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A Precise Integrated GPS/Gravity Geoid Model for Egypt

BY

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Abstract

A precise geoid is a crucial demand in many scientific and practical fields of applications as: determination of the size and shape of the Earth, the geoid is the datum for a height system, the geoid undulations are used in geodynamics monitoring applications. The rapid growth of the Global Positioning System (GPS) applications in Egypt has resulted in an increasing need for precise geoid models in order to transform the GPS-based ellipsoidal heights to orthometric heights needed for engineering applications.

Two high resolution geoid models for Egypt (32^o <latitude> 22^o, 37^o <longitude> 25^o) have been developed based on the utilization of the Earth Geopotential Model (EGM96) as the most recent long-wavelength representation of the Earth geopotential, incorporated with the recent precise geodetic data base available in Egypt. Both EGM96 and OSU91A were used to compute the geoid undulations over 195 known GPS/Levelling stations. The obtained results show that the mean and RMS values of EGM96 undulations exhibit a better consistency with the measured undulations than the OSU91A corresponding values. A 5’x5’ geoid solution, called SRI2001A, is a gravimetric geoid model utilizing 240 first-order gravity stations, a local Digital Elevation Model (DEM); and the EGM96 in a remove-compute-restore Fast Fourier Transform (FFT) processing methodology. Then, another 5’x5’ geoid model, named SRI2001B, is developed based on integrating the SRI2001A model to an undulation datum defined by GPS/Levelling observed undulations. SRI2001B has a minimum undulation value of 9.44 m and a maximum value of 21.39 m with an average of 13.62 m. The values of the estimated undulations from the SRI2001B model have been compared against the pure GPS undulation of some independent GPS/Levelling stations. The differences range from –0.01m to –0.28 m with an average of –0.10 m. Consequently, it can be concluded that SRI2001B is the most precise geoid model in Egypt based on the current available data.

1. Introduction

The geoid is the equpotiential surface of the Earth’s gravity field approximating mean sea level in an optimum way, and extended under the continents. A precise geoid is a crucial demand in many scientific and practical fields of applications as: determination of the size and shape of the Earth, the geoid is the datum for height systems, the geoid undulations are used in geodynamics monitoring applications. Recently, the rapid growth of the Global Positioning System (GPS) applications in Egypt has resulted in an increasing need for precise geoid models in order to transform the GPS-based ellipsoidal heights to orthometric heights needed for engineering applications [Alnaggar and Dawod, 1999].

In September 1998, the Egyptian National Gravity Standardization Network (ENGSN97) has been established, by the Survey Research Institute (SRI), as the accurate gravity datum for Egypt. ENGSN97 consists of 5 absolute gravity stations and 145 high-precision relative gravity stations, which GPS 3-D coordinates besides orthometric heights are measured. Moreover, a new first-order High Accuracy Reference GPS Network (HARN) has also established, by the Egyptian Survey Authority (ESA), to furnish the New Egyptian Datum (NED95). Both networks constitute the most recent and accurate geodetic database in Egypt. The global Earth Geopotential Model (EGM96) is considered as the most precise spherical harmonic model representing the potential of the Earth.

The main objectives of this research study are: to investigate the performance of EGM96 geopotential model in Egypt, to use this recent global model in developing a new precise gravimetric geoid solution for Egypt based on utilizing the available Egyptian precise gravity data bas; and to integrate all available geodetic measurements to develop a high-precision combined GPS/Gravity geoid model for Egypt. Therefore, a brief description of ENGSN97 network in given in section two, followed by a summary of the three basic methods of geoid determination methodologies, namely: gravimetric, geometric satellite; and spherical harmonic expansion. Section four sums up the characteristics of both OSU91A and EGM96 geopotential models, while section five presents the available data and the procedures of developing the two gravimetric and GPS/Gravity geoid solutions for Egypt. The summary and obtained conclusions are given in section six.

2. The Egyptian National Gravity Standardization Net of 1997 (ENGSN97)

The establishment and re-calibration of the Egyptian National Gravity Standardization Net (ENGSN97) is a project executed by the Survey Research Institute during 1994-1998, and has aimed to: the establishment of a fundamental national gravity network for Egypt, the updating of the gravity-anomalies maps of Egypt; and accurate definition of the figure of the earth, the geoid, in Egypt. According to the ENGSN97 project’s goals, the field observation campaigns include the collection of three types of measurements: relative gravity, GPS coordinates, and precise levels. This is, of course, beside the necessary absolute gravity measurements at some selected stations.

Five absolute gravity stations have been established and observed to serve as an absolute gravity framework for the ENGSN97 network. The locations of these sites are: Giza, Helwan, Marsa Matrouh, Aswan, and El-Kharga (Figure 1). The measurements have been carried out using the FG5 absolute gravity meter. The FG5 instrument has a higher level of robustness, reliability and an instrumental uncertainty estimate of 0.0011 mGal. The ENGSN97 gravity network contains 145 relative gravity stations. Seven LaCoste and Romberge (LCR) gravimeters have been used in measuring the relative gravity values of the ENGSN97. The main observation schemes that have been applied in ENGSN97 are the step method and the profile method. Both techniques are useful in controlling the gravimeters’ drift. A number of dual-frequency GPS receivers have been used to obtain accurate coordinates of the ENGSN97 stations. In each gravity loop in the ENGSN97 project, the orthometric height of each station is determined by the precise levelling technique. Wild N3 precise levels, with a precision of 0.1 mm, and Invar rods are used in levelling routes, each starts from a first-order bench mark [Dawod, 1998].

The final solution of the ENGSN97 contains 1045 observations for the 150 gravity stations, after removing 44 outlier observations. A number of 133 virtual gravimeters was used in terms of estimating the orientation and the drift unknowns for each virtual gravimeter. Hence, there were 408 unknowns to be estimated, and 632 degrees of freedom. The most essential information of the final adopted solution of the ENGSN97 network is summarized in table 1. Concerning the estimated gravity values at the network 150 stations, the obtained results indicate that the minimum adjusted gravity value was 978679.776 mGal at Abu-Sombol station while the maximum adjusted gravity value was 979504.981 mGal at Balteem station. Therefore, the gravity range over Egypt is 825.205 mGal with an average gravity value of 979126.005 mGal. As an indication of the precision of the ENGSN97, the standard deviations of the adjusted gravity values range from 0.002 mGal to 0.048 mGal [Shaker et al, 2001].

