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МАТЕМАТИЧЕСКИЕ МЕТОДЫ ВЫЧИСЛЕНИЯ ВЫСОТ ГЕОИДА ПО GPS ДАННЫМ

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MATHEMATICAL TECHNIQUES TO COMPUTE GEOID UNDULATION WITH GPS DATA

Определение геоида было одним из главных направлений научных исследований в области геодезии в течение многих десятилетий. Глобальные навигационные спутниковые системы широко используются в геодезии, и в связи с этим большое внимание уделяется точному определению локального геоида, чтобы заменить геометрическое нивелирование GPS-технологиями.

В данной статье предлагаются различные математические методы для вычисления высот геоида в Египте с помощью GPS данных. Также представлено сравнение точности этих моделей.

Determination of geoid has been one of the main research areas in science of Geodesy for decades. According to the wide spread using of GPS in geodetic applications, great attention is paid to the precise determination of local and regional geoid to replace the geometric leveling, which is very onerous measurement work, with GPS surveys. There are several methods for geoid determination; these methods depend mainly on the available data and the used mathematical models. GPS and leveling method, which is also known as geometric method, is one of these methods. This paper investigates several mathematical techniques for determination the geoid undulation in Egypt from GPS data. Comparison between the accuracy of these models is also presented.

The global positioning system is a satellite based radio positioning system designed for accurate positioning information in three dimensions (Latitude, Longitude and Height), that is, vertical as well as horizontal information is provided. The satellites continuously send radio signals that can be received by suitable equipments. Most topographic elevations are referred to the height above mean sea level (MSL), these heights are determined by leveling and GPS. GPS elevations are called geometric heights and are referenced to the center of earth mass [1].

The geoid is an equipotential surface of the earth that coincides with the undisturbed mean sea level. Therefore it describes the actual shape of the earth. The geoid can be considered as the reference surface for most height networks since leveling gives the heights above the geoid. In geodesy, these heights are called orthometric [3].

The geoidal undulation can be defined as the separation of the reference ellipsoid with the geoid surface measured along the normal ellipsoid. The combined use of GPS, leveling, and geoid height information has been used as key procedure in various geodetic applications. Although these three types of height information are considerably different in terms of physical meaning, reference surface definition, observational methods, accuracy…etc., they should fulfill the following simple geometrical relationship [5]:

$$N = h - H \quad (1)$$

Where: N is the geoid undulation, h is the ellipsoidal height, H is the orthometric height.

Among the geoid modeling techniques, there is one, which is based on the reference points that have been chosen in the most critical locations for representing of the geoid, is one of the most common used method. In which, representing geoid heights is considered as an analytical surface and deriving the geoid undulation values in new points, which are measured with GPS technique, according to the mathematical formulation of this surface constitutes the basic idea of this technique.

This paper investigates a solution to compute the adjusted values of geoid undulation in Egypt from the data available. The suggested mathematical methods used the least square techniques to solve the derived equations of each suggested method. The suggested mathematical techniques are:

1) Multiple regression equation (MRE);
2) Least square collocation (LSC);
3) Minimum curvature surface (MCS).

According to Erol and Celik, (2004), the important factor which affected on the accuracy of GPS/leveling geoid model is the number of reference stations and its distribution [6]. These stations must be distributed homogeneously to the cover the studied area to detect any changes of geoid surface. The first suggested mathematical model is multiple regression equation (MRE) summarizes as following:

MRE or polynomial approximation functions are subjected to variations which may be achieved by different polynomial functions. The general form of multiple regression equation has the form:

$$N = A_0 + A_1U + A_2V + A_3U^2 + A_4UV + A_5V^2 + \ldots\ldots\ldots\ldots A_{nn}U^nV^n, \quad (2)$$

Where: $A_0, \ldots, A_{nn}$ are the coefficients; N is the geoid undulation; U, V are the available data, that is, the coordinates of points.

The following flowchart (figure 1) explains the MRE technique to compute the geoid undulation. To find out the best suitable degree of polynomial for determination the geoid undulation in Egypt, distortions values of several check points are calculated from known points.

