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Fitting gravimetric local and global quasi-geoids to GPS/levelling data: The role of geoid/quasi-geoid variations in Egypt

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Abstract

Recently, the GPS technique has been extensively utilized in surveying and mapping applications in Egypt, which results in a crucial need for a precise geoid model. Traditionally, global and local gravimetric quasi-geoid models are determined and, then, fitted to GPS/leveling data. In the context of precise geoid determination, a quasi-geoid determination has to be converted to a geoid model prior to the fitting process takes place. This transforming step has not been considered in most geodetic literature in Egypt, leading to the fact that the effects of such a correction being absorbed in the fitting residuals that, hence, may produces misleading results and interpretations. The geoid/quasi-geoid separations in Egypt have been computed, based on real geodetic data, and found to range from -0.074 m to 0.367 m, with an average of 0.060 m and a standard deviation equals 0.049 m. Additionally, when the quasi-geoid to geoid conversion is performed, it has been found that the undulation of the EIGEN-CG01C global quasi-geoid model when compared to true undulations of HARN stations, have been decreased, in the mean sense, from 0.52 m to 0.46 m. That means an average improvement of accuracy of about 11%. Based on the obtained results, it can be concluded that the conversion of the quasi-geoid to the geoid is quite significant and it should be a regular step in precise geoid determination in Egypt.

Key Words: Quasi-geoid, Geoid Model, Global Geopotential Model, GPS, Egypt

الملخص العربي:

مع انتشار تطبيقات النظام العالمي لتحديد المواقع في مشروعات المساحة والخرائط في مصر فقد تزايدت الحاجة للحصول علي نموذج دقيق لسطح الجيويد وخاصة لتحويل الارتفاعات الجيوديسية إلي المناسيب المناظرة. تقليديا يتم استنباط نماذج "شبيه الجيويد" باستخدام قياسات الجاذبية الأرضية ثم يتم ضبطها لتتوافق مع نقاط معلوم عندها قيم حيود الجيويد الناتجة من النظام العالمي لتحديد المواقع و الميزانيات الأرضية. لكن يجب قبل إجراء عملية التوافق أن يتم تحويل نموذج شبيه الجيويد إلي نموذج جيويد حقيقي ، وهذه الخطوة لم تؤخذ في الاعتبار في معظم الدراسات الجيوديسية في مصر. بناءا علي قياسات جيوديسية حقيقية عند ١٠٨٠ نقطة فقد تم حساب المسافات بين شبيه الجيويد و الجيويد ووجد أن قيمها تتراوح بين -٠,٠٧٤ متر و ٠,٣٦٧ متر بمتوسط يبلغ ٠,٠٦٠ متر و انحراف معياري يبلغ ٠,٠٤٩ متر. وبعد إجراء هذا التحويل فأن الدقة المتوسطة لأحد نماذج الجيويد العالمية - التي تم اختبارها ووجد أنه أحسن نموذج عالمي لتمثيل مجال الجاذبية في مصر - قد تحسنت من ٠,٥٢ متر إلي ٠,٤٦ متر بتحسن يبلغ ١١% في المتوسط. تدل النتائج التي تم الحصول عليها أن عملية التحويل من نماذج شبيه الجيويد إلي نماذج الجيويد تعد خطوة مؤثرة ويجب أخذها في الاعتبار في أية محاولة مستقبلية لتطوير نموذج جيويد دقيق في مصر.

1. Introduction

Developing a precise nation-wide geoid model becomes a crucial geodetic task in Egypt in recent decades. It is mainly due to the extensive utilization of the Global Positioning System (GPS) technique, and in order to transform the GPS-derived ellipsoidal heights to orthometric heights used in surveying and mapping. Geoid development in Egypt has been addressed by a variety of researchers (e.g. Alnaggar 1986, Saad and Dawod 2002, and Abdelmotaal 2006). The geoid solution of Alnaggar (1986) is considered the pioneering nation-wide (22^{o}N < latitude < 32^{0}N, 25^{o}E < longitude < 37^{o}E) geoid, developed using a least-squares collocation technique and heterogeneous geodetic data including terrestrial gravity, astronomic deflections of the vertical, and Doppler/levelling undulations. A generation of geoid models for Egypt on a five-minute grid has been developed by Saad and Dawod (2002): SRI2001A is a gravimetric geoid computed using local gravity, a local Digital Terrain Model (DTM) and the EGM96 GGM, using a remove-compute-restore Fast Fourier Transform (FFT) processing methodology. SRI2001B is a geoid based on surface-fitting of the SRI2001A model to GPS/levelling (ibid). Abdelmotaal (2006) developed and applied a high-degree tailored reference model by merging the available gravity anomalies with the EGM96 Global Geopotential Model (GGM) in order to obtain better residual gravity anomalies. Moreover, there are other studies that have developed geoid models for particular areas of interest in Egypt (e.g. Tscherning *et al.* 2001a).

