THE SIMPLE POLYNOMAIL TECHNIQUE TO COMPUTE THE ORTHOMETRIC HEIGHT IN EGYPT

M.Kaloop[1], M. EL-Mowafi [2], M.Rabah[3]
[1] Assistant lecturer, Public Works Engineering Department, Faculty of Engineering, El-Mansoura University, EGYPT,2008
Email:razankaloop@yahoo.com
[2] Assistant Professor, Public Works Engineering Department, Faculty of Engineering, El-Mansoura University, EGYPT,2008
Email: mmewafi1@excite.com
[3] Assistant professor, Geodynamic Department N. R. I. Helwan., EGYPT,2008
Email: mostafa_rabah@yahoo.com

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Abstract

The first main motivation of this thesis is using the GPS data to compute the orthometric height by simple polynomial techniques and filter the data by least square method using Matlab program. Control vertical networks consisting of ellipsoidal, orthometric and geoid height data are investigated. Although the theoretical relationship between these height types is simple in nature discrepancies among the combined height data, its practical implementation has proven to be quite challenging due to numerous factors that cause, with particular emphasis on (i) modeling a technique to compute the geoid undulation from GPS and precise leveling, (ii) correcting the observed geoid undulation by least square method.

To address these challenges, a general procedure involving empirical and statistical tests for assessing the performance of selected parametric models is developed. Additional numerical studies include the obtained of geoid models (local geoid), scaling the GPS-derived ellipsoidal height matrix, and evaluating the orthometric heights obtained from national/regional adjustments of leveling data. Finally, the used technique with special mathematical and adjustment models gives good results for the middle and east of Egypt.

Introduction

The geoidal undulation can be defined as the separation of the reference ellipsoid with the geoid surface measured along the normal ellipsoid as shown in fig.1. The combined use of GPS, leveling, and geoid height information has been used as key procedure in various geodetic applications. Although these three types of height information are considerably different in terms of physical meaning, reference surface definition, observational methods, accuracy, etc., they should fulfill the simple geometrical relationship [7] is:

$$N = h - H \tag{1}$$

Where: N is the geoid undulation, h is the ellipsoidal height, H is the orthometric height.

The GPS technique has benefits of high accuracy and simultaneous 3-D positioning in Geodetic aims, however, GPS derived ellipsoidal heights must be transformed to orthometric heights) to have any physical meaning in a surveying or engineering applications.

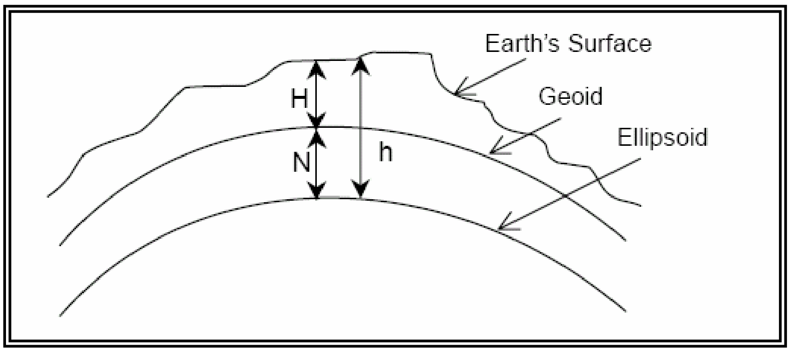


Fig. 1 Orthometric, geoid, ellipsoid height

Some previous trials for orthometric height determination have been conducted, for small areas in Egypt, by many researchers, depending on utilizing local geoid models, or global geopotential model. In the first test, conducted by Baraka & Eman [1991], the field surveys were performed by Egyptian Survey Authority (ESA) and Survey Research Institute (SRI). 28 GPS stations were observed, from which a subset of 14 stations were known to be the first order vertical control GPS stations, and cover an area of 72 x 72 km. All such GPS stations have both orthometric and ellipsoidal heights. The used geoid is the geopotential model developed by the National Geodetic Survey (NGS) of order 360, with relative accuracy of 2-3 PPM, for points separated by 10 km. the results of this test notified that error in orthometric height from about 12 cm to 37 cm was reached, on absolute basis [1]. Concerning the second test, conducted by Shaker et al. [1996], two test areas were chosen. The first one was in Helwan (12 stations), and the second one was in Al-Abour city (7 stations). In areas, spirit leveling and GPS measurements were conducted. The geoid was determined by two different ways, the first one was through geometric satellite technique, while the other one is gravimetric geoid. The results, in terms of accuracy, declared that the leveling still yields better results and the GPS-geoid method supplied an accuracy of orthometric heights, in absolute sense, from 0.891 m to 0.899 m. These results were improved when using the relative height difference approach, 1.3 cm to 1.2 cm, when one station with known ellipsoidal, orthometric height and geoidal undulation, was employed as a reference [1].