Table 1: Essential Information for the Final Adjustment of the ENGSN97 Network

Number of stations	150	
Number of observations	1045	
Number of loops	51	
Number of relative gravimeters	7	
Average station separation	66	Km
Number of unknowns	408	
Number of degrees of freedom	637	
Minimum standard deviation of gravity values	0.002	mGal
Maximum standard deviation of gravity values	0.048	mGal
Minimum adjusted gravity value	978679.776	mGal
Maximum adjusted gravity value	979405.981	mGal

3. Geoid Determination

The geoid is the equpotiential surface of the Earth's gravity field approximating mean sea level in an optimum way, and extended under the continents. The determination of the geoid is an old problem of physical geodesy and a numerous number of geoid evaluations have been carried out world wide, and in Egypt [e.g., Nassar et al, 1993; and Shaker et al, 1997]. The geoid is determined using several techniques based on a wide variety of using one or more of the different data sources such as: Gravimetric method using surface gravity data, Satellite positioning based on measuring both ellipsoidal heights for stations with known orthometric heights, Geopotential models using spherical harmonics coefficients determined from the analysis of satellite orbits, Satellite altimetry using satellite-borne altimetric measurements over the oceans, Astrogeodetic method using stations with measured astronomical and geodetic coordinates; and Oceanographic levelling methods used mainly by the oceanographers to map the geopotential elevation of the mean surface of the ocean relative to a standard level surface. Only the first three methods are used in this research study, while the other methods are found in several literatures [e.g. Nassar, 1986].

Stockes' boundary value problem (BVP) is the gravimetric determination of the geoid. BVP deals with the determination of a potential field, harmonic outside the masses, from gravity anomalies given everywhere on the geoidal surface. A lot of reference materials are available for this subject [e.g. Heskanien and Moritz, 1967]. The final formula of the geoid undulations, N, is given as [Sideris, 1994]:

$$N = \left({R}/{4\Pi\gamma} \right) \times \iint_{\sigma} (\Delta g + \delta\Delta g + \delta A) \times S(\Psi)\ d\sigma + \left({1}/{\gamma} \right) \delta T \tag{1}$$

where R is the mean radius of the Earth, Δg is the free-air gravity anomaly, γ is the normal gravity, δΔg is the indirect effect on gravity, δA is the attraction change, δT is the indirect effect on the potential, σ denotes the Earth's surface, dσ is the infinitesimal surface element; and S(Ψ) is the Stokes' function. There are several processing techniques for geoid determination, such as the Fast Fourier Transformation (FFT) and the Least-Squares Collocation (LSC). The adopted mathematical processing technique, in the current research study, is taken as the FFT technique. The effect of a global geopotential model, as assumed to represent coarse-scale smoother geoid, is removed from the observed gravity measurements. The contributions of the topography are also removed since they are implicitly included in the Stokes' equation to be evaluated. The residual gravity anomalies are used as input to the FFT routine to obtain fine-scale geoid. The final geoid height model is the sum of the coarse-scale and fine-scale models along with the indirect effect or the terrain contribution. This is called "the remove-compute-restore" processing strategy.

The idea of geoid computations, from geometric satellite geodetic results, is to make benefit from the derived satellite ellipsoidal height h, to compute the geoid undulation N at the same point. This necessitates that, the orthometric height H of the same point of interest to be known, since the relationship among those three quantities, is given as:

$$N = h - H \tag{2}$$

Of course, for precise determination of N, using this technique, in the order of the same precision of the satellite vertical component positioning, which can nowadays reach few tenth of a centimeter, the orthometric height H must be determined with at least the same precision. The best accurate method for determining H will be the use of the precise levelling technique.

The geoid undulations may be computed using the following spherical harmonic expansion [Heskanien and Moritz, 1967]:

$$N = \left({GM}/{r\gamma} \right) \times \sum_{n=2}^{360} \left({a}/{r} \right)^n - \sum_{m=0}^{n} ((C^{-}_{nm} \times \cos m\lambda) + (S^{-}_{nm} \times \sin m\lambda)) \times P_{nm}(\sin\phi) \tag{3}$$

where: n is the maximum degree of the model, m is the maximum order of the model, γ is the normal gravity of the reference ellipsoid, r is the geocentric radial distance of the computation point projected on the ellipsoid, G is the Newtonian gravitational constant, M is the mass of the Earth, a is the semi-major axis, ϕ is the geocentric latitude, λ is the geocentric longitude, C^{-}_{nm} and S^{-}_{nm} are the fully normalized harmonic coefficients, and P_{nm} is the fully normalized associated Legendre polynomial.

4. EGM96 and OSU91A Global Geopotential Models

The Earth Geopotential Model (EGM96) and the Ohio State University (OSU91A) are examples of the recent global geopotential models representing the Earth gravitational potential as spherical harmonic coefficients. Both models are complete to degree and order 360. Therefore, the shortest wavelength of these models is one degree, and their resolution is one-half degree (about 50 km). The geoid undulations of OSU91A and EGM96 models over Egypt are represented as contour maps in figures 2 and 3, respectively.

Smith and Milbert [1997a] stated that there is an error in the order of one meter in geoid undulation determination using OSU91A model and Geodetic Reference System (GRS80) as the chosen reference field. This effect is due to the difference between the normal potential of GRS80 and the potential of the geoid, and in the case of OSU91A model, it is due to the fact that its implemented value for the gravitation-mass constant does not equal the corresponding value of the GRS80. In the case of EGM96-based geoid undulations computed by NIMA, a constant bias of 0.41 m was taken into account [Smith and Milbert, 1997b]. This result was also evident when comparing two sets of EGM96 undulations: the first set is computed using spherical harmonic processing software, and the second set contains the corresponding values interpolated from the global undulation grid distributed by the U.S. NIMA. Over 5’x5’ grid covers Egypt, the differences between the two sets of EGM96 undulations have an average value of 0.46 m. This is an important note when dealing with EGM96 in geoid determination. As a primarily comparison, both EGM96 and OSU91A were used to compute the geoid undulations over 195known GPS/Levelling stations. The obtained results are tabulated in table 2. From this table, it can be noticed that, for EGM96 model, the undulation differences range from –0.11 m to 1.89 m with an average of –0.43 m and RMS of 0.78 m. The corresponding undulation differences values of the OSU91A model have a minimum of –0.53 m, a maximum of –1.97 m, an average of –1.53 m; and RMS of 1.86 m. In addition, it can be noticed that the mean and RMS values of EGM96 undulations exhibit a better consistency with the measured undulations than the OSU91A corresponding values. Hence, it may be concluded that, although both global geopotential models do not represent the short wavelength of the potential over Egypt, the EGM96 is more precise than OSU91A in the sense of geoid determination.

