STUDYING THE APPLICATION ACCURACY OF LINEAR POLYNOMIAL ADJUSTMENT TECHNIQUE IN MONITORING THE STRUCTURAL DEFORMATION

By

Zaki Mohamed Zeidan
Associate professor, Faculty of Engineering, Mansoura University, Egypt,
e-mail:zmze283@yahoo.com
Ashraf A. A. Beshr
Assistant lecturer, Faculty of Engineering, Mansoura University, Egypt,
e-mail:eng.aaabeshr@yahoo.com
Mosbeh R. Kaloop
Assistant lecturer, Faculty of Engineering, Mansoura University, Egypt,
e-mail: razankaloop@yahoo.com

Abstract

The movements of an engineering structure, which serves the human life of today's modern world, are exhibiting safe behaviors. So, a lot of deformation monitoring studies for determining and analyzing different kinds of engineering structures such as high-rise buildings, dams, bridges, industrial complexes etc., are necessary. Monitoring and analyzing deformations of these structures constitutes a special branch of geodesy science. Polynomial surfaces are a very useful tool in order to represent the geomagnetic field over small area of the earth's surface. Nevertheless, the method has not always been applied with enough precision. This paper investigates the applicability of the linear polynomial adjustment technique to the data array from monitoring the structural deformation by determining the coordinates accuracy of unknown points on the monitoring structure from measured points. The comparison study between the resulted accuracy from polynomial technique and measured values for computation the structural deformations is introduced and discussed. The results of the practical measurements, calculations and analysis of these results using least squares theory and computer programs are presented.

Keywords: Deformation, monitoring, polynomial technique, total station.

1. Introduction

Engineering structures are subjected to external loads that cause deformation of the structure itself, as well as its foundations. Any indication of abnormal behavior may threaten the safety of the structure. Careful monitoring of the loads on a structure and its response to them can aid in determining abnormal behavior of that structure [1,2]. Polynomial adjustment technique can be applied to various fields of study. Most extensively it is used in business and economic situations, but can also be utilized for some engineering problems such as determination of the coordinates and the accompanied accuracy of several points, and then it can be used in monitoring the structural deformation analyses [3]. In mathematics, a polynomial is an expression that is constructed from one or more variables and constants, using only the operations of addition, subtraction, multiplication, and constant positive whole number exponents.

Polynomials are built from terms called monomials, which consist of a constant (coefficient), multiplied by one or more variables. Each variable may have a constant positive whole number exponent. The exponent on a variable in a monomial is equal to the degree of that variable in that monomial. A monomial with no variables is called a constant monomial. The degree of a constant term is 0. The coefficient of a monomial may be any number, including fractions, irrational numbers, and negative numbers.

A system of polynomial equations is a set of equations in which a given variable must take on the same value everywhere it appears in any of the equations. Systems of equations are usually grouped with a single open brace on the left. In elementary algebra, methods are given for solving a system of linear equations in several unknowns. To get a unique solution, the number of equations should equal the number of unknowns. If there are more unknowns than equations, the system is called underdetermined. If there are more equations than unknowns, the system is called overdetermined. This important subject is studied extensively in the area of mathematics known as linear algebra. Overdetermined systems are common in practical applications.

2. Pre-analysis study of the used surveying techniques

Pre-analysis of the surveying measurements is the analysis of the component measurements before the project is actually undertaken [5]. Main items to be considered in the pre-analysis study of a certain survey project are: Possible surveying techniques, and thus the corresponding mathematical model, and available instruments (cost, simplicity and the precision of a single measurement).

2.1 One total station technique

From figure (1), the Y-axis is chosen arbitrary as a horizontal line in the direction of the base of the monitoring building, where the X-axis is a horizontal line perpendicular to the building base direction and positive in the direction towards the monitoring object, and the Z- axis is a vertical line determined by the vertical axis of the instrument at occupied station. There is a known point (A), and these coordinates are (X_A, Y_A, Z_A). From this point, we can monitor the movements of any point (B) in space in order to determine its local coordinates (X_B, Y_B, Z_B) and accuracy. This case has a unique solution, so the multivariate propagation technique will be used.

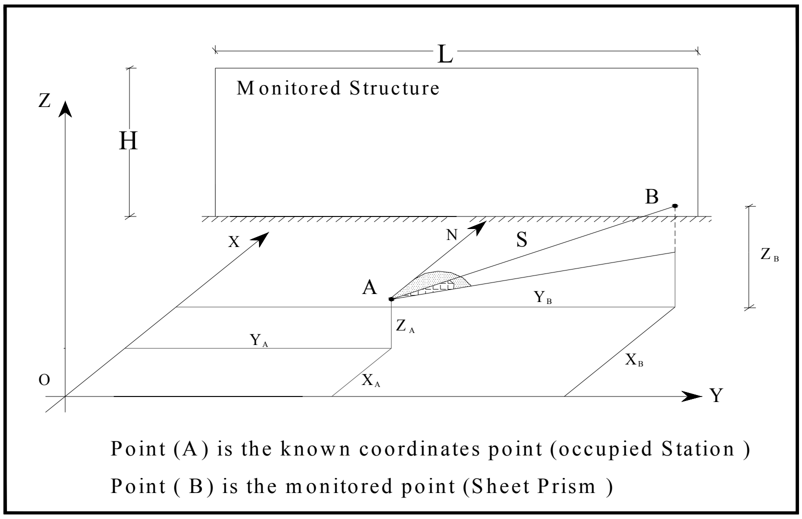

Figure (1) The geometry of one total station technique

2.2 Two total station technique

The two total stations technique employees the intersection process in three dimensions to determine the spatial coordinates of a specific target. Figure (2) illustrates the geometry of the two total stations technique. A local three-dimensional rectangular coordinates system is needed to calculate the spatial coordinates of any target points. There are two known occupied coordinates points (X_A, Y_A, Z_A) and (X_C, Y_C, Z_C). From these two known points (A and C), the coordinates of unknown point (B) can be determined.