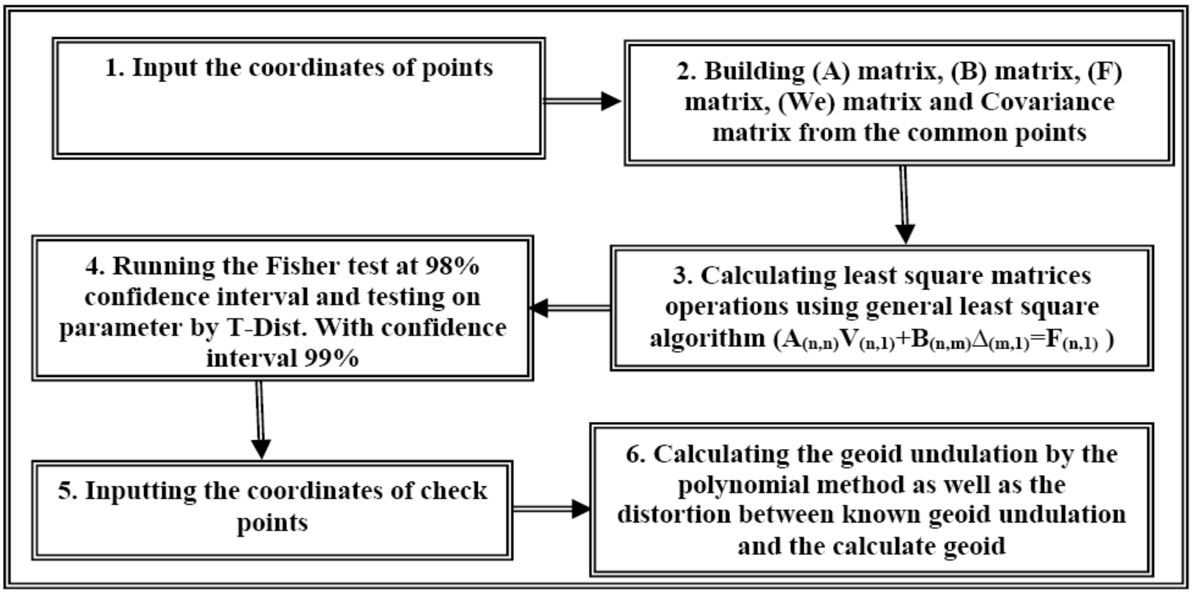

Fig. 1. Flowchart of the solution steps by using polynomial technique

The second suggested technique is least square collocation (LSC). The purpose of using LSC technique is to supplement the GPS/Leveling observations which are made at only a relatively few points. A trial was made to compute the geoid undulation in Egypt by using least square collocation technique as following:

– The first step of the program module is dividing the Egypt map to grids 0.50' x 0.50';

– The second step is designing the covariance matrix between the data points and the computation points. Then the distances between data points and the geoid undulation are computed. The procedure of solution by using LSC is explained as following:

Divide all the distances between every pair of points to equal distance (r_k). So, for the distance r_1 the covariance between every pair of points can be calculated according to the following equation:

$$C_1 = \frac{1}{n_1}\sum U_i U_J \quad , \tag{3}$$

Where: $\boldsymbol{C_1}$ is the covariance between i,j; n_1 is the total number of pairs which satisfy the distance $r_{i,j}$; U_i ,U_j are the measurements at points i,j.

For the general case for every distance r_k the general equation is as follows:

$$C_k = \frac{1}{n_k}\sum U_i U_J \tag{4}$$

A histogram can be drawn showing the relation between the distance and the covariance between points as shown in figure 1.The curve function can be expressed as:

$$C_{i,j} = ae^{(-br_{ij})} \tag{5}$$

Where: a, b are unknown parameters and Cij are observations of geoid undulation.

Using least square method, we may calculate the parameters a, b (figure 2).