Most of gravimetric Egyptian geoids are based on the FFT technique in a remove-compute-restore strategy that utilizes a GGM to represent the long wavelengths of the Earth gravity field. Furthermore, these geoid models are fitted to GPS/levelling data to enhance their integrity and accuracy. A key issue in the utilization of GGMs is that their direct results are the height anomalies (not the geoid undulations) representing a quasi-geoid surface, not the geoid itself. Neglecting the geoid/quasi-geoid separation, among other factors, affect the fitting process between the gravimetric local or global quasi-geoids and the GPS/levelling data (Featherstone 2006). This geoid/quasi-geoid transforming step has not been considered in most geodetic literature in Egypt, leading to the fact that the effects of such a correction being absorbed in the fitting residuals that, hence, may produces misleading results and interpretations. The main objectives of the current research study are: (1) the assessment of the geoid/quasi-geoid separations, based on real data; and (2) investigating the expected increase of accuracy in fitting quasi-geoids to GPS/levelling data towards the development of a precise geoid model for Egypt.

2. Geoid/Quasi-Geoid Separations

The well-known Stocks' integral for computing geoid undulations from gravity anomalies is (Heiskanen and Moritz, 1967, pp. 94):

$$N = (R / 4 \pi G) \iint_{\sigma} \Delta g \, S(\psi) \, d\sigma \qquad (1)$$

where N is the geoid undulation, Δg is the gravity anomaly, R is a mean radius of the earth, $d\sigma$ denotes the surface element of the unite sphere, G s a mean gravity value over the earth, $\iint_{\sigma}$ represents an integral extended over the whole unit sphere, and $S(\psi)$ is the Stoles' function given by (ibid):

$$S(\psi) = [1/(\sin \psi/2)] - 6 \sin(\psi/2) + 1 - 5 \cos \psi \\ - 3 \cos (\psi/2) \ln [(\sin (\psi/2) + \sin^2 (\psi/2))] \qquad (2)$$

where ψ is the spherical distance between the computational point and the running point.

The remove-restore-compute technique separate three different frequency contributions in the gravity anomalies, so that it may be written that (Sideris and Footopoulus, 2005):

$$\Delta g = \Delta g_{FA} + \Delta g_{GGM} + \Delta g_H \quad (3)$$

and

$$N = N_{\Delta g} + N_{GGM} + N_H \quad (4)$$

where Δg_{FA} is the medium wavelength contribution of the local free-air gravity anomalies, Δg_{GGM} is the long wavelength contribution as represented by a GGM model, Δg_H is the short wavelength contribution as represented by the topography or a DTM, $N_{\Delta g}$ is the local gravity medium wavelength component, N_{GGM} is the GGM long wavelength contribution, and N_H is the topography short wavelength component.

The long wavelength component, as represented by a GGM, can be evaluated in the FFT spherical approximation by (ibid):

$$N_{GGM} = R \sum^{n=max}_{n=2} \sum^{n}_{m=0} [C_{nm} \cos m\lambda + S_{nm} \sin m\lambda]\, P_{nm} (\sin \phi) \quad (5)$$

where C_{nm} and S_{nm} are the fully normalized harmonic coefficients, P_{nm} is the fully normalized associated Legendre polynomial, n and m are the degree and order of the GGM model respectively, and ϕ and λ are the geocentric latitude and longitude respectively.