Table 2: Statistics of Geoid Undulations from GPS/Levelling, OSU91A, and EGM96

Item	Minimum	Maximum	Average	RMS
N (GPS/Levelling)	9.42	17.66	14.44	2.15
N (EGM96)	9.43	18.40	14.87	2.04
N (OSU91A)	9.95	18.51	15.96	2.42
N (GPS/Levelling) –N (EGM96)	-0.11	1.89	-0.43	0.78
N (GPS/Levelling) –N (OSU91A)	-0.53	-1.97	-1.53	1.86

5. Data Used and Obtained Results

The available data include gravity, GPS; and precise levelling measurements. A total of 240 gravity stations have been used in generating this geoid solution (Figure 4). The data used consists of the 150 ENGSN97 stations, 67 stations of the National Gravity Standard Base Network of 1977 (NGSBN77); and some gravity stations observed by the Survey Research Institute. These gravity values used are considered as first-order stations. Less-accurate available gravity measurements were not considered in this study in order to develop a precise geoid. All point gravity measurements have been corrected first to the terrain effect before generating the 5’x5’ free-air gravity anomaly grid. A total of 195precise GPS stations, with known orthometric heights, have been collected (Figure 5). They include the Egyptian National High Accuracy Reference Network (HARN) observed by the Egyptian Survey Authority to form the New Egyptian Datum 1995 (NED-95). These data sets have been used to develop two geoid models for Egypt.

5.1 Developing a Gravimetric Geoid Model (SRI2001A)

The FFT technique is utilized in developing a gravimetric geoid (called SRI2001A) for Egypt, i.e., from 22^{o} to 32^{o}N in latitude, and from 25^{o} to 37^{o} E in longitude. The available 240 gravity stations have been used to generate a 5’x5’ grid of free-air gravity anomalies. Table 3 presents the statistics of the obtained free-air gravity anomalies. From this table, it can be seen that the obtained free-air gravity anomalies range from a minimum of –122.42 mGal to a maximum of 128.65 mGal with a mean value of –2.93 mGal and Root Mean Square (RMS) error of 29.51 mGal. The EGM96 geopotential model was used as the reference global model in the FFT processing of the SRI2001A geoid. The corresponding free-air gravity anomalies of EGM96 have a minimum of –130.36 mGal, a maximum of 152.99 mGal, an average of 6.19 mGal; and an RMS equals 24.59 mGal. The differences between the two sets of free-air gravity anomalies range from –129.91 mGal to 254.84 mGal with a mean value of 9.17 mGal and RMS of 40.16 mGal.

Table 3: Statistics of used anomalies from both gravity measurements and EGM96 global model

Item	Minimum	Maximum	Average	RMS
Free-Air Gravity Anomalies from Terrestrial Gravity Data	-122.42	128.65	-2.93	29.51
Free-Air Gravity Anomalies from EGM96	-130.36	152.99	6.19	24.59
Differences in Free-Air Gravity Anomalies	-129.91	254.84	9.17	40.16

The obtained results of the developed gravimetric geoid are three 5’x5’ grids of the geoid undulations and the two components of the deflection of the vertical. Table 4 summarizes the obtained results, and figures 6, 7, and 8 present the findings in three contour maps. It can be noticed from these figures and table, that the computed geoid undulations range from 5.42 m to 22.40 m with a mean value of 14.54 m and RMS equals 2.96 m. The obtained values of the deflection of the vertical in the meridian component have a minimum of –20.53”, a maximum of 21.88”, an average of –0.92”; and RMS of 4.26”. The deflection of the vertical in the prim vertical direction ranges from –24.93” to 25.48” with a mean value of 1.11” and RMS equals 3.98”.

Several comparisons have been carried out between the obtained gravimetric geoid model, SRI2001A, and the global geopotential models OSU91A and EGM96. Table 5 presents the results of these comparisons. From this table, it can be seen that the SRI-GEOID98 undulations range from 7.22 m to 22.55 m with an average of 15.31 m and RMS equals 3.10 m. The corresponding minimum, maximum, average, and RMS values of the EGM96 undulations are 5.90, 21.52, 14.16, and 2.92 m respectively. The OSU91A model gives undulations range from 6.77 m to 23.12 m with a mean value of 14.81 m and RMS equals 3.29 m. The developed geoid model differs from the EGM96 model by a mean value equals to 0.38 m with RMS equals 2.75 m, while it differs from OSU91A model by an average of –0.27 m and RMS of 3.03 m. A similar comparison shows that the undulation differences between the EGM96 and a gravimetric geoid of the USA range from –1.86 m to 3.34 m [Smith and Milbert, 1997b].

Table 4: Statistics of the SRI2001A gravimetric Geoid

Item	Minimum	Maximum	Average	RMS
Geoid Undulations (m)	5.42	22.40	14.54	2.96
Deflection of the vertical in the meridian direction (“)	-20.53	21.88	-0.92	4.26
Deflection of the vertical in the prime vertical direction (“)	-24.93	25.48	1.11	3.98

Table 5: Statistics of Comparisons of the SRI2001A and Two Global Geoid Models

Item	Minimum	Maximum	Average	RMS
SRI2001A	5.42	22.40	14.54	2.96
EGM96	5.90	21.52	14.16	2.92
OSU91A	6.77	23.12	14.81	3.29
SRI2001A – EGM96	-10.13	7.54	0.38	2.75
SRI2001A – OSU91A	-10.62	9.30	-0.27	3.03

5.2 Developing a Combined GPS/Gravity Geoid (SRI2001B)

The issue of combining gravity and GPS data in developing high-precision geoid models gains a lot of attention in the last few years. Several research studies have handled this point and investigated different fitting polynomials (e.g. Shaker et al, 1997, Smith and Milbert, 1997b, Veronneau, 1997; and Denker, et al, 1996). Forty five common stations, between gravity and GPS available data sets, have been used, to relate the obtained gravimetric geoid model to a precise geoid undulation datum as obtained from GPS measurements on precise levelling known stations. The developed combined geoid model, depicted in figure 9, has a minimum undulation value of 9.437 m and a maximum value of 21.39 m with an average of 13.623 m and RMS of 2.62 m.