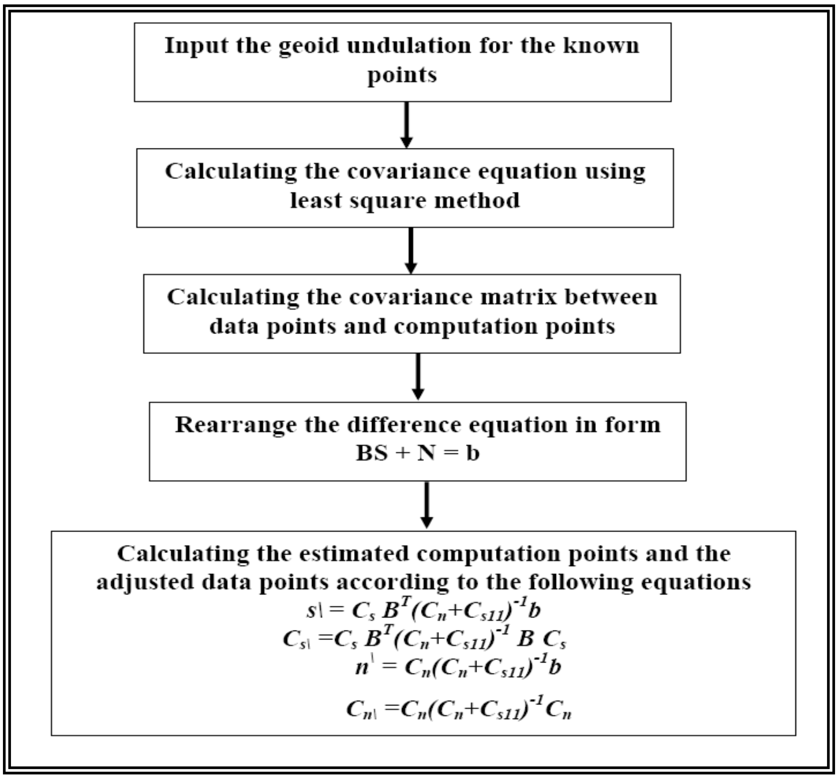

Fig. 2: Flow chart of LSC model

Then the computation value of geoid undulation at the grid can be obtained from the following equations as a special case of least square collocation where the system will be modeled by the signal S and noise n.

$$\left.\begin{aligned} S^{\backslash} &= C_s B^T (C_n+C_{s11})^{-1} b, \\ Cs^{\backslash} &= Cs\, B^T (C_n+Cs11)^{-1} B\, C_s, \\ n^{\backslash} &= C_n (C_n+C_{s11})^{-1} b, \\ C_n^{\backslash} &= C_n (C_n+C_{s11})^{-1} C_n, \end{aligned}\right\} \tag{6}$$

Where: C_n is the covariance matrix of observations.

The third suggested technique is minimum curvature surface (MCS). MCS is an old and over-popular approach for constructing smooth surface from irregularly spaced data. The surface of minimum curvature corresponding to the minimum of the Laplacian power or, in alternative formulation, satisfies the biharmonic differential equation.

In most of practical application cases, the minimum-curvature technique produces a visually pleasing smooth surface. However, in case of large changes in the surface gradient, the method can create strong artificial oscillations in the unconstrained regions [2]. Explanation for how to convert the system of Laplace and Possion equations by MATLAB program is demonstrated by using the same divided grids in the LSC. The four arms about the nodes may be not completed. So, equation 7 can be rearranged as follows to be suitable in our special case such as:

$$\frac{2\varphi_a}{k_1(k_1+k_2)}+\frac{2\varphi_c}{k_1(k_1+k_2)}+\frac{2\varphi_b}{l_1(l_1+l_2)}+\frac{2\varphi_4}{l_1(l_1+l_2)}-(\frac{2}{(l_1l_2)}+\frac{2}{(k_1k_2)})\phi_0=\begin{cases}0, Laplace\\ h^2 f_0, Poisson\end{cases} \qquad (7)$$

Where: K_1, K_2, l_1, l_2 are the ratio from the complete grid arm (h) as shown in figure 3; f_0 - function of unknown value φ_0.

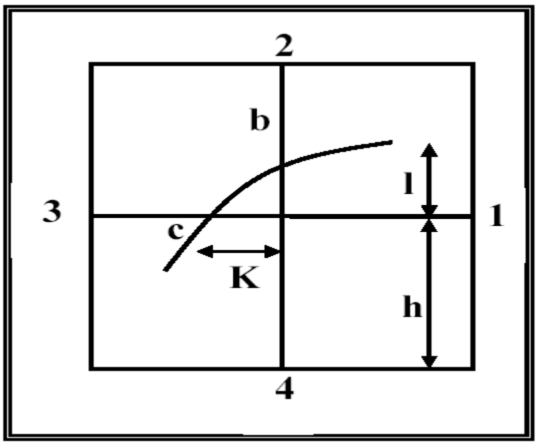

Fig. 3: The case of uncompleted of grid arms

It can be noticed from this formula the position effect of stations on the neighbor nodes.

– For using the grid, there is no information about true difference at the boundaries of the grid to be used in the solution. Unified least square technique will overcome this obstacle and the boundaries difference will be predicted and adjusted according to the available data.

– The number of nodes. This obstacle can be solved by designing a program using MATLAP program. Unified least square can be applied to equations with Laplace equation.