GGMs, when used in a spherical harmonic expansion, produce quasigeoid (sometimes called co-geoid) not geoid solutions since the processing yields height anomalies not geoid undulations (e.g. Featherstone 2006, Tscherning et al, 2001b, and Ivan, 1998). Consequently, the conversion of the obtained quasi-geoid (height anomaly) to the geoid (geoidal undulation) is a regular step in precise geoid determination worldwide (e.g. Denker 1998, Iliffe et al., 2003, Preijatna, 2004, and Featherstone, et al, 2007). Neglecting this procedure, among other factors, affect the fitting process between the gravimetric local or global quasi-geoids and the GPS/levelling data (Featherstone 2006).

Several researchers have considered this issue and have presented solutions to convert height anomalies to geoid heights. Rapp (1997) suggested that potential coefficient models be used first to calculate a height anomaly and then that a correction term, represented by a high degree spherical harmonic expansion, be applied to give the geoid undulation. Heiskanen and Moritz (1967, p.326, Eq. 8-100) provide a formula for computing the geoid/quasi-geoid separation as:

$$N = \xi + [(g^{-} - \gamma^{-}) / \gamma^{-}] H \qquad (6)$$

where: N is the geoid undulations, ξ is the height anomaly or undulations of quasi-geoid, g^{-} is the mean gravity along the plumb line between geoid and ground, γ^{-} is the mean normal gravity along the normal plumb line between ellipsoid and telleroid, and H is the orthometric height

For an average density = 2.67 g/cm^3, g^{-} can be computed as (ibid, Eq. 4-24):

$$g^{-} = g \text{ (in gal)} + 0.0424 \, H \text{ (in Km)} \qquad (7)$$

where g is the measured surface gravity. Additionally, the quantity γ^{-} may be replaced by a constant average value (ibid, pp.327).

The maximum geoid/quasi-geoid separation (N-ξ) can be up to 2 m (Tenzer et al, 2005). In the Mt. Blance area in the Alps, where the height equals 4877 m, this separation equals 1.8 m (Heiskanen and Moritz,1967, p.328). Consequently, applying such a transformation is crucial in the context of developing a precise geoid model for Egypt.

3. Available Data

The local geodetic data used in this study was held in two databases (Table 1). Firstly, terrestrial gravity values have been measured at 1138 points (Figure 1). The date of these observations and their accuracy vary greatly between the most recent Egyptian National Gravity Standardization Network of 1997 (ENGSN97) (Dawod 1998) and older gravity surveys carried out several decades ago. The accuracy of ENGSN97 gravity values is 0.022 mGal, while the accuracy estimate for older gravity data is 0.5 mGal on average (Dawod and Alnaggar 2000). As can be seen from Figure 1, gravimetric data distribution is not homogeneous over Egypt, with significant gaps, particularly in the Eastern and Western deserts (ibid). An evaluation of the existing gravity data, based on comparing the gravity value of each point to values at the nearest four stations, was carried out to identify any large discrepancies. This process identified 58 points (about 5%) as suspected outliers and these were subsequently removed. The statistics of the remaining 1080 gravity stations are presented in Table 2, where the observed gravity values range from 978588.29 mGal to 979509.69 mGal, with an average equals 979089.18 mGal and a standard deviations of 159.68 mGal. The orthometric heights of these stations vary from -253.23 m to 1433.52 m, with a mean value of 169.34 m and a standard deviations equals 142.26 m.

Table 1: Available local geodetic data

Data Type	No. of points
Terrestrial Gravity: 1- ENGSN97 Network 2- Older gravity points	 150 988
GPS/levelling HARN	15

Table 2: Statistics of the available 1080 gravity stations

Item	Min	Max	Mean	RMS
Observed Gravity (mGal)	978588.29	979509.69	979089.18	159.68
Orthometric Heights (m)	-253.23	1433.52	169.34	142.26