Several comparisons have been made in order to investigate the reliability of the developed GPS/Gravity geoid. The first comparison has been made between the developed geoid and a previously determined GPS/Gravimetric geoid, SRI-GEOID98, based on OSU91A geopotential model [Dawod, 1998]. The differences between the two geoids range from –5.40 m to 5.71 m with a mean of –0.57 m and RMS equals 1.88 m. In order to investigate the accuracy of SRI2001B, fifteen independent GPS/Levelling stations have been used. The values of the estimated undulations from the SRI-GEOID98 model have been compared against the pure GPS undulation of these stations. These results are summarized in table 6. From this table, it can be seen that the differences range from –0.01m to –0.28 m with an average of –0.10 m and RMS of 0.49 m. The corresponding differences of the previous SRI-GEOID98 model have a minimum of –1.69 m, a maximum of –0.05 m, a mean of 0.41 m; and RMS of 0.79 m. Consequently, it can be concluded that SRI2001B is more precise than SRI-GEOID98, which indicates that the EGM96 global geopotential model represents the most precise geopotential model to be used for geoid determination in Egypt.

Table 6: Statistics of Comparisons of the SRI2001B and SRI-GEOID98 Geoid Models

Item	Minimum	Maximum	Average	RMS
SRI2001B	9.44	21.39	13.62	2.62
SRI-GEOID98	7.22	22.55	15.31	3.10
(SRI2001B) – (SRI-GEOID98)	-5.40	5.71	-0.57	1.88
N (SRI-GEOID98) – N(pure GPS)	-1.69	-0.05	-0.41	0.79
N (SRI2001B) – N(pure GPS)	-0.28	-0.01	-0.10	0.49

6. Summary and Conclusions

The EGM96 is the most recent and precise spherical harmonic model representing the potential field of the Earth. New terrestrial gravity and satellite altimetric data have been used in the development of EGM96 and lead to significant improvements in the precision of geoid determination using this model. Compared to the OSU91A geopotential model, EGM96 proves to be more precise and reduces the uncertainties (in terms of RMS) in geoid undulations over Egypt from 1.86 to 0.78 m, when compared with GPS/Levelling undulations. Two precise geoid models have been developed for the entire Egyptian territory. The first geoid, called SRI2001A, is a gravimetric geoid model utilizing the most recent and accurate first-order gravity measurements, and is based on the GRS80 reference datum. The EGM96 global geopotential spherical harmonic model is used to provide the long wavelength of the Earth gravitational field, along with a local DEM in the remove-compute-restore FFT processing technique. The obtained geoid undulations range from 5.42 m to 22.40 m with a mean value of 14.54 m and RMS equals 2.96 m. A GPS/Levelling data set of 195precise stations have been used to generate a geometric-satellite geoid model. A second-order polynomial as a function of the distance from the network origin is found to be the best fitting function to integrate gravimetric undulations and GPS/Levelling undulations. Therefore, a combined GPS/Gravity geoid for Egypt, SRI2001B, has been generated. It has a minimum undulation value of 9.437 m and a maximum value of 21.39 m with an average of 13.623 m and RMS of 2.62 m. The SRI2001B geoid model is compared to a previously determined GPS/Gravimetric geoid, SRI-GEOID98, based on OSU91A geopotential model. The differences between the two geoids range from –5.40 m to 5.71 m with a mean of –0.57 m and RMS equals 1.88 m. The values of the estimated undulations from the SRI-GEOID98 model have been compared against the pure GPS undulation of some independent GPS/Levelling stations. The differences range from –0.01m to –0.28 m, with an average of –0.10m, and RMS of 0.49 m. The corresponding differences of the previous SRI-GEOID98 model have a minimum of –1.69 m, a maximum of –0.05 m, a mean of 0.41 m; and RMS of 0.79 m.

From the obtained results, it can be concluded that:

* The EGM96 global geopotential model represents the most precise geopotential model to be used for geoid determination in Egypt.
* The developed gravimetric geoid SRI2001A is more precise than previously determined gravimetric geoid solutions in Egypt, mainly because of the utilization of the precise ENGSN97 gravity values. Moreover, it gives smaller RMS compared with the gravimetric geoid SRI-GEOID98 that was developed based on OSU91A model.
* The developed combined GPS/Gravity SRI2001B geoid model is the most precise geoid model for Egypt based on the available data.

Based on the previous conclusions, some recommendations may be suggested:

* It is highly recommended that the ENGSN97 gravity network being incorporated into any new global geopotential models in order to increase the accuracy of those models in representing at least the medium wavelength of the gravity field over Egypt.

* The accuracy of the geoid determination in Egypt can be significantly improved by making additional geodetic measurements in void areas, especially the south-west part of the western desert.

* The established ENGSN97 gravity network should be used in the process of redefinition of the Egyptian geodetic datum and the associated reduction and computations of geodetic quantities needed for surveying and mapping activities.