Methodology and Models

The multiple regression equation (MRE) is the mathematical technique used for solution some problems in all branches of science. This traditional technique only accommodates coordinate transformations relating to two datum's. In many instances, particularly for many classical local datum's, there are known datum and can be changed into realized unknown datum. For example, the ellipsoidal surface is known datum and the geoid surface is unknown datum.

Various methods have been proposed to address this problem, and one of the most popular is the multiple regression formula. In this branch, the best technique used are the polynomial techniques, that given best solution in some searches for this problem and others. In simple terms, they are polynomial functions which represent the variations, as a function of position, of the difference of latitude, longitude and height (or X, Y and Z coordinates) [3].
Depending on the degree of variability in the distributions, approximation may be carried out using 2^{nd}, 3^{rd}, higher degree polynomials. In the case of geoid undulation, you can use any degree that makes limitation from the less distortion in the check points. For example in Turkey, the fifth degree is given the best solution [4].
Polynomial approximation functions themselves are subjected to variations, as different approximation characteristics may be achieved by different polynomial functions. The simplest of all polynomials is the general polynomial function [5].
The polynomial technique can be classified into two models, the first is a real number polynomial model and the second is a complex number polynomial model.

The first model is the general model, the formula is:

$$N=A_0+A_1U+A_2V+A_3U^2+A_4UV+A_5V^2+\ldots\ldots\ldots\ldots A_{nn}U^nV^n \tag{2}$$

Where $A_0,\ldots, Ann$ the coefficients , N the geoid undulation, U,V the available data

This model is using in most researches with mean value which used U and V relative to central evaluation points.

Polynomial Technique Program

The mathematical formula of the polynomial model for general case is outlined in Eq. (2) [4]:

$$N = \sum_{i=0}^{n} \sum_{j=0}^{i} A_{ij} * \lambda^{i-j} * \phi^{j} \tag{3}$$

Where: **N** the geoid undulation, $\mathbf{A_{ij}}$ the parameter, **Φ** geodetic latitude, **λ** geodetic longitude, **n** the degree of polynomial model.

By considering the coordinates (Φ,λ,h) as an observation equation for every data point, so a system of equations can be written for every point, hence we will have n equations for n points where the parameters (A_{00}, A_{10}, A_{11}, A_{20}, A_{21}, A_{22}, A_{30},………A_{nn}) are the unknowns. Thus by using the Least Square Technique, this system of equations can be solved, where the above system can be reconstructed in a matrix form as follows [3].

$$A_{(n,n)}V_{(n,1)}+B_{(n,m)}\Delta_{(m,1)}=F_{(n,1)} \tag{4}$$

Where **A** is the coefficients matrix of residuals, **V is** Vector of residuals, **B** is Design matrix of parameters, Δ **is** Vector of parameters, **F** is Vector of constants, **n** is Number of observations = Number of equations , **m** is Number of parameters = Number of unknowns

The degree of freedom "r" can be calculated by **r = n-m**. The aforementioned algorithm is used by MATLAB to find the required solution of the above system of equation by LSA. In addition, the developed program will compute the distortion between the check points (known point from the S.Powell report) and the output values from the program. The steps of program as following:

1-Inputting the known data (Φ, λ, N) from the S.Powell report where the coordinates of points are WGS84 geodetic coordinate.
2-Building A matrix,
3-Calculating B matrix.
4-Calculating F matrix
5-Building Covariance matrix and calculating the equivalent weight matrix (We).
6- Calculating least square matrices operations using general least square algorithm.
7-Running the Fisher test at 98% confidence interval.
8-Inputting the check points from the S.Powell report.
9-Calculating the geoid undulation by the polynomial method as well as the distortion between known geoid undulation and the calculate geoid undulation.