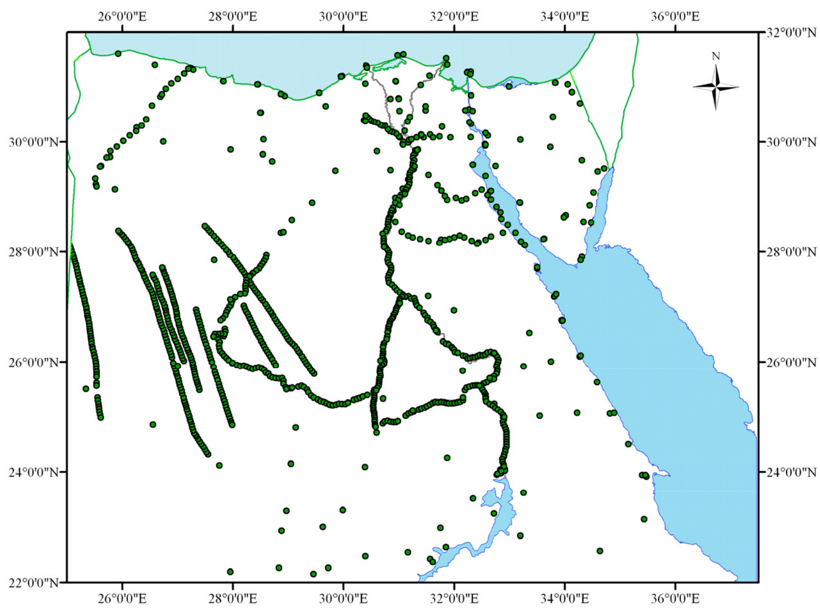

Figure 1: Available local gravity stations

The second compiled geodetic data base consists of 15 GPS/levelling points (Figure 2). This dataset belongs to the High Accuracy Reference Network (HARN), which furnishes the national Egyptian GPS datum established by the Egyptian Survey Authority (ESA) in 1995. Although the HARN network consists of 30 stations, only those 15 points have observed orthometric heights, and consequently true geoid undulations. The statistics of the geoid undulations over these stations are presented in Table 3, where the orthometric heights range from 11.76 m to 556.42 m, with an average equals 146.49 m and a standard deviations of 150.70 m. The geoid undulations of these stations vary from 10.86 m to 21.14 m, with a mean value of 15.47 m and a standard deviations equals 3.57 m.

Table 3: Statistics of the available 15 GPS/Levelling stations

Item	Min	Max	Mean	RMS
Orthometric Heights (m)	11.76	556.42	146.49	150.70
True Undulations (m)	10.86	21.14	15.47	3.57

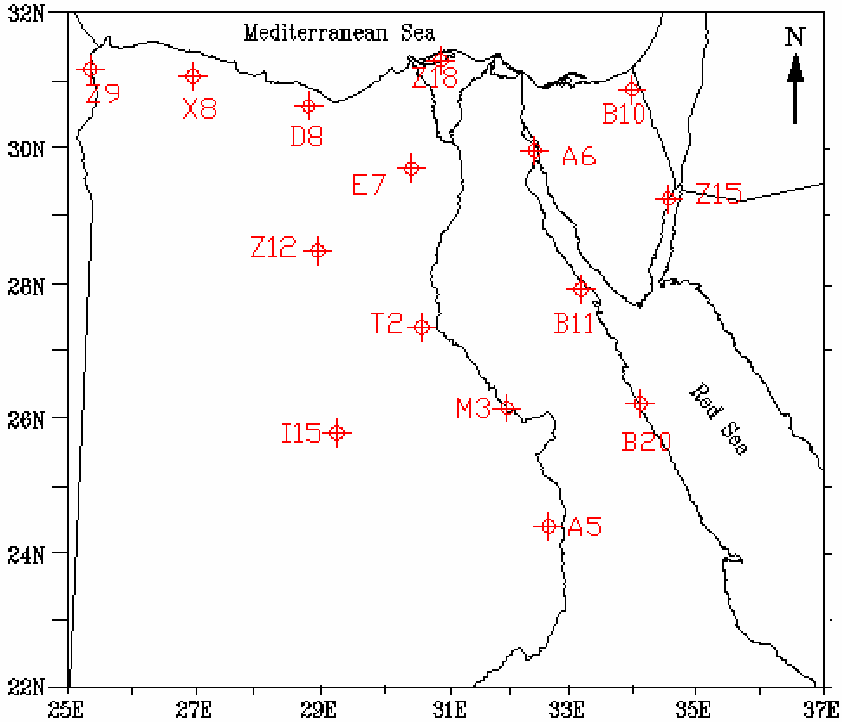

Figure 2: Available stations with known geoid undulations

4. Processing and Results

The main objectives of the current research study in the assessment of the geoid/quasi-geoid separations, and investigating the expected increase of accuracy in fitting quasi-geoids to GPS/levelling data towards the development of a precise geoid model for Egypt Therefore, the data processing involves two steps.