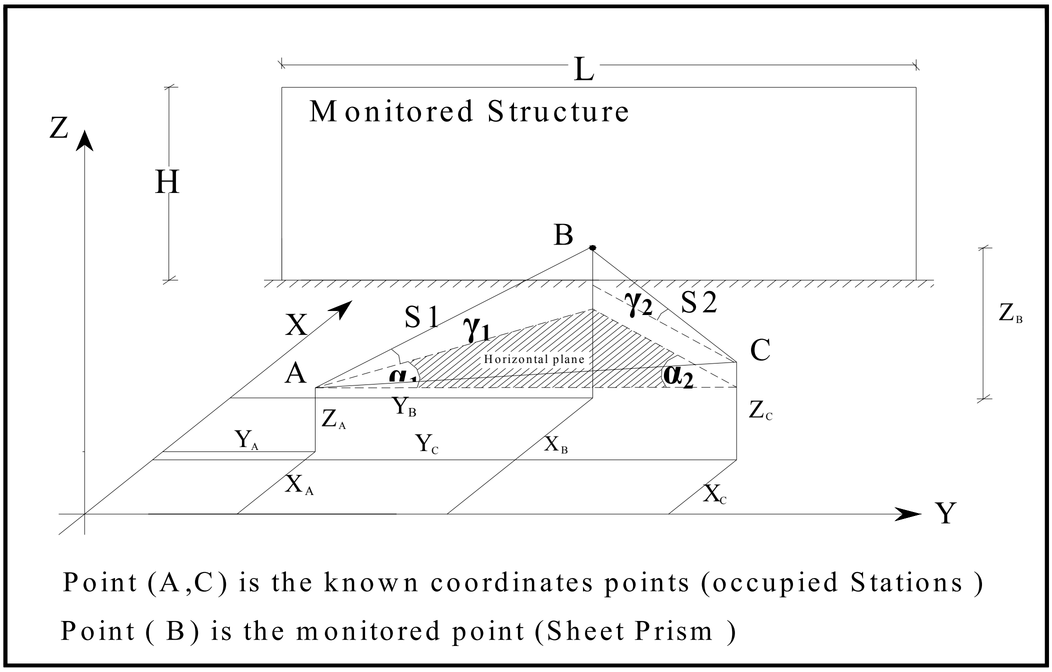

Figure (2) The geometry of two total stations technique

From figure (2), there are three unknowns (X_B, Y_B, Z_B) and six observations. Then the least squares adjustment technique will be used to calculate the coordinates of point (B) and its accuracy. The observations equation technique will be used.

3. Mathematical model of linear polynomial technique

The mathematical formula of the polynomial model for this case is illustrated in equation (1) as following [4]:

$$\boldsymbol{\sigma_i = A_0 + A_1X_i + A_2Y_i + A_3\,Z_i.} \tag{1}$$

Where: (σ_i) the standard deviation of point (i), $A_{(0,\,1,\,2,\,3)}$ the parameters, (X_i, Y_i, Z_i) the coordinates of point (i). Using general least square technique to solve this system for every point altogether and notice that, the coordinates (X,Y, Z) will be considered as observations and the unknowns will be considered as (A_0, A_1, A_2, A_3). So the system of equations can be written for every point and we will have n equations for n points. According to general least square technique, the system can be reconstructed in a matrix form as follows:

$$\boldsymbol{A_{(n,n)}V_{(n,1)}+B_{(n,m)}\Delta_{(m,1)}=F_{(n,1)}} \tag{2}$$

Where: (A) The coefficients matrix of residuals, (V) Vector of residuals, (B) Design matrix of parameters, (Δ) Vector of parameters, (F) Vector of constants, (n) Number of observations = Number of equations, (m) Number of parameters = Number of unknowns. Matrix (A) represents the differentiation of equations for n observations point. In this case, (A) matrix is equally the unity matrix as follow $\mathbf{A_{(n,n)}}$ **= -**

[I]. Matrix (B) represents the differentiation of unknown parameters for n points. The steps of program will be done as shown in flowchart in figure (3).

