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HOW CAN YOU ESTIMATE THE GEOID UNDULATION BY LEAST SQUARE COLLOCATION TECHNIQUE AND GPS DATA?

Abstract

Levels have been normally derived by conventional surveying methods and their values are related to the geoid surface. Hence, in order to derive heights above the geoid by GPS, it is necessary to know the height of the ellipsoid above the geoid where over wide areas of the world. This relation is not known with a great precision. Nowadays, various mathematical models of the geoid determination are existed. Most of these models have been derived for individual countries based upon satellite observations and terrestrial gravity data. Others models are derived based on astronomical and geometrical observations and others utilized a combination of all types of the available data. This paper investigates the application of using least square collocation (LSC) model in determination the geoid undulation in Egypt from GPS observations.

Introduction

The geoid is an equipotential surface of the earth that coincides with the undisturbed mean sea level. Therefore one might say that it describes the actual shape of the earth. The geoid is also the reference surface for most height networks since leveling gives the heights above the geoid. In geodesy, these heights are called orthometric heights (H), but they are the ordinary heights above the sea level [7].

The geoid is determined by using several techniques based on a wide variety of using one or more of the different data source such as: Gravimetric method using surface gravity data, Satellite positioning based on measuring both ellipsoidal heights for stations with known orthometric heights, Geopotential models using spherical harmonic coefficients determined from the analysis of satellite orbits, Satellite altimetry using satellite-borne altimetric measurements over the ocean [6].

The ellipsoid is the mathematical figure generated by the revolution of an ellipse about one of its axes. The ellipsoid is dividing into two types; local and international ellipsoid according to the study and the country.

The geoid defines the astronomic coordinate system while the reference ellipsoid defines the geodetic coordinate system. The relationship between these two systems is relative and can be fully described at any point in space by specifying the magnitude of the linear separation and the angular deviation between the two systems at the same point [4]. The separation denoted by N, is known as goidal undulation, separation or geoid height (positive or negative depending on the relation in equation1.

$$N=h-H \quad (1)$$

Where N- is the geoid undulation, h – is the ellipsoidal height and H – is the orthometric height.

The purpose of using the Least-Square Interpolation Technique in the geoid undulation determination is to supplement the GPS/Leveling observations, which are made at only a relatively few points, by dense points which their values of geoid undulation are estimated across a surface covering a specific region.

Methodology and models

The adoption of the least square collocation (LSC) technique for geoid determination requires the solution of a set of linear equations with dimension equal to the number of observations [2]. In its simplest form, the least square collocation can be considered as a direct extension of least square prediction. In this technique, it can determine the quantities at the computation points which are not generally the same as those being measured at the data points. Furthermore, the general collocation model is also able to take into account measurement errors at the data points and the possible requirement to compute certain parameters during the prediction process [3]. But at the first, it must be known that how data can be prepared from old observations especially if the precision of this data was not known. So, the process has two steps:

First step: The covariance matrix

Let there are old observations at points 1, 2, 3,..., n points, and the covariance matrix between these measurements relative to the distance between these points (d_{ij}) will be calculated. Divide all the distances between every pair of points to equal distance (r_k). So, for the distance (r_1) the covariance between every pair of points can be calculated according to the following equation:

$$C_1 = \frac{1}{n_1}\sum U_i U_J \qquad (2)$$

Where C_1 is the covariance between i,j , n_1 - is The total number of pairs which satisfy the distance $r_{i,j}$ and U_i ,U_j - are the measurements at points i,j.

For the general case for every distance r_k the general equation is as follows:

$$C_k = \frac{1}{n_k}\sum U_i U_J \qquad (3)$$

After that, a histogram can be drawn showing the relation between the distance and the covariance between points as shown in figure 1.The curve function can be expressed as:

$$C_{i,j} = ae^{(-br_{ij})} \qquad (4)$$

Using least square method to calculate parameters a, b. after that the covariance matrix can be expressed as

$$\begin{bmatrix} C_{s1} & C_{s12} \\ C_{s21} & C_{s2} \end{bmatrix} \qquad (5)$$

Where

C_{s1}	The covariance matrix of the original data
C_{s2}	The covariance matrix of the computational points
C_{s21},C_{s12}	The covariance matrix between data points and the computational points.

Second step: General case of least square collocation

In order to calculate the equation for the least square collocation, first the observation equation which contains parameters and observations will be linearised by Taylor's series is as follows

$$B(l+v)+A\Delta=b \quad (6)$$

Where:

- $\boldsymbol{B}$ The coefficient matrix of observations.
- $\boldsymbol{l}$ The observations at data points
- $\boldsymbol{v}$ The vector of errors
- $\boldsymbol{A}$ The design matrix of parameters.
- Δ The vector of unknown parameters
- $\boldsymbol{b}$ The vector of constants.