In the above program module, two trials are made; the first trial is done by using the coordinates of points as outlined above. While the other is using the average of the coordinates of points according to the following equation:

$$N = \sum_{i=0}^{n} \sum_{j=0}^{i} A_{ij} * (\lambda - \lambda_o)^{i-j} * (\phi - \phi_o)^{j} \tag{5}$$

Where $\boldsymbol{\Phi_o}$ is The mean of the geodetic Latitude for known points, $\boldsymbol{\lambda_o}$ is The mean of the geodetic longitude for known points.

The only difference between the second trial and the first trial is that: the input known data are (Φ, λ, N) from the S.Powell report, where the coordinates of points are WGS84 geodetic coordinate. Then the mean values of the ellipsoidal coordinates are calculated, and the rest of the program is done as indicated in the first trials.

Alternative solution can also be made by using the Cartesian coordinates according to the next form of equation:

$$N = \sum_{i=0}^{n} \sum_{j=0}^{i} A_{ij} * X^{i-j} * Y^{j} \tag{6}$$

Where: the (***X, Y***) is the Cartesian coordinates for the known points which are obtained by converting the geodetic coordinates to Cartesian coordinates. If they are not available in the Cartesian format. By solving it by the same program module by changing the input data by (X,Y,Z). As alternative solution, the mean value of the Cartesian coordinates can be used according to the following equation:

$$N = \sum_{i=0}^{n} \sum_{j=0}^{i} A_{ij} * (X - X_o)^{i-j} * (Y - Y_o)^{j} \tag{7}$$

Where: the (X_0,Y_0) is the mean of the Cartesian coordinates for known points and the most solution steps are still the same only the following steps are updated:
1. Inputting the known data (Φ, λ, N) from the S.Powell report where the coordinates of points are WGS84 geodetic coordinate.
2-Converting the geodetic to Cartesian coordinates.
3- Calculating the mean of the Cartesian coordinates. The conclusion of the trails as shown fig. 3