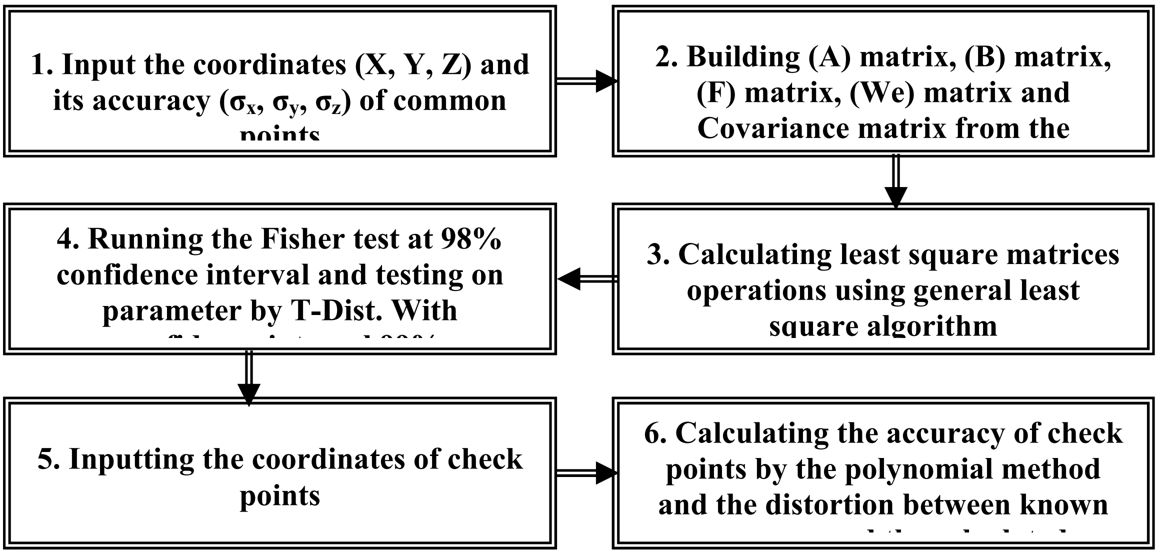

Figure (3) Flowchart describe the steps of solution by using polynomial technique and least square theory

4. Experimental program

The dataset used in this analysis consists of a sample of points that have been monitored using the discussed surveying techniques. To achieve that goal, the monitoring of the vertical wall is done. A mesh of twelve monitoring points on the (7.7m x 3.0m) wall is distributed for coordinating a building façade as shown in figure (4). A local three-dimensional rectangular coordinates system is needed to calculate the spatial coordinates of any target points on the mesh. Two total stations (DTM 850-Nikon and SET300-Sokkia) and sheet prisms of diameter 1 cm are used in the field of measurements. The accuracy of all instruments and effect of the systematic errors are taken into considerations during the practical measurements. The coordinates of all points and its accuracy are calculated.

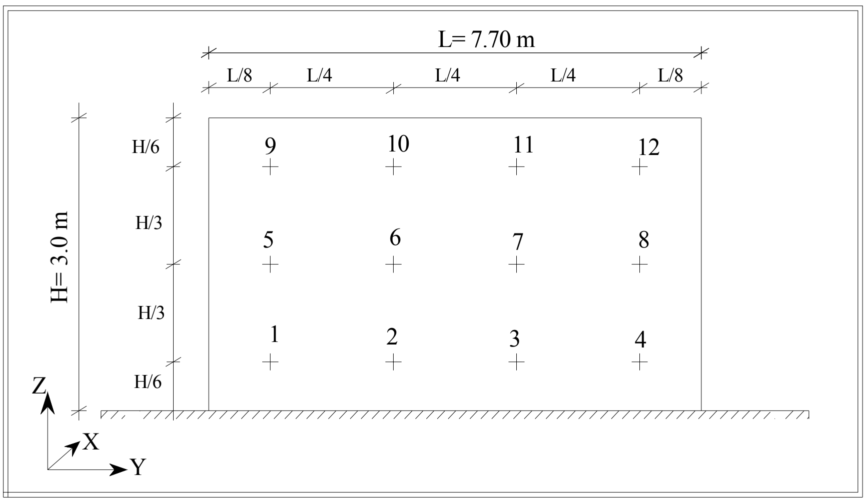

Figure (4) Geometric layout of the wall and monitoring points

5. Results and conclusions

The adjusted coordinates and surveying accuracy of each monitoring point on the wall are calculated from one total station and two total stations.

5.1 For one total station.

It is obvious from the obtained accuracy of monitoring points that there is optimum distance minimize the standard deviations in three dimensions but when the distance from the instrument to the monitoring wall increases, both "σ_X" and "σ_Z" will decrease, but "σ_Y" will increase.

5.2 For two total stations:

To find the best position of the used two instruments and the best locations of the monitoring points for this technique, some test measurements are carried out in the wall zone. The values of b0 and d0 which minimize the values of standard deviations are called the best parameters, the values of b0 and d0 as a function of the object dimensions for two total station instruments, the best parameters were determined graphically, and they are:

$$\mathbf{B_0 = 0.7545\ L \qquad D_0 = 0.242\ L.}$$

Where: (L) the width of the building

5.3 Application of polynomial adjustment technique

The adjusted coordinates and its associated accuracy of each point in the monitoring mesh are calculated by using Matlab program and the linear polynomial technique. Different cases are taken into consideration for one and two total stations, the results can be summarized as following:

1. For one total station

First case: In this case, points (1,4, 9 and12) will be considered as common points but points (2,3,5,6,7,8,10 and 11) will be check points. The comparison between the resulted standard deviations of the coordinates from linear polynomial interpolation and from observations can be indicated as shown in table (1) and figures (5, 6 and 7).