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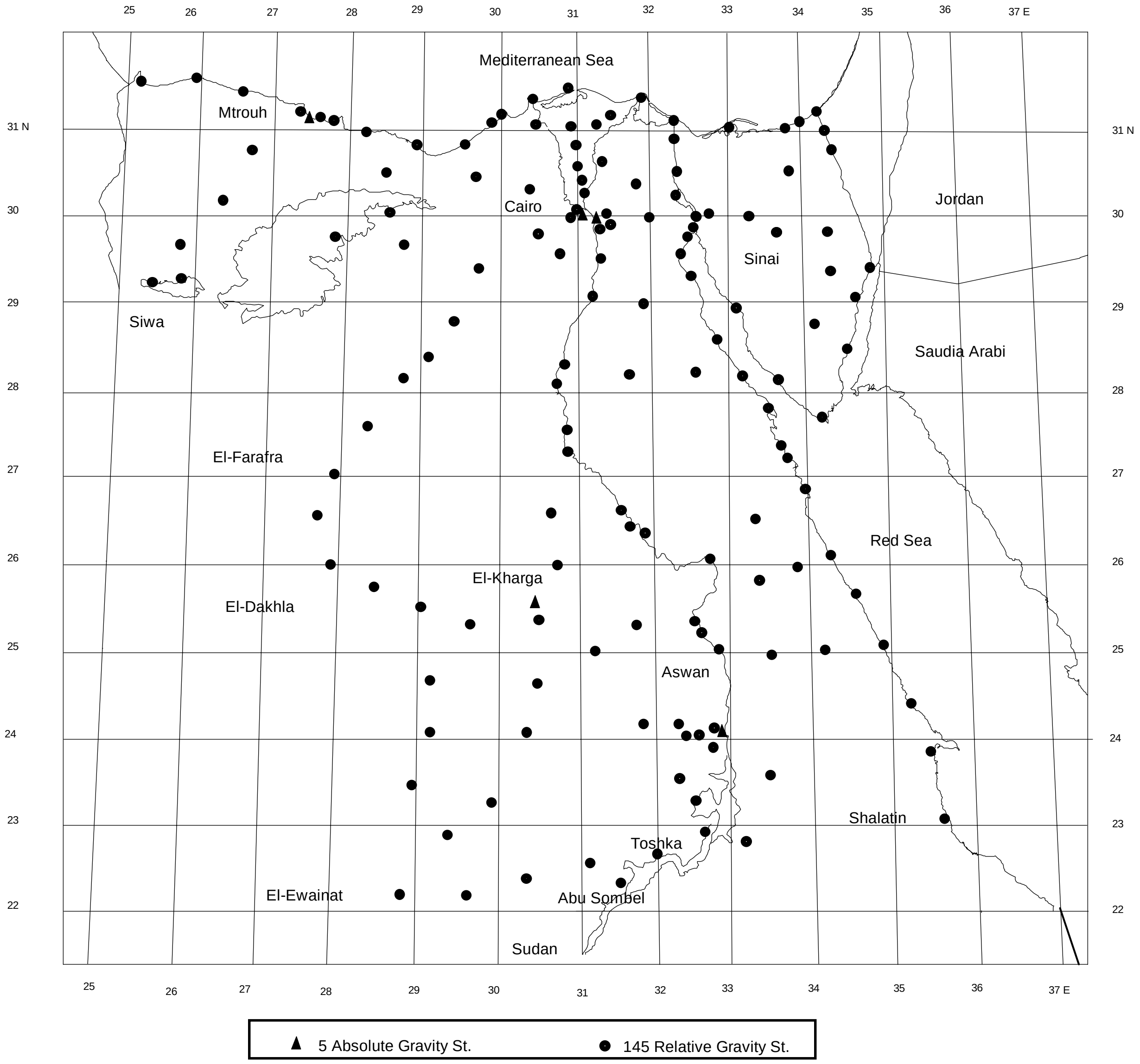

Figure 1: The Egyptian National Gravity Standardization Network of 1997 (ENGSN97)