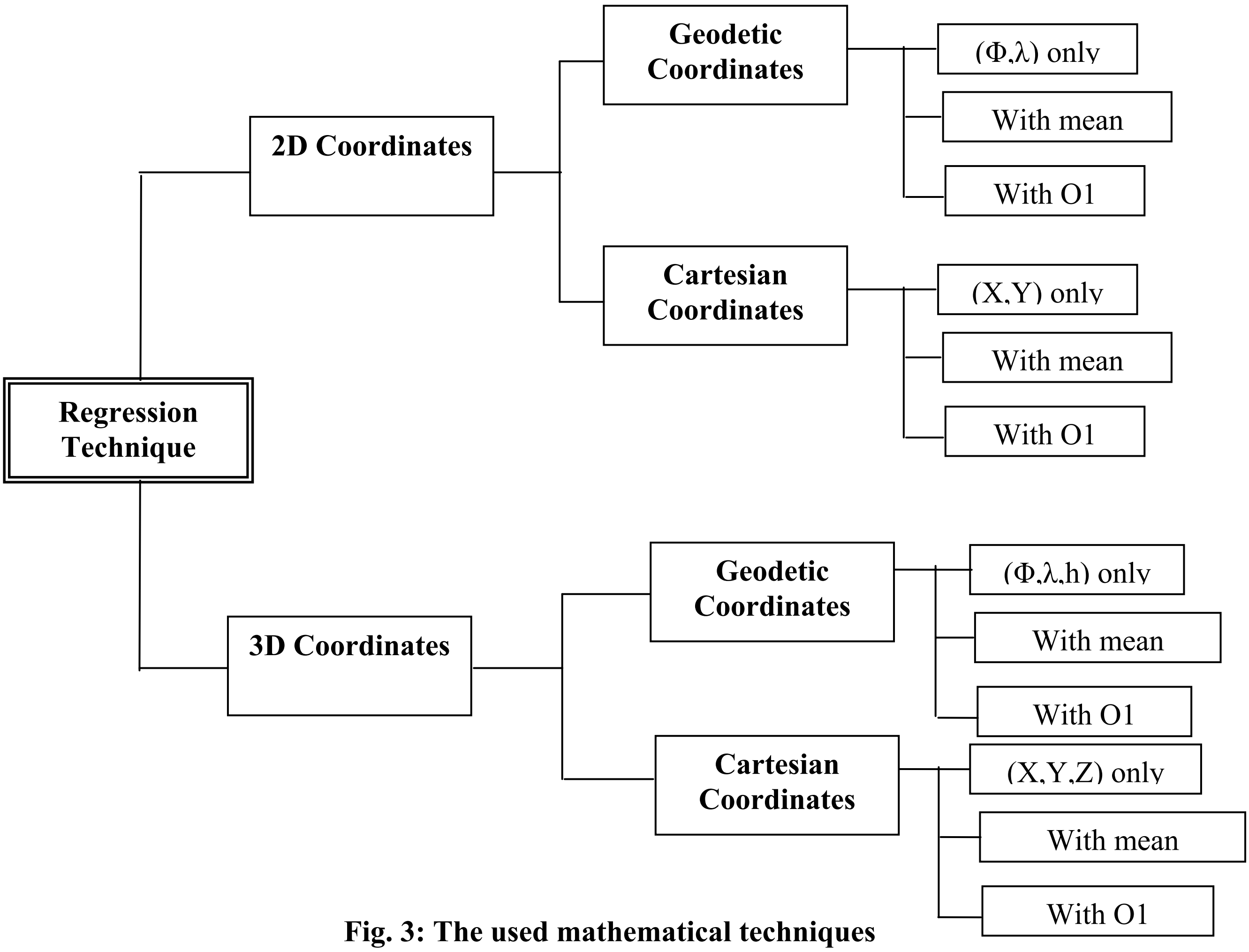


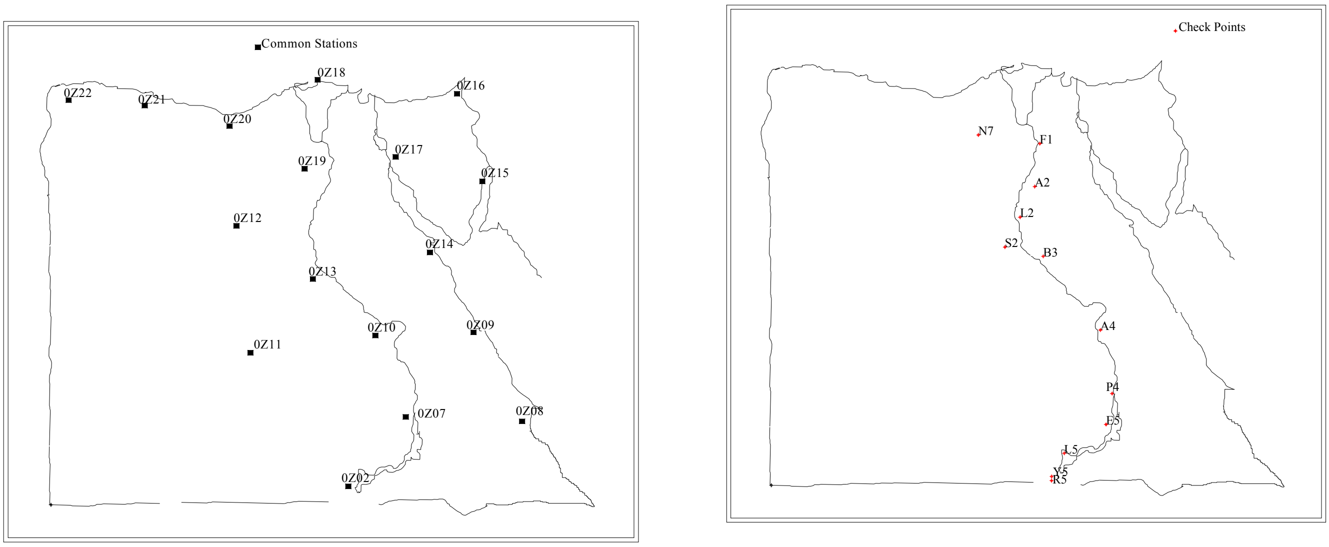
Fig. 3: The used mathematical techniques