Points	Coordinates from one total station observations (m)			Standard deviations for points in X (mm)		Residual of accuracy (mm)	Standard deviations for points in Y (mm)		Residual of accuracy (mm)	Standard deviations for points in Z (mm)		Residual of accuracy (mm)
	X	Y	Z	σ_{X0}	σ_{X1}	V_X	σ_{Y0}	σ_{Y1}	V_Y	σ_{Z0}	σ_{Z1}	V_Z
1	-3.072	3.5722	1.1918	1.2738	1.2736		1.481	1.4809		0.4946	0.4946	
2	-1.152	3.6027	1.1807	0.585	1.2355	-0.6505	1.8293	1.5093	0.32	0.5998	0.4962	0.10354
3	0.7631	3.6815	1.3246	0.3856	1.1785	-0.7929	1.8582	1.5321	0.3261	0.6688	0.542	0.12679
4	2.7133	3.7058	1.1826	1.1525	1.1524		1.5739	1.574		0.5027	0.5024	
5	-3.089	3.5623	2.2082	1.1961	1.1815	0.0146	1.3791	1.364	0.0147	0.8551	0.822	0.0324
6	-1.156	3.6034	2.205	0.5317	1.1414	-0.6097	1.6563	1.394	0.2617	1.0136	0.825	0.18793
7	0.772	3.6537	2.2042	0.3588	1.1002	-0.7414	1.6961	1.426	0.2693	1.0234	0.828	0.195
8	2.6983	3.6953	2.1996	1.0712	1.0603	0.0109	1.4669	1.457	0.0097	0.8734	0.830	0.0426
9	-3.055	3.5555	3.2047	1.0853	1.090		1.2631	1.251		1.1386	1.144	
10	-1.154	3.6023	3.2049	0.4694	1.0496	-0.5802	1.4641	1.282	0.1818	1.3026	1.147	0.1551
11	0.7593	3.6435	3.2217	0.3113	1.0080	-0.6967	1.4914	1.310	0.1813	1.3188	1.1567	0.162
12	2.6882	3.6895	3.1996	0.9727	0.9692		1.3349	1.343		1.1577	1.153	

Where: (σ_{X0}, σ_{Y0} and σ_{Z0}) are the standard deviations from observations, (σ_{X1}, σ_{Y1} and σ_{Z1}) from polynomial technique, V_x residual in accuracy ***($V_x = \sigma_{X0} - \sigma_{X1}$)***

Table (1) Comparison between regression analysis from polynomial technique and observations for first trial