Firstly, the geoid/quasi-geoid separations, through equations 6 and 7, have been computed at the available known 1080 gravity stations. The attained results are presented in Table 4, and a contour map is depicted in Figure 3. It can be seen that the geoid/quasi-geoid separations range from -0.074 m to 0.367 m, with an average of 0.060 m and a standard deviation equals 0.049 m. That means that such a correction to the gravimetric quasi-geoids can reach up to 37 centimeters at mountainous areas in Egypt (actually this value occurred at Saint Katreen area, south of Sinai peninsula). Moreover, although the topography of Egypt is fairly flat in general, the geoid/quasi-geoid correction has an average value of 6 centimeters. This value is quit significant when a precise geoid model is concerned.

Table 4: Statistics of geoid/quasi-geoid separations at known 1080 gravity stations (m)

Min	Max	Mean	RMS
-0.074	0.367	0.060	0.049

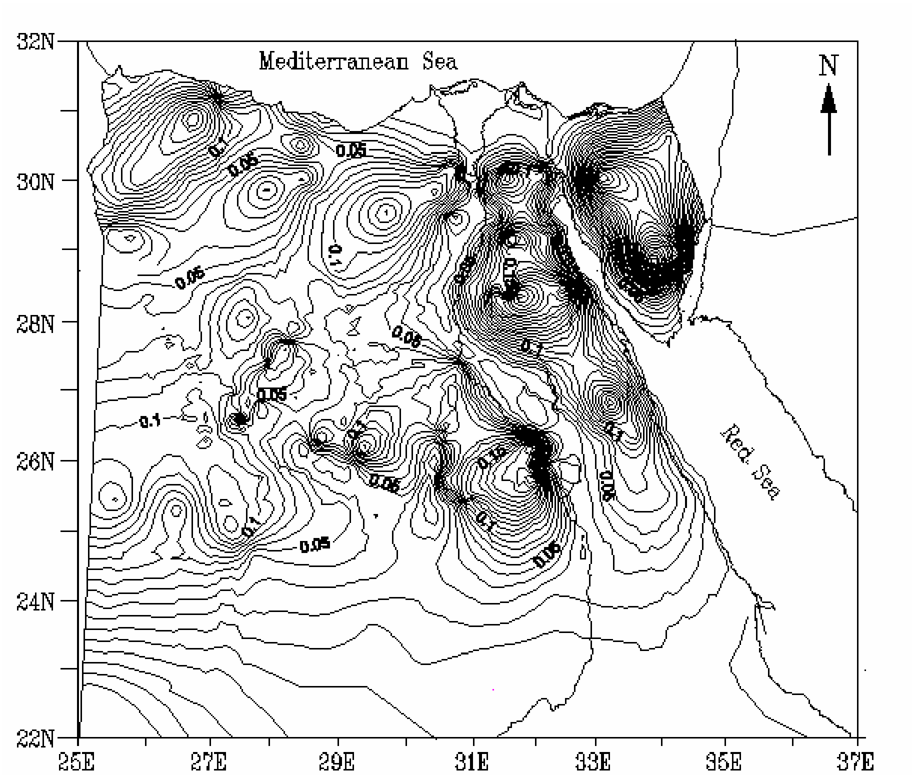

Figure 3: Geoid/quasi-geoid separation contour map

The second step in the data processing is the assessment of the expected accuracy of fitting quasi-geoids to GPS/levelling data. It should be emphasize that neglecting the geoid/quasi-geoid separations will be absorbed in the residuals of the fitting process, which causes misleading results. Dawod (2008) concluded that, out of eight tested recent GGMs, the EIGEN-CG01C (Reigber et al., 2006) model is best at representing the long and medium wavelengths of the gravity field in Egypt. Therefore, the height anomaly at the known 15 HARN GPS/levelling stations have been computed using the Secondly, the program SHS360 (Spherical Harmonic Solution to degree and order 360 computer program (Tscherning 1976). The corresponding geoid/quasi-geoid separations at these stations have been interpolated from the contour map. Furthermore, the geoid undulations have been computed by adding the geoid/quasi-geoid separations to the EIGEN-CG01C height anomalies. Two comparisons, of both height anomalies and GGM-based geoid undulations against known geoid undulations, have been carried out. The achieved results are tabulated in Table 5.