Results and Discussion

Actually, the full results are shown in M. Sc. thesis [8]. The important and vital results are displayed. The next part is devoted to illustrate the applicable regression models which estimate the geoid undulation parameters for the studying area (Egypt). Regression models can be classified into two main parts: first regression model is in two dimensions and the second regression model is in three dimensions. In the first and second regression models, Cartesian and the Geodetic coordinates are applied. Using the mean value of the data available and, point O1 (Φ = 29.85936392, λ = 31.34369133, h = 135.2113) on the WGS84 datum, to be the mean value of observations for Egypt, which were applied in the two regressions models. The value of coefficients can be estimated by using four linear regressions models (first, second, third and fourth) using the common points at WGS84 datum. The used common points and check points are extracted from HARN network which mentioned in the final report of the new adjusted national geodetic network [6] as shown in Fig. 4.

On the other hand, the geoid undulation is extracted from the report of (6), the common points (oz2, oz7, oz8,oz9 ,oz10,oz11,oz12 ,oz13,oz14,oz15oz16, oz17,oz18,oz19,oz20, oz21,oz22) and check points (oy27,oy35,oy36,oy41,oz32,oz44,oz52,oz66,oz68,oz70,oz74,oz97).

Seventeen of these points were taken as modeling pins (common points) with known geoid undulation. These points are distributed as shown in fig. 4.a, and the rest 12 points were chosen for testing the model (check points) as shown in fig. 4.b. While choosing these test points, the homogenous distribution and topographic properties were considered and the availability of the data. The geographic coordinates including heights (h) are identified in WGS84 datum and the practical heights (H) are identified from geometric leveling in the datum of Egypt, OED-30 datum.



(a) Common points. **(b) Check points.**

Fig. 4: The distribution of the used common and check points

In general the solution divided into two trials, the first trial used a technique to determine the geoid undulation by a linear multiple regression model in two dimensions for the geodetic coordinates of the points, and the second is to determine it by Cartesian coordinates. To obtain the Cartesian coordinates, we should convert the geodetic coordinates of points to the Cartesian coordinates because all available data are geodetic coordinates. The second trials use the mean value of known data and point (O1) as an average point of Egypt.

Two Dimensional polynomial techniques with plane geodetic coordinates

The used polynomial model in the regression solution is shown as Eq. (3). Trial uses the geodetic coordinate in two dimensions (Φ,λ) only and uses it in the polynomial model (first, second, third and fourth) degree. In the second trial, it uses the mean value of longitude and latitude in the same equation of polynomial as Eq. (5). Also using the point O1 is equally the mean value in the Eq. (5). The results of the used polynomial model with least square method are shown in table 1.

Degree	Test	Max.dist.	Min.dist.	S.D.dist.	Average dist.
First Degree	Φ,λ only	1.265862723	0.067587077	0.356054367	0.620689001
	With mean	1.265862723	0.067587077	0.356054367	0.620689001
	With O1	1.265862723	0.067587077	0.356054367	0.620689001
Sec. Degree	Φ,λ only	0.706943399	0.058985627	0.232491278	0.303259486
	With mean	0.850853984	0.066258689	0.266230491	0.300319663
	With O1	0.850853984	0.066258689	0.266230491	0.300319663
Third Degree	Φ,λ only	0.788438563	0.066823542	0.249446916	0.328392631
	With mean	0.784895727	0.071875784	0.247107709	0.327490099
	With O1	0.784895727	0.071875784	0.247107709	0.327490099
Fourth Degree	Φ,λ only	261.9146256	5.591150034	95.85310137	136.1270782
	With mean	1.076674039	0.058514803	0.345641922	0.393025862
	With O1	1.076674039	0.058514803	0.345641922	0.393025862