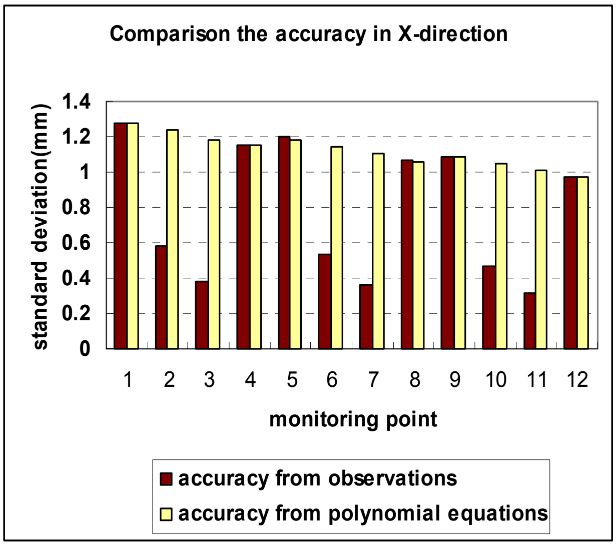

Figure (5) Comparison between regression analyses for σ_X

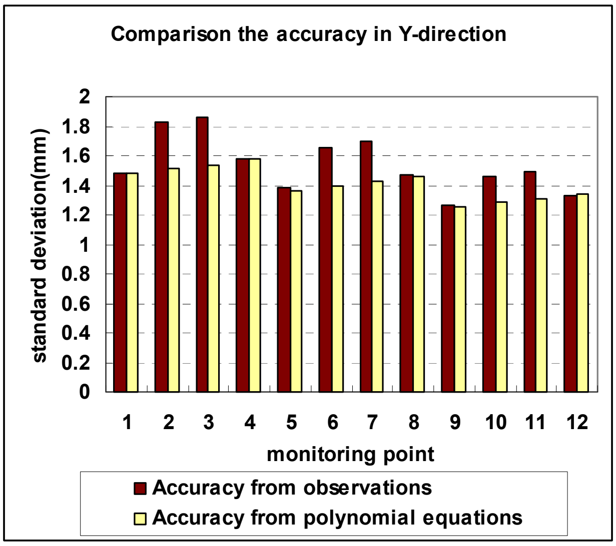

Figure (6) Comparison between regression analyses for σ_Y

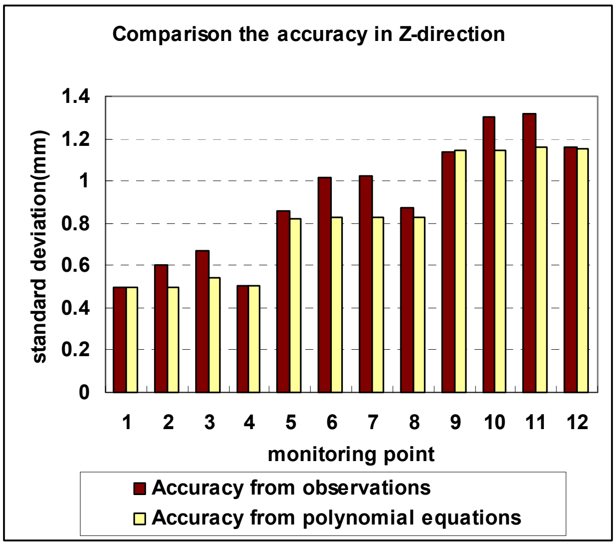

Figure (7) Comparison between regression analyses for σ_Z

From table (1) and figures (5,6 and 7) , it is obvious that the resulted standard deviations from using the linear polynomial technique are close to that from observations, the differences between the two techniques are small. For this case, maximum difference for σ_X is 0.8 mm and minimum is 0.01mm, but for $\sigma_{Y,}$ maximum value is 0.33mm and minimum value is 0.01 mm and for σ_Z maximum difference value

is 0.2 mm and minimum is 0.03 mm. Four cases are done for polynomial techniques to reach the best method which achieve the best accuracy for monitoring points, summary of the results are given in table (2) and figures (8,9 and 10).

Points	**First Case:** *Points (1,9,12,4) common points* Residual of accuracy (mm)			**Second Case:** *Points (1,4,9,12,6,7) common points* Residual of accuracy (mm)			**Third Case:** *Points (1,4,9,12,5,8) common points* Residual of accuracy (mm)			**Fourth Case:** *Points* (1,2,3,4,9,10,11,12) *common points* Residual of accuracy (mm)		
	V_X	V_Y	V_Z	V_X	V_Y	V_Z	V_X	V_Y	V_Z	V_X	V_Y	V_Z
1												
2	-0.6505	0.32	0.1035	-0.422	0.2221	0.0369	-0.740	0.2124	-0.1456			
3	-0.7929	0.3261	0.1267	-0.583	0.2577	0.0716	-0.713	0.4956	0.1948			
4												
5	0.0146	0.0147	0.0324	0.237	-0.0841	-0.033				0.3149	-0.107	-0.033
6	-0.6097	0.2617	0.18793				-0.655	0.2236	0.1018	-0.2893	0.137	0.1205
7	-0.7414	0.2693	0.195				-0.760	0.2779	0.2084	-0.3915	0.1431	0.1256
8	0.0109	0.0097	0.0426	0.239	-0.0706	-0.019				0.3813	-0.119	-0.028
9												
10	-0.5802	0.1818	0.1551	-0.353	-0.0949	0.09	-0.592	0.1968	0.194			
11	-0.6967	0.1813	0.162	-0.466	0.2221	0.097	-0.723	0.1767	0.1468			
12												

Table (2) The results of comparison between regression analysis from polynomial technique and observations from one total station for all trial cases

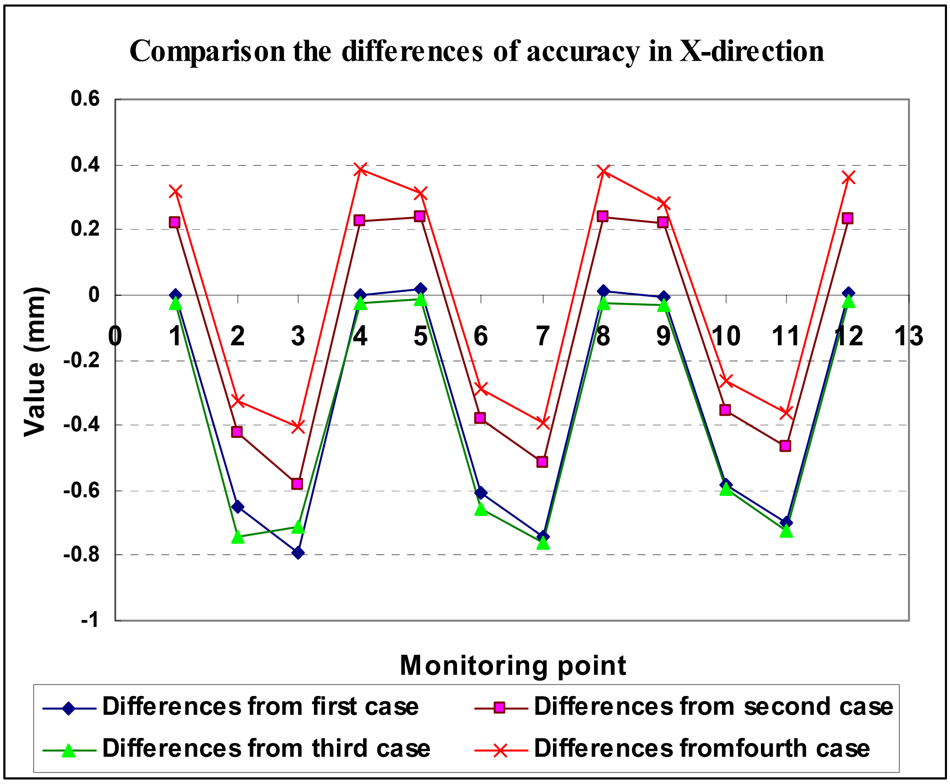

Figure (8) Comparison the differences of accuracy from one total station and polynomial for X-direction

