] [A(A^TPA)^{-1} A^TP] = [A(A^TPA)^{-1} A^TP] \tag{4}$$

This implies that the redundancy matrix, R, is also idempotent. It is known (Koch 1988) that the trace of an idempotent matrix equals its rank. Therefore, the sum of the diagonal elements of the redundancy matrix can be written as:

$$\Sigma_{i=1}^{n} (r_i) = \text{trace } (R) = \text{rank } (R) = n-m \tag{5}$$

where n is the number of observations, m is the number of estimable parameters, and r_i is the i-th diagonal element of the redundancy matrix, i.e., r_i is the **redundancy number**. Since (n-m) is the degree of freedom, the redundancy number r_i reflects the contribution of the i-th observation to the total redundancy in the network (Milbert 1985).

It can be shown that (Leick 1990)

$$0 \leq r_i \leq 1 \tag{6}$$

Many investigators (e.g. Caspary 1987) find that the redundancy number of a no-check observation equals zero. Therefore, any gross error in this uncontrolled observation will be directly transferred to the estimated parameters. However, a more efficient algorithm for detecting no-check observations can be developed if we include some additional information about possible errors that might be present in the observations.

3. THE SUGGESTED METHOD FOR DETECTING NO-CHECK OBSERVATION

Concerning possible setup errors (those errors associated with setting up antennas relative to ground marks), a no-check observation will not reveal these errors in the residuals. This concept is the key for the proposed algorithm.

The residuals vector, V, can be written as:

$$V = [\ I - A(A^T PA)^{-1}\ A^T\ P\ ]\ Y \tag{7}$$

Instead of using the usual observation equations model

$$Y = A\ X + \varepsilon \tag{8}$$

we will use the model which is often used in hypothesis testing for outliers (Kok 1984):

$$Y = A\ X + B\ \tau + \varepsilon \tag{9}$$

where τ is a vector containing the setup errors assumed to be present in the observations, B is a matrix of derivatives of the observations with respect to the assumed errors such that rank [A B] $\leq$ n, and ε is the vector of random observational errors.

Substituting equation (9) in equation (7) and differentiating w.r.t the setup errors vector, τ, yields:

$$[\delta V/\delta \tau^T] = [\ I - A\ (A^T PA)^{-1}\ A^T\ P\ ]\ B \tag{10}$$

With the concept that a no-check observation is characterized by "uncontrollable" setup errors, we can conclude that the submatrix of $[\delta V/\delta \tau^T]$ corresponding to a no-check observation will be zero.

After detecting no-check observations in a GPS networks, it is recommended that additional baselines are observed to provide some independent checks in this weak part of the network.

4. MERITS OF THE PROPOSED METHOD

The test for detecting no-check observations is carried out based on the computed matrix $[\delta V/\delta\tau^T]$, which is the product of two matrices: the redundancy matrix, R; and the matrix B. This fact gives the algorithm three advantages over the redundancy number concept. First, the test includes some other information, throughout the matrix B, about any possible setup error that might be present in the observations. The second advantage is that the matrix $[\delta V/\delta\tau^T]$ may contain some zero elements even if one of the two matrices (R and B) has non-zero elements on the diagonal. The third advantage of the new method is that $[\delta V/\delta\tau^T]$ reveals the influence of the off-diagonal elements of the redundancy matrix, R. Consequently, the suggested algorithm identifies no-check observations more efficiently.

5. SOME PRACTICAL CONSIDERATIONS

The matrix B, as appeared in equation 10, has 'n by d' size, where n is the number of observations, and d is the number of total possible setup errors which is three times the number of total stations occupations in the GPS campaign. It would not be appropriate to construct B with this relatively huge size. However, the test could be performed with some modifications considering the idea that the presence of setup errors in a GPS station will affect only those baselines that pass by this station and are observed in the same session. Therefore, the test may be carried out session by session and baseline by baseline. For each point in a baseline, n becomes the number of baselines connected to this point in the current session, and the number of possible setup errors, d, is always 3. This procedure reduces the size of the required matrices involved in the test (equation 10) and, hence, facilitate the test from a practical point of view.

6. CONCLUSIONS

No-check observations are due to lack of observations in a GPS observation campaign. This type of observation defects must be detected since no-check observations can dramatically decrease the quality of the results. Based on analyzing the least-squares adjustment results, a new algorithm is developed to detect no-check observations. The method has some advantages over the redundancy number concept as far as the detection of no-check observations is concerned. The main advantage of the proposed algorithm is its simplicity which enables it to be easily integrated in any least-squares adjustment procedures without additional computations. Detecting observation defects could be thought of as an indicator for reliable GPS networks.

REFERENCES

Caspary, W., 1987, *Concepts of networks and deformation analysis,* Monograph No. 11, School of Surveying, University of New South Wales, Australia.

Koch, K-R., 1988, *Parameters estimation and hypothesis testing in linear models*, Springer-Verlag, New York.

Kok, J., 1984, *On data snooping and multiple outlier testing,* NOAA Technical Report NOS NGS 30, U.S. Department of Commerce.

Leick, A., 1990, *GPS Satellite Surveying*, John Wiley & Sons, Inc., New York.

Milbert, D., 1985, A note on observation decorrelation, variance of residuals, and redundancy numbers, *Bulletin Geodesique*, 59, pp. 71-80.

Pope, A., 1976, *The statistics of residuals and the detection of outliers,* NOAA Technical Report NOS 65 NGS 1, U.S. Department of Commerce.