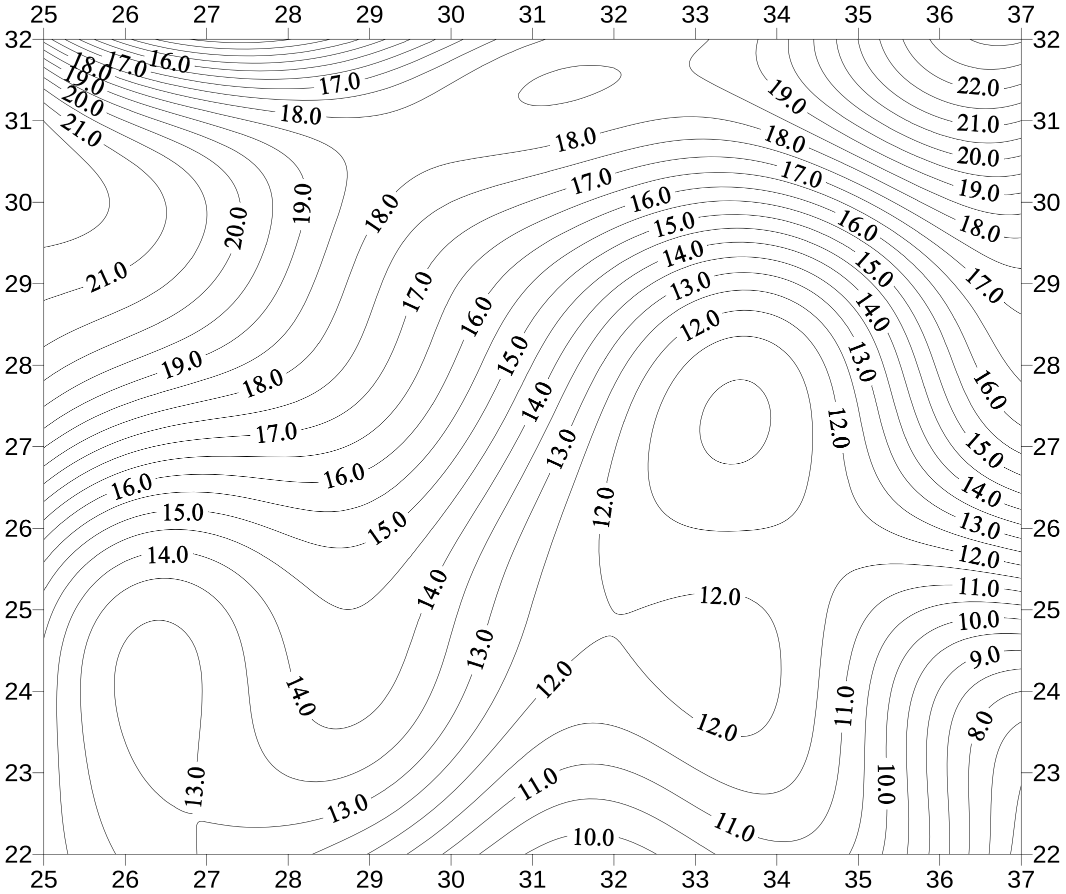

Figure 2: The Geoid Undulations of the OSU91A Global Model

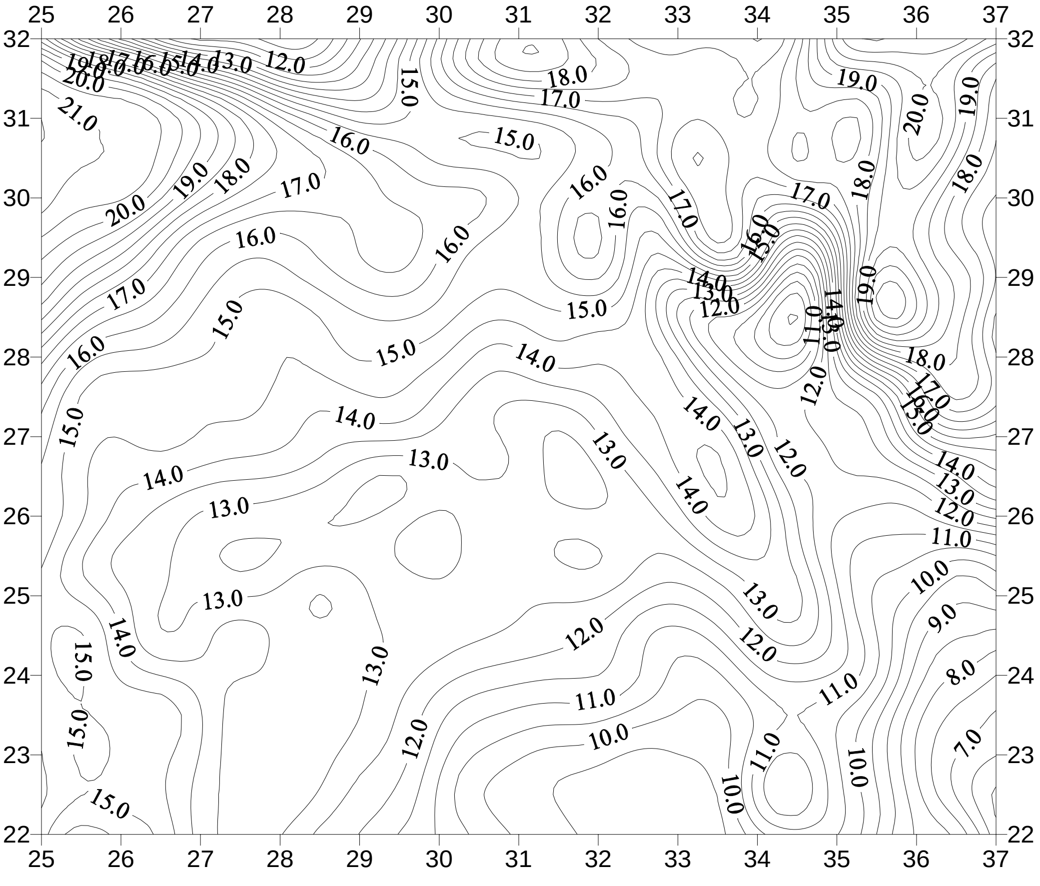

Figure 3: The Geoid Undulations of the EGM96 Global Model

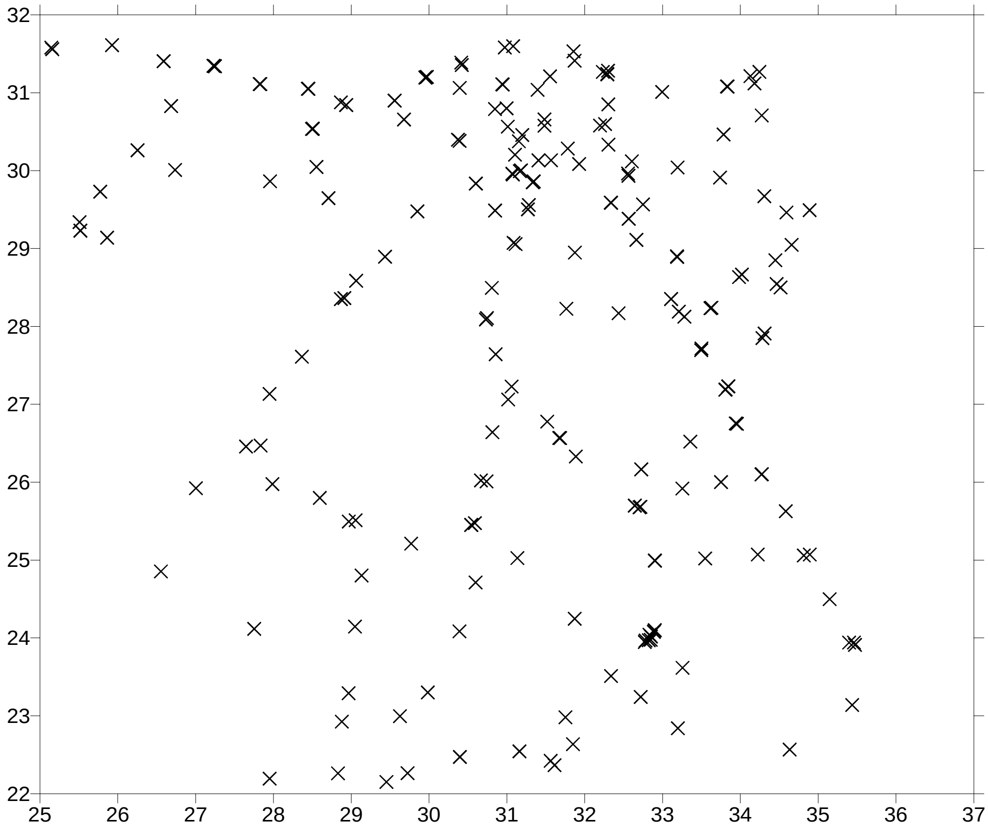

Figure 4: The Available Gravity Stations