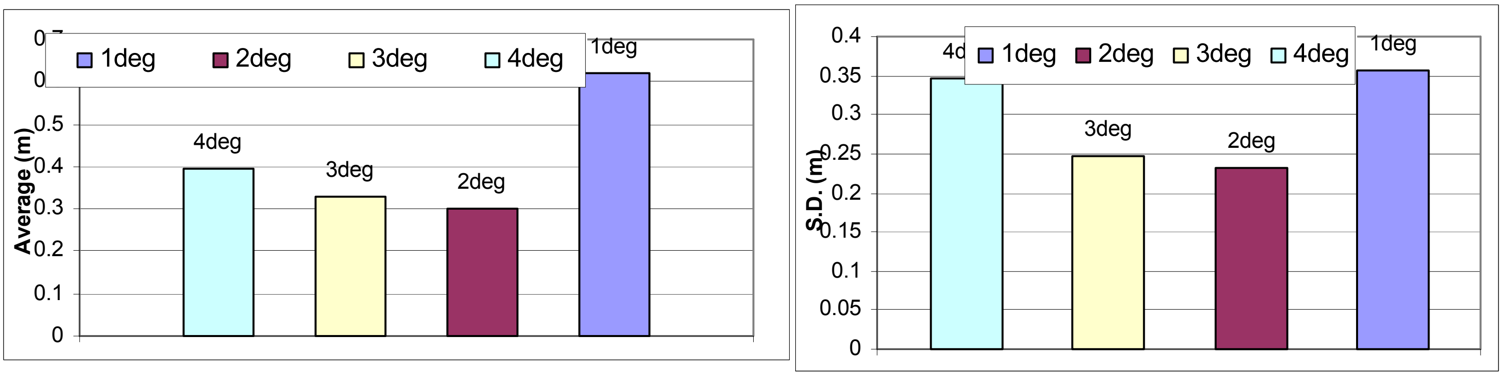
Table 1: Comparison of various polynomial degrees (Φ,λ). Unit (m)

From table 1, it is obvious that:-

*The first degree: all trials (Φ,λ only, With mean and With O1) have the same maximum, minimum, average distortion and standard deviation of distortion.

* The second degree of solution: the (Φ,λ only) is the best solution because , it is less than distortion and standard deviation for other.

* In the third degree the method of using (with mean, with O1) showed the best solution in comparison with first and second and the same results were obtained for fourth degree.
*The results clearly show that by using the points O1, for the mean of Egypt, gave the same values of distortion when using the mean value of data point. The comparison between the average of distortions of the best models and the standard deviation of the best model is illustrated in fig. 5.



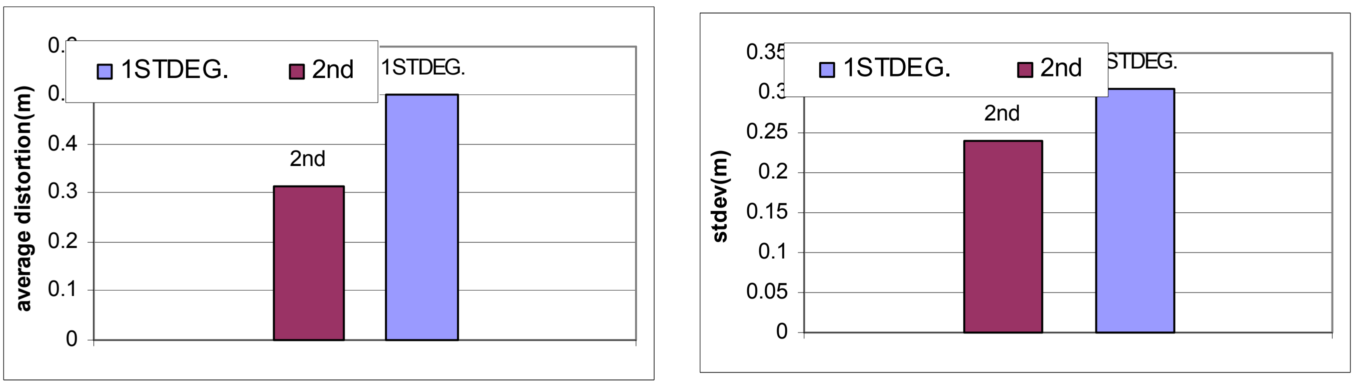
(a) Average distortion **(b) Standard deviation of distortion**

Fig. 5: The average distortion and standard deviation at check points by (Φ,λ) polynomial technique

From the obtained figures, the second degree (Φ,λ only) of polynomial method is the best solution, because it gives the minimum distortion and high accuracy as compared with the other degrees of solutions, and gives the best value of geoid undulation in Egypt.