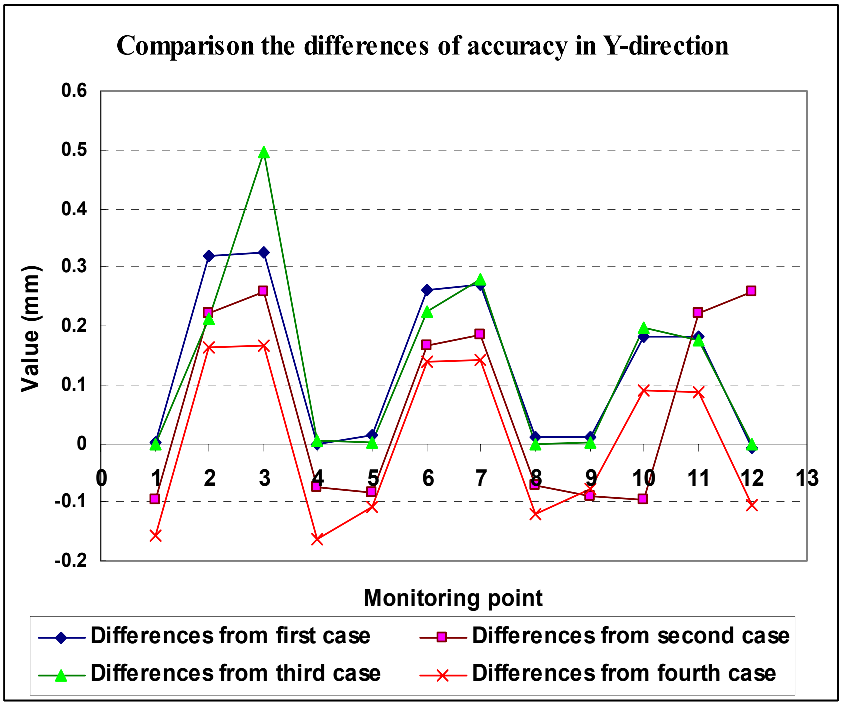

Figure (9) Comparison the differences of accuracy from one total station and polynomial for Y-direction

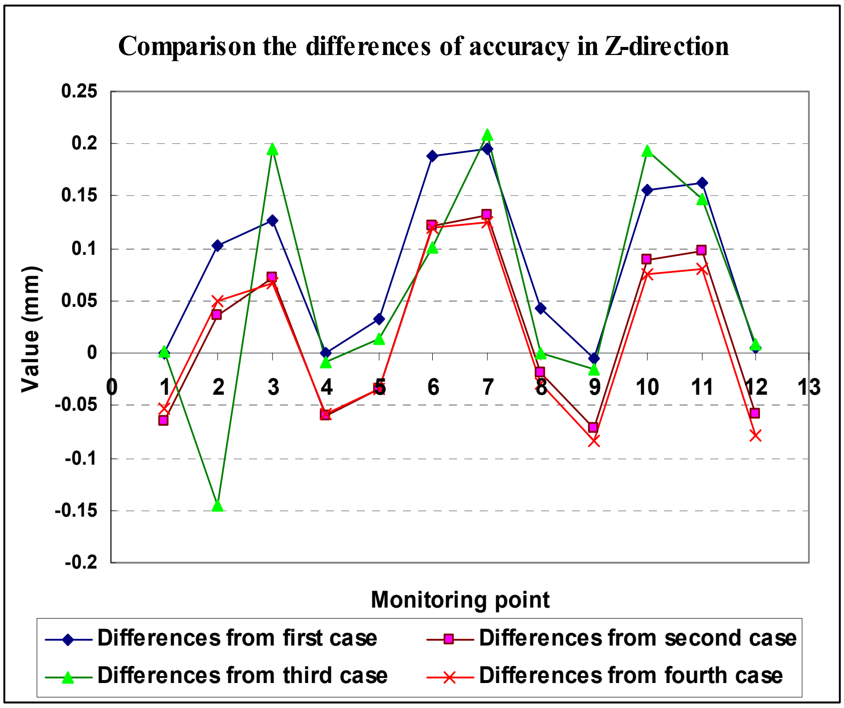

Figure (10) Comparison the differences of accuracy from one total station and polynomial for Y-direction

From table (2) and figures (8, 9 and 10), it is obvious that the differences between the standard deviations from polynomial technique and observations are very small. The best arrangement for monitoring points on this wall is the second case in which points (1, 4, 9,12,6 and 7) are common points. For this case, maximum difference σ_X is 0.58 mm and minimum is 0.23mm but for σ_Y maximum is 0.26mm and minimum is 0.07mm and for σ_Z maximum value for standard deviation is 0.097 mm and minimum value is 0.019mm.

2. For two total stations:

Four cases are done for polynomial techniques to reach the best method which achieve the best accuracy for monitoring points. The comparison between the resulted standard deviations of the coordinates from linear polynomial interpolation and from observations is done Summary of the results are given in table (3) and figures (11, 12 and 13).