Let a new set observations l_e may be written as:

$$l_e+v_e=(-Bl+b)+Bv \quad (7)$$

Where:

$\boldsymbol{l_e = (-Bl + b)}$

$\boldsymbol{v_e = (Bv)}$

A special case can be appeared (observation equation) in the least square technique as following:

$$v_e+A\Delta=l_e \quad (8)$$

And the equation of the covariance matrix for the equivalent observation as follows

$$C_{le}=(BCB^T) \quad (9)$$

The above equation can be obtained from the low of error propagation [5].So, every equation in the main system can be written as follow

$$A\Delta-l_e+GS + n = 0 \quad (10)$$

Where

$$v_e=GS+n \quad (11)$$

And

- S The signal of original data and the computation data = $[S_1 \mid S_2]$
- n The noise in the observations
- S_1 Signals for original data points
- S_2 Signals for computation points
- G The coefficient matrix for signals $[I/0]$

So, the parameters can be calculated as follows

$$\Delta^{\backslash}= [A^T(C_n+C_{s1})^{-1}A]^{-1} A^T(C_n+C_{s1})^{-1} l_e \quad (12)$$

But we can notice that

$$C_{le} =(C_n+C_{s1}) \quad (13)$$

Vector of signals (S)

$$S^{\backslash}= C_s\, G^T(C_n+C_{s1})^{-1}(l_e - A\Delta^{\backslash}) \quad (14)$$

Vector of noise (n)

$$n^{\backslash} = C_n(C_n+C_{s1})^{-1}(l_e - A\Delta^{\backslash}) \quad (15)$$

Covariance matrix for signals

$$C_{s\backslash} = C_s\, G^T(C_n+C_{s1})^{-1}\, G\, C_s - C_s\, G^T(C_n+C_{s1})^{-1}A[A^T(C_n+C_{s1})^{-1}A]^{-1}A^T(C_n+C_{s1})^{-1}\, G\, C_s \quad (16)$$

Covariance matrix for noise

$$C_{n\backslash} = C_n(C_n+C_{s1})^{-1}C_n - C_n(C_n+C_{s1})^{-1}A[A^T(C_n+C_{s1})^{-1}A]^{-1}A^T(C_n+C_{s1})^{-1}C_n \quad (17)$$

For solving these equations, a program using least square collocation technique to compute the geoid undulation in Egypt was designed. The first stepin program is division the region map of Egypt to grids (0.50' x 0.50') as shown in figure 1. The second step is computation the distances between data points and the geoid undulation