Two Dimensional polynomial techniques with plane Cartesian coordinates

This technique is done by using Cartesian coordinates with applying (X, Y) coordinate of common and check points. Using the polynomial regression model is as Eq. (6). The Cartesian coordinates in two dimensions ((X,Y) only) are used in the polynomial model (first, second, third and fourth) degree as a first trial. In the second trial, the mean values of X and Y were used in the same equation of polynomial method as Eq. (7). Also using the point O1 is equally the mean value the Eq. (7). The results of the used polynomial model with least square method are shown in fig. 6.
The average of distortions of the best models and the standard deviation of the best models is illustrated in fig. 6. In these figures, the third and fourth degrees are neglected, because the distortion of it is greater than other method.



(a) Average distortion **(b) Standard deviation of distortion**

Fig. 6: The average distortion and standard deviation at check points by(X,Y) polynomial technique

From the obtained figures, the second degree with mean point O1 is the best solution of this method, because it can give the minimum distortion and high accuracy than the other degrees.

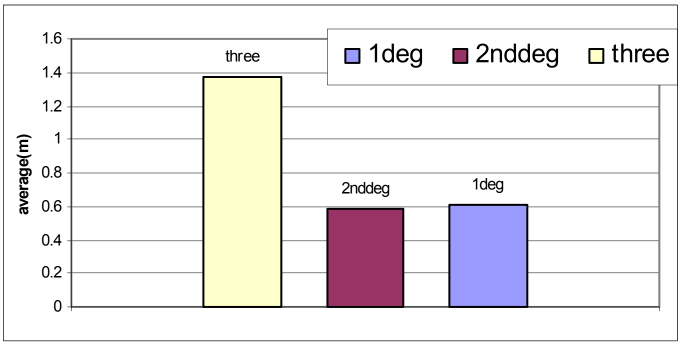
The three dimensions polynomial technique

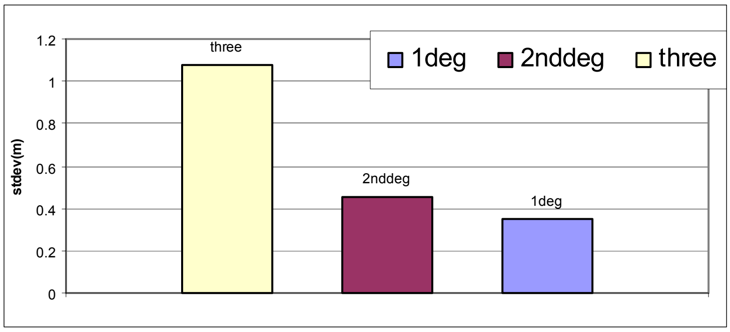
The second trial uses the three dimensions with polynomial and least square technique. The geodetic and Cartesian coordinates are used to compute the geoid undulation by regression methods (first, second and third degree). This trial used the coordinates directly, the coordinates with mean values and the coordinates with the mean of O1 point.

The three dimensions polynomial with geodetic coordinates

Firstly, this trial uses the geodetic coordinate (longitude, latitude and height (Φ, λ, h)).

The comparison between the best solutions obtained from the three degrees, the average of this distortion and the standard deviation of distortion is shown in fig. 7.





(a) Average distortion **(b) Standard deviation of distortion**

Fig. 7: The average distortion and standard deviation at check points by (Φ,λ,h) polynomial technique

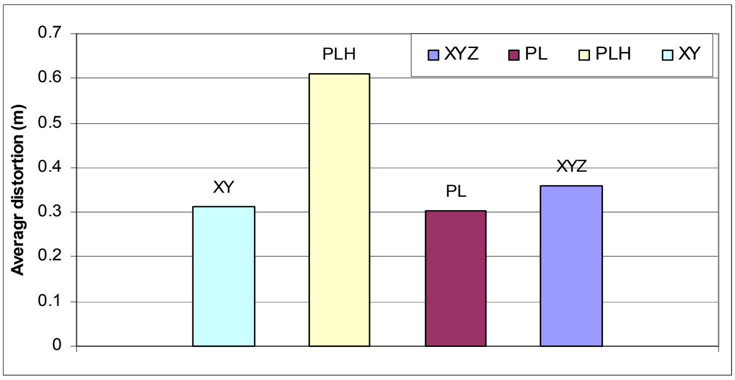
From the fig. 7 clearly show that:

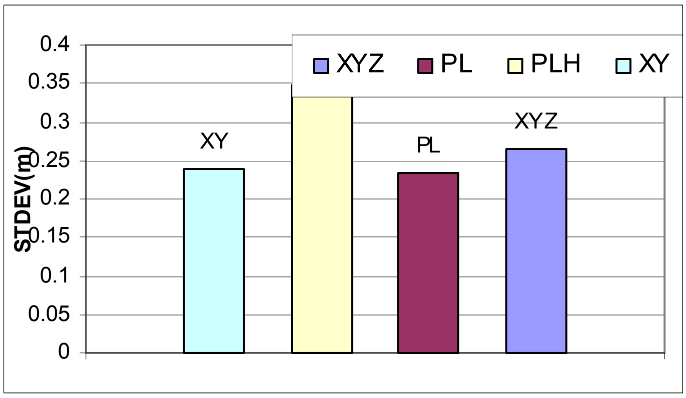
- The second degree with three dimensions is the best solution.
- In general the ellipsoidal height (h) in this models, gives high distortion due to the errors in the vertical coordinate high.

The three dimensions polynomial with Cartesian coordinates

As the same trial uses the Cartesian coordinates (X,Y,Z) that converted from known geodetic coordinate by MATLABE program. This trial applied the Cartesian coordinates directly, the coordinates with the mean values and the coordinates with the mean of O1 point.

Comparing the best solutions obtained from the all degrees, the average of distortion and the standard deviation of distortion is shown in fig. 8.





(a) Average distortion **(b) Standard deviation of distortion**

Fig. 8: The average distortion and standard deviation at check points by polynomial technique

The best solution in this case is the second degree in the plane coordinate with geodetic coordinate with (Φ,λ only). This best model can be represented by the Eq. (8).

$$N = A_{00} + A_{10}\lambda + A_{11}\Phi + A_{20}\,\lambda^{2} + A_{21}\,\lambda\,\Phi + A_{22}\,\Phi^{2} \quad (8)$$

The geoid undulation of Egypt by using the best model with HARN points is represented in fig. 9. It is shown that the results are different in the western part of Egypt because the available data in this region is smaller and less than that for other regions.

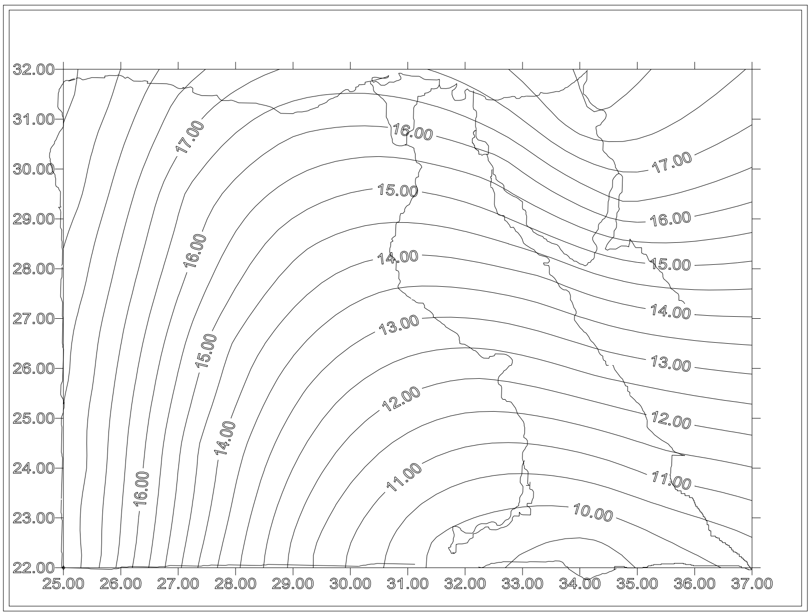


Fig. 9: The geoid undulation by using second degree polynomial model in two directions (geodetic coordinates) in the HARN data

Summary

This paper used the simple polynomial techniques to compute the geoid undulation in Egypt using 12 trails. These trails are dependent on the coordinates observed by GPS and precise leveling (report of S.Powell, 1997), and used the least square method to adjust these observations.

From these trials, the results obtained from the second degree in the plane with geodetic coordinates is the best technique for compute the geoid undulation in Egypt, so compute the othometric height by the simple equation H=h-N. This technique gives the best solution on the middle and east of Egypt.

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