Points	**First Case:** *Points (1,9,12,4) common points* **Residual of accuracy (mm)**			**Second Case:** *Points (1,4,9,12,6,7) common points* **Residual of accuracy (mm)**			**Third Case:** *Points (1,4,9,12,5,8) common points* **Residual of accuracy (mm)**			**Fourth Case:** *Points* **(1,2,3,4,9,10,11,12)** *common points* **Residual of accuracy (mm)**		
	V_X	V_Y	V_Z	V_X	V_Y	V_Z	V_X	V_Y	V_Z	V_X	V_Y	V_Z
1												
2	-0.348	0.288	-0.183	-0.313	-0.333	-0.153	-0.349	-0.363	-0.168			
3	-0.519	1.262	-0.289	-0.532	-0.148	-0.279	-0.533	-0.120	-0.293			
4												
5	-0.022	0.560	-0.092	0.033	0.121	-0.050				0.219	0.194	0.047
6	-0.288	-0.100	-0.189				-0.281	-0.151	-0.165	-0.139	-0.144	-0.099
7	-0.150	-2.099	-0.072				-0.121	-0.261	-0.031	-0.437	-0.699	-0.214
8	-0.039	-2.480	-0.054	0.113	0.042	0.013				-0.443	-0.675	-0.259
9												
10	-0.332	0.337	-0.214	-0.276	0.173	-0.175	-0.328	0.121	-0.192			
11	-0.339	-0.452	-0.223	-0.259	0.190	-0.179	-0.326	0.125	-0.200			
12												

Table (3) The results of comparison between regression analysis from polynomial technique and observations from two total station for all trial cases

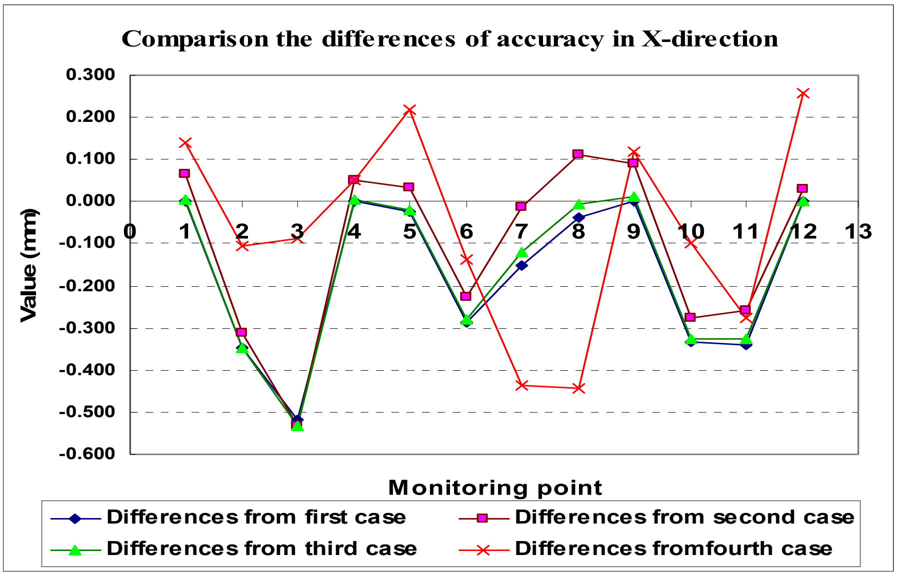

Figure (11) Comparison the differences of accuracy from two total station and polynomial for X-direction

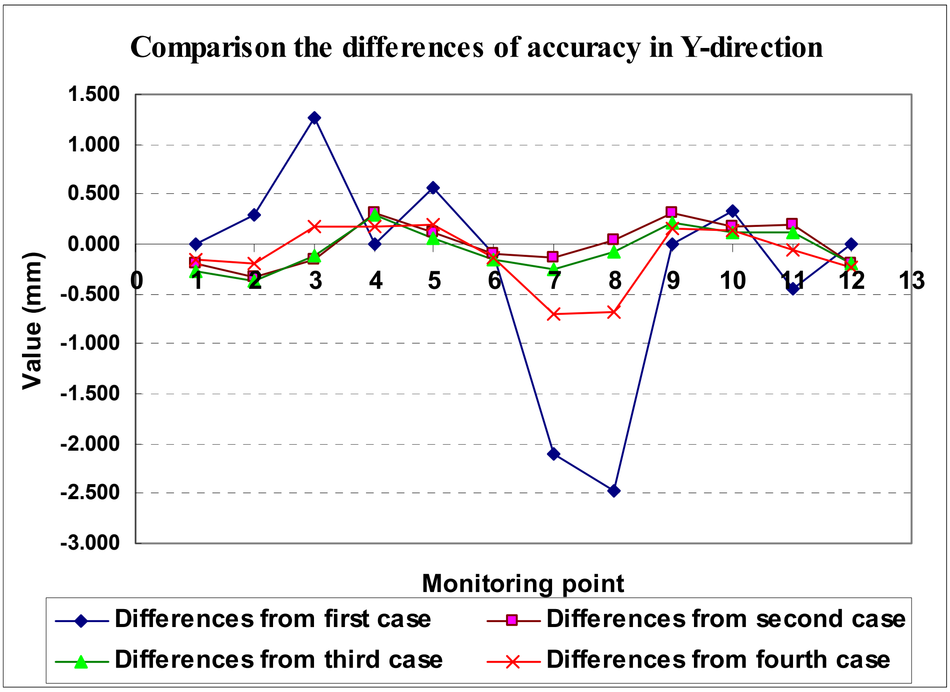

Figure (12) Comparison the differences of accuracy from two total station and polynomial for Y-direction