It can be seen that the height anomalies of the EIGEN-CG01C GGM range from 10.98 m to 20.26 m with a mean value equals 14.95 m and a standard deviation of 2.43 m. When compared against true geoid undulations, the differences vary from -4.82 m to 4.86 m, with an average of 0.52 m and a standard deviation equals 2.42 m. The interpolated geoid/quasi-geoid separations at those stations range from 0.002 m to 0.16 m with a mean value equals 0.06 m and a standard deviation of 0.05 m. After the quasi-geoid to geoid conversion, the corresponding EIGEN-CG01C-based geoid undulations vary from 11.00 m to 20.28 m with a mean value of 15.01 m and a standard deviation equals 2.42 m. The computed undulation differences, between the GGM and true values, now have a minimum value of -4.84 m and a maximum value of 4.86 m, with an average of 0.46 m and a standard deviation equals 2.41 m. Consequently, the mean difference value of geoid undulation, of the EIGEN-CG01C GGM, has been decreased from 0.52 m to 0.46 m, which means an average improvement of accuracy of 11% approximately. However, it should be realized that the utilized known HARN stations exist mostly at flat or moderate-topography areas, and it is expected that the attained findings may be greatly changed for mountainous areas in Egypt.

Table 5: Statistics of height anomaly and geoid undulations at the known 15 GPS/Levelling stations (m)

Item	Min	Max	Mean	RMS
EIGEN-CG01C height anomaly	10.98	20.26	14.95	2.43
True geoid undulation – EIGEN-CG01C height anomaly	-4.82	4.86	0.52	2.42
Geoid/quasi-geoid separation	0.002	0.16	0.06	0.05
EIGEN-CG01C geoid undulation	11.00	20.28	15.01	2.42
True geoid undulation – EIGEN-CG01C geoid undulation	-4.84	4.86	0.46	2.41

5. Summary and Conclusions

The extensive utilization of GPS in surveying and mapping applications in Egypt has necessitated the development of a precise geoid model. Global and local gravimetric geoid determinations utilize GGMs, in the FFT spherical approximation processing, produce height anomalies represent a quasi-geoid surface, not the geoid itself. The geoid/quasi-geoid transforming step has not been considered in most geodetic literature in Egypt, leading to the fact that the effects of such a correction being absorbed in the fitting residuals that, hence, may produces misleading results and interpretations. In order to evaluate the geoid/quasi-geoid separation in Egypt, 1080 gravity stations have been utilized. The orthometric height of these stations, which is the key factor in geoid/quasi-geoid conversion, vary from -253.23 m to 1433.52 m, with a mean value of 169.34 m. The attained results show that the values of the quasigeoid-to-geoid transformation range from -0.074 m to 0.367 m, with an average of 0.060 m and a standard deviation equals 0.049 m. Hence, it can be concluded that this conversion step is substantial when a precise Egyptian geoid model is concerned.

The second utilized dataset consists of 15 HARN GPS/levelling points, whose true geoid undulations are known. This dataset has been used to investigate the expected accuracy of fitting a global quasi-geoid GGM model to GPS/levelling points. The EIGEN-CG01C has been found to be the best GGMs at representing the long and medium wavelengths of the gravity field in Egypt (Dawod, 2008). The EIGEN-CG01C height anomaly at those stations have been computed and the corresponding geoid/quasi-geoid separations have been interpolated. Two comparisons, of both GGM-based height anomalies and geoid undulations against known geoid undulations, have been carried out. The results show that when compared the quasi-geoid GGM against true geoid undulations, the differences have an average of 0.52 m. After the quasi-geoid to geoid conversion, the corresponding undulation differences have been decreased, in the mean sense, to 0.46 m, which means an average improvement of accuracy of 11% approximately. Nevertheless, it should be recognized that the exploited HARN stations exist generally at flat or moderate-topography areas, and it is anticipated that the accomplished findings may be greatly changed for mountainous areas. Based on the obtained results, it can be concluded that the conversion of the quasi-geoid to the geoid is quite significant and it should be a regular step in precise geoid determination in Egypt.

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