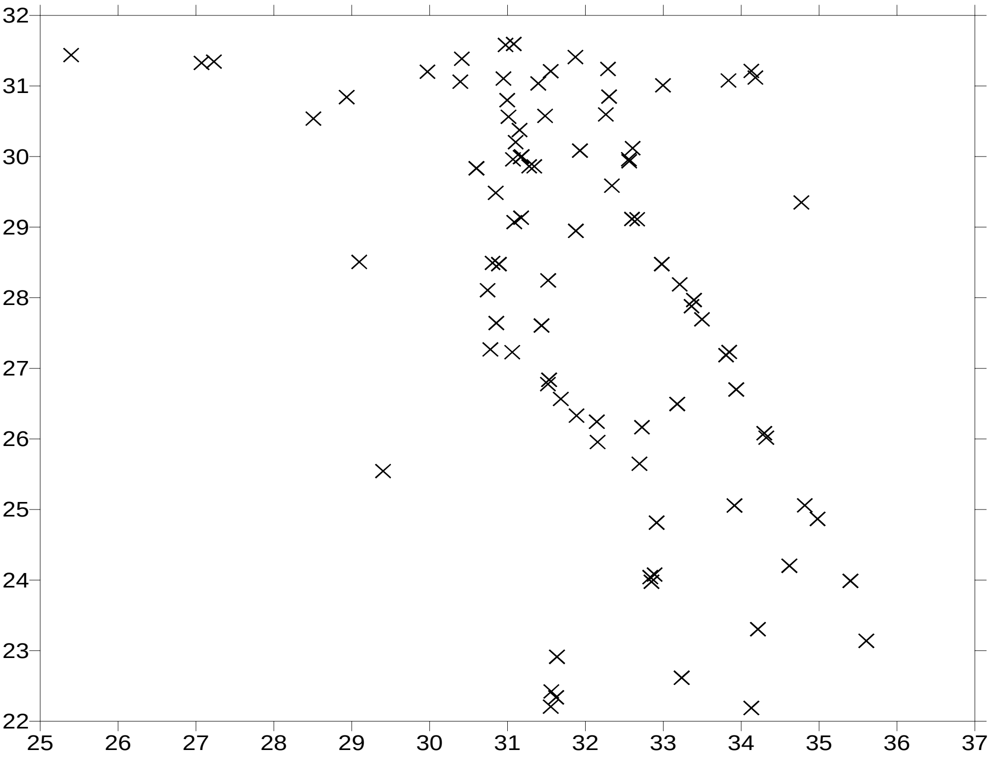

Figure 5: The Available GPS Stations

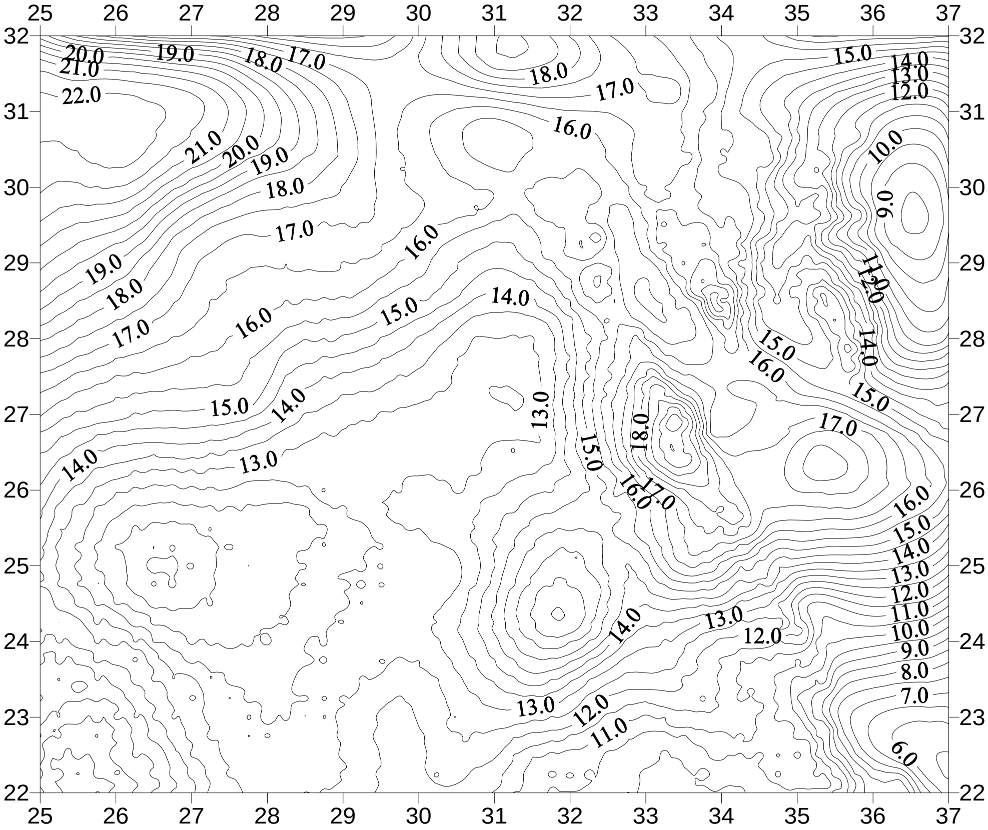

Figure 6: The Geoid Undulations of SRI2001A Gravimetric Geoid Model

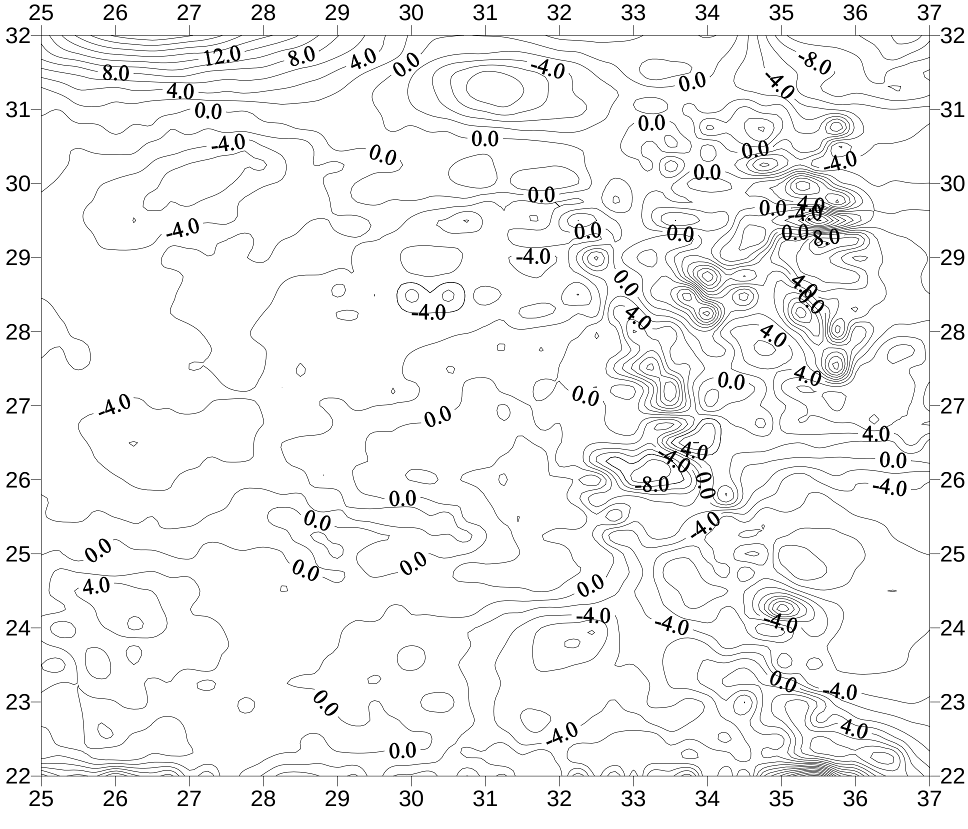