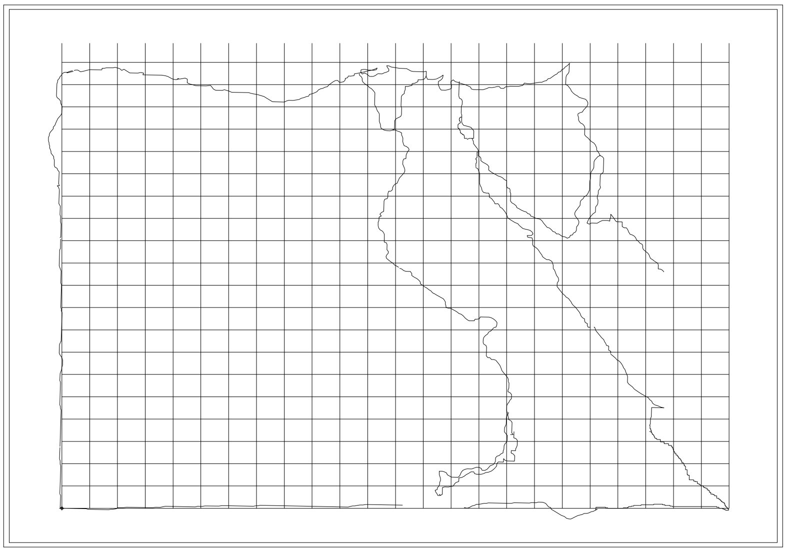

Figure 1. Grids of Egypt region map

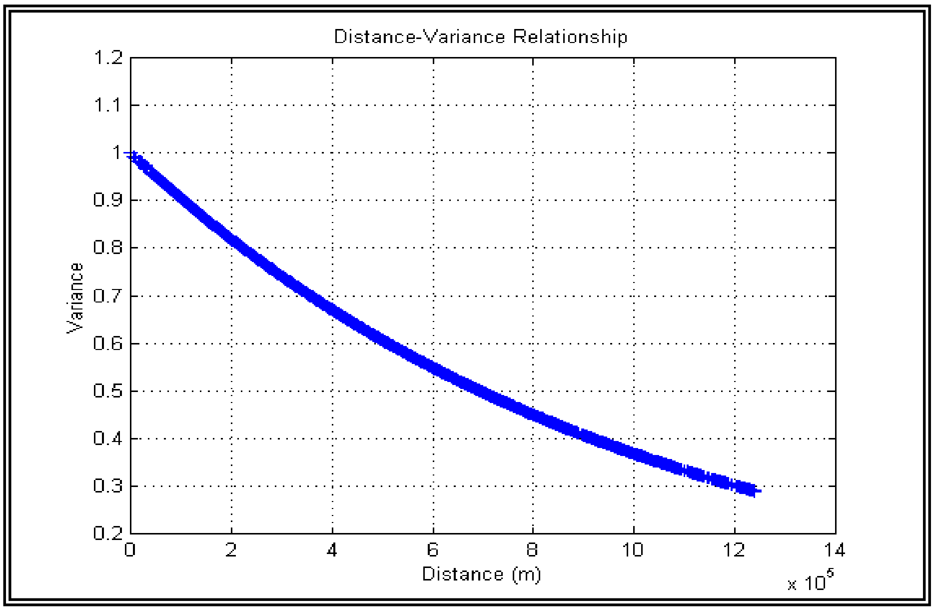

Figure 2. The distance – Covariance relationship

Then, the system of equations can be arranged in the form as following

$$S + N = b \qquad (18)$$

Where :

B=[I|0] where the unit matrix has a (n,n) dimension, and the null matrix has a (n,q) dimension.

S=[S11|S22]T where S11 is the data vector (n,1) dimension, and S22 is the computation vector (q,1) dimension. b is the vector of constants (n,1) dimension.

Then the computation value of geoid undulation at the grid can be obtained from the following equations as a special case of least square collocation where the system will be modeled by the signal S and noise n.

$$\left.\begin{aligned} S^{\backslash} &= C_s\, B^T(C_n+C_{s11})^{-1}b \\ Cs^{\backslash} &= Cs\, B^T(C_n+Cs11)^{-1}\, B\, C_s \\ n^{\backslash} &= C_n(C_n+C_{s11})^{-1}b \\ C_n^{\backslash} &= C_n(C_n+C_{s11})^{-1}C_n \end{aligned}\right\} \qquad (19)$$

Where: C_n is the covariance matrix of observations.

Results and conclusion

Using the above solution steps and the available data for Egypt with the designed Matlab program, a statement about geoid undulation in Egypt is possible. The obtained results show that the values of distortion geoid undulation in Egypt for available data are in the range of 1.9 m. The contour map of the geoid undulation for the Egyptian grids by using LSC model is shown in figure 3.

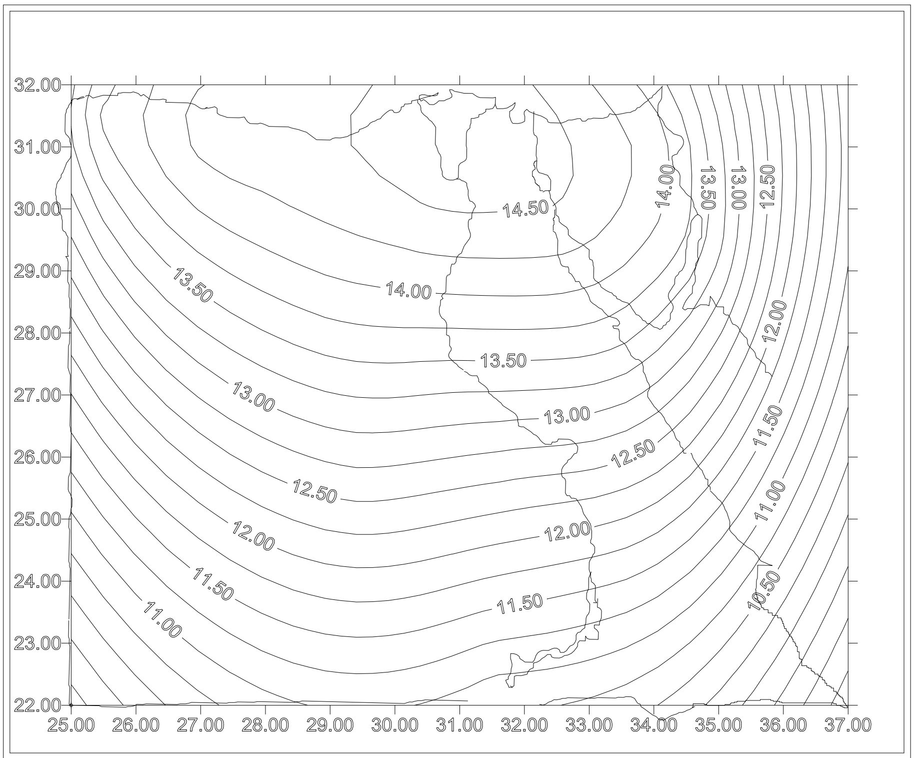

Figure 3. The geoid undulation by using LSC model in two directions in Egypt

As shown in figure 3, the values of geoid undulation in the study area between 10 m and 14.5 m. These values approximately are good for this region because the distribution of most of these points is in north Egypt, and similar to the gravimetric geoid in Egypt.

The accuracy of this new methodology for computation the geoid undulation approaches $\pm$0.62 m, so this method can be used to estimate approximate geoid.

Least square collocation with few common points can determine the approximate orthometric height at any other points. It depends on the mish grid of the studied region and its known points to compute the unknown points.

When the number of common points is increased, the obtained accuracy from this technique will be improved.

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