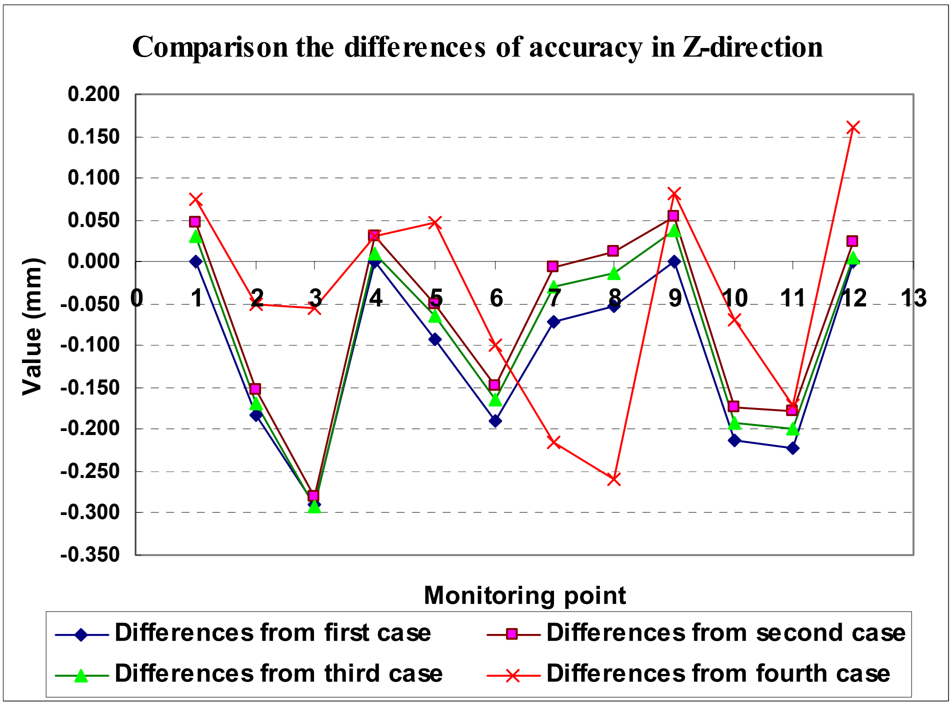

Figure (13) Comparison the differences of accuracy from two total station and polynomial for Z-direction

From table (3) and figures (11, 12 and 13), it is obvious that the differences between the standard deviations from polynomial technique from two total stations and observations are very small. The best arrangement for monitoring points on this wall is the second case in which points (1, 4, 9,12,6 and 7) are common points. For this case, maximum difference σX is 0.53 mm and minimum is 0.03 mm but for σY maximum is 0.33 mm and minimum is 0.04 mm and for σZ maximum value for standard deviation is 0.28 mm and minimum value is 0.013 mm.

From the previous analysis and numerical results obtained, the following conclusions can be summarized:

1. The two used surveying techniques (one total station and two total stations) can provide valuable data on the deflection of the structural members and movement of buildings.

2. Linear polynomial technique can be used to determine the coordinates of points and its associated accuracy from known points, the differences in accuracy are very small. So this technique can be used in monitoring the structural deformation. If you want to monitor any engineering building, you must observe some points (coordinates and accuracy) on the building, and by this technique you can determine the coordinates of several points and its accuracy.

3- The best positioning for the common points are around and middle the structural monitoring.

REFERENCES

1. **Ashraf A. A. Beshr** “Accurate surveying measurements for monitoring the structural deformation”. The third international exhibition and scientific congress “GEO-SIBERIA-2007” 25-27 April 2007, Novosibirsk, Russia Vol.1 p.112-117.
2. **Ashraf A. A. Beshr**, H. Abou Halima and Z. Zeidan “Application of Auto-correlation technique in monitoring the structural deformation”. The Fifth International Engineering Conference (IEC 2006), Mansoura – Sharm El-Sheikh, Egypt. 27-31 March 2006, p.501-516.
3. **J. Ardizone and M. Herraiz** “Application of the polynomial adjustment to the aeromagnetic survey of the Spanish Mainland; Requirements and shortcomings” Earth Planets Space, 2000, 52, p.183–196.
4. **M.Kaloop, 2006** "Precise leveling by using global positioning system", M.Sc. thesis, Public Work Department, Faculty of Engineering, Mansoura University, Egypt.
5. **Mikhail, E.M. and Gracie**, "Analysis and Adjustment of Survey Measurements", Van Nostrand Reinhold Company, New York. U.S. 1981.
6. **Wan Aziz, W.A.Othman Z**. and Najib H., 2001, “ Monitoring high rise building deformation using Global Position System”, Department of Geomatics, Faculty of Engineering, University technology Malaysia, Skudai, Malaysia.