Figure 7: The SRI2001ADeflection of the Vertical in the Meridian Plane

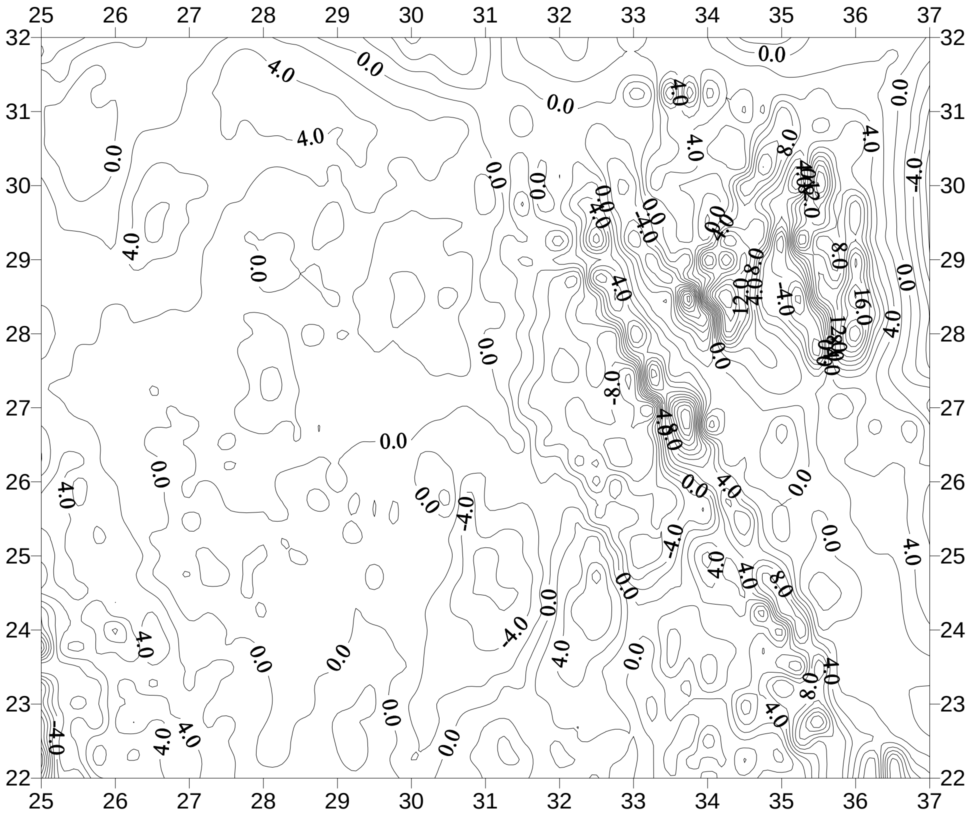

Figure 8: The SRI2001A Deflection of the Vertical in the Prime Vertical Plan

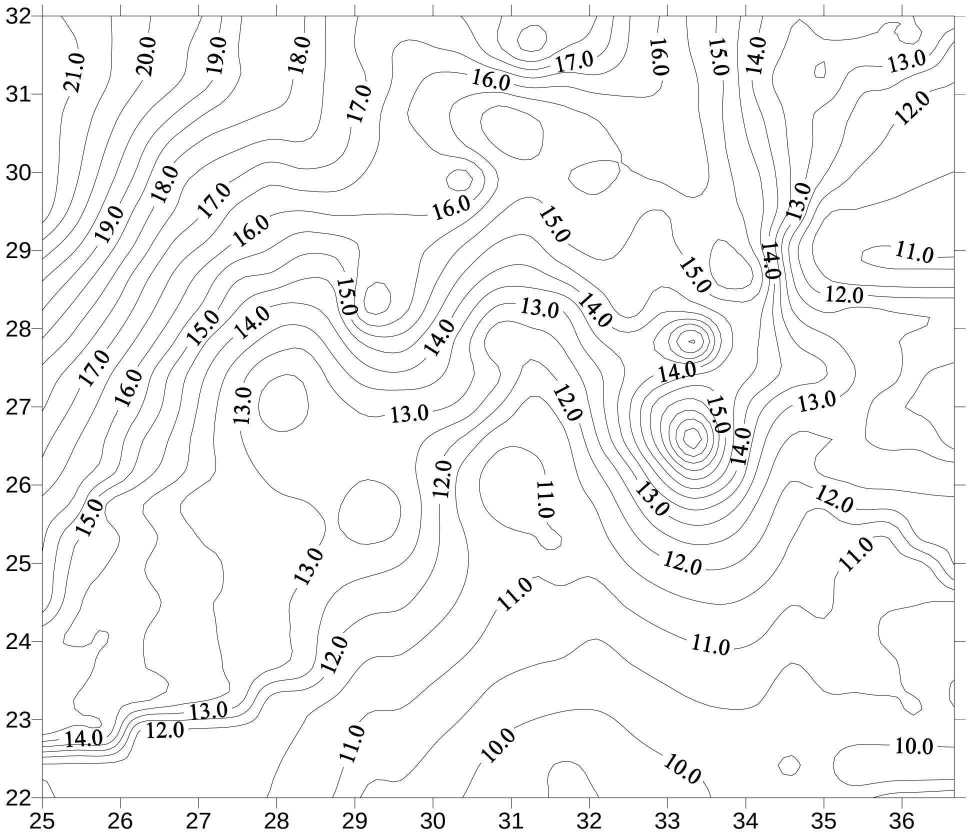

Figure 9: The Geoid Undulations of SRI2001B Combined Geoid Model