The work is done for the data available from report of S.Powell, 1997 (oz_2, oz_7, oz_8,oz_9 ,oz_{10},oz_{11},oz_{12} ,oz_{13},oz_{14},$oz_{15}$$oz_{16}$,$oz_{17}$,$oz_{18}$,$oz_{19}$,$oz_{20}$,$oz_{21}$,$oz_{22}$) are common points and (oy_{27} ,oy_{35}, oy_{36}, oy_{41}, oz_{32}, oz_{44}, oz_{52},oz_{66},oz_{68},oz_{70},oz_{74},oz_{97}) are check points (at these points, geoid undulation are known) [4]. The comparison between the resulted distortion of MRE, LSC and MCS models was done. The distortion values at check points are calculated. Distortion at check points is shown in figure 3 and the standard deviation of the distortion is shown in figure 4.

From the results, it is obvious that the MCS model has the minimum distortion with high accuracy. This means that the MCS is the best model to compute the geoid undulation in Egypt. The distortion values at check points are calculated. Distortion at check points is shown in figure (5) and the standard deviation of the distortion is shown in figure (5).

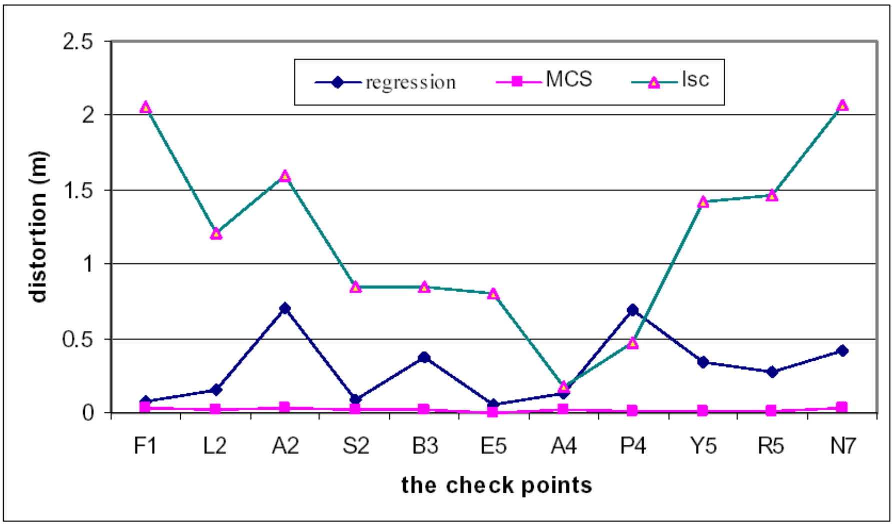

Fig. 4: The distortion at check points by different mathematical models

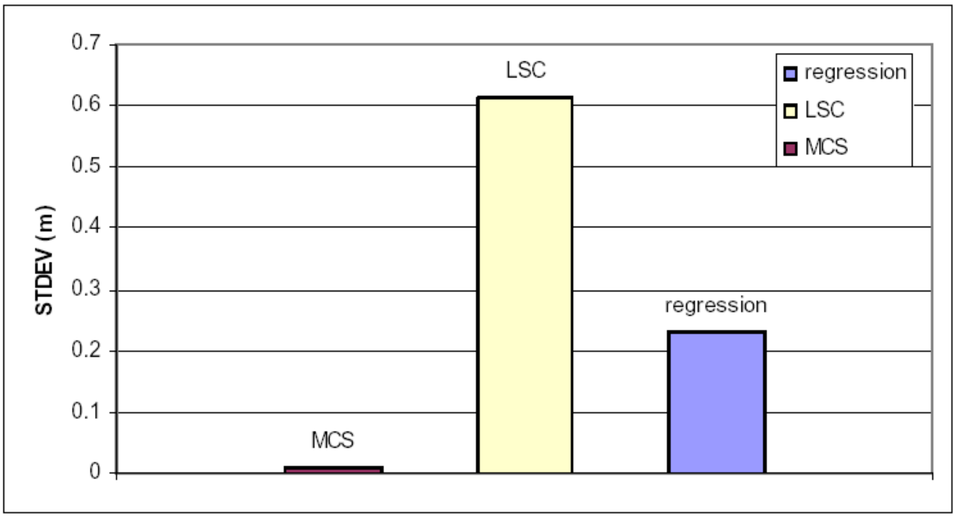

Fig. 5: The standard deviation of the distortion by different mathematical models

As a conclusion, the three suggested method is suitable for determination geoid undulation in Egypt. MCS technique gives the best results for geoid undulation rather than MRE and LSC techniques. It is recommended to use MCS technique to compute the geoid undulation in Egypt.

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