

# Investigating the Precision of Recent Global Geoid Models and Global Digital Elevation Models for Geoid Modelling in Egypt

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## Abstract

A national geoid model is crucial for a wide range of surveying and civil engineering applications worldwide, with Egypt being of no exception. Although there are many researchers attempted to develop a national Egyptian geoid model, the limitations of available geodetic data sized the precision of such models. In order to increase the accuracy of a geoid model, a precise Global Geopotential Model (GGM) along with a precise Digital Elevation Model (DEM) is needed. This papers aims to quantify the precision of most-recent released GGM and global DEM models based on a precise local geodetic dataset (gravity and GPS/Levelling data) covers the Egyptian territories. The attained results show that, out of seven investigated recent GGMs, the EGM2008 is the most precise global models over Egypt, that produces a mean standard deviation of geoid undulation differences equals  $\pm 0.23$  meter over observed 1074 GPS/Levelling stations. Additionally, the EGM2008 model gives a mean standard deviation of gravity differences that equals  $\pm 25.1$  mGal when examined over 941 observed gravity points. Furthermore, it has been shown that the SRTM3 DEM produces a mean standard deviation of  $\pm 4.3$  meter when compared over 1227 points of observed orthometric heights. Consequently, it is recommended to apply these two specific models in the undergoing development of a national geoid model of Egypt.

**Keywords:** Geoid, Gravity, GPS, GGM, DEM, Egypt

## 1. Introduction

A precise geoid model constitutes one of the most challenging research subjects of geodesy, particularly since 1980s. Geoid modelling deal with the determination of geoid undulations between the geodetic heights obtained from the Global Navigation Satellite Systems (GNSS) techniques and the orthometric heights, or levels, relative to the Mean Sea Level (MSL). Thus, geoid models are crucial for the utilization of GNSS (particularly the Global Positioning System: GPS) in civil engineering projects. Over the last two decades, national precise geoids have been developed all over the world, e.g. in Indonesia (Pahlevi et al. 2015), Uganda (Ssengendo et al. 2015), Korea (Lee and Kim, 2012), New Zealand (Classens et al. 2011), Sudan (Abdalla and Fairhead, 2011), and Italy (Corchete 2011).

In Egypt, since the development of the first pioneer national-scale geoid (Alnaggar 1986), several geoid researches have been carried out. For example, Dawod (1998) has developed a national geoid model based on the data of the Egyptian National Standardization Network of 1997 (ENGSN97) along with GPS/levelling data. Saad and Dawod (2002) have developed a national geoid model

based on the EGM96 global geopotential model along with GPS/levelling datasets. In addition, Abd-Elmotaal (2008) has developed a gravimetric geoid model utilizing high-degree tailored reference geopotential model. Recently, Rabah and Kaloop (2013) have developed a local geoid model utilizing the minimum curvature surface technique. and Al-Karargy et al. (2014) have investigated the utilization of GPS/levelling in developing geoid models for small areas in Egypt.

The geoid undulation (N) can be computed from gravity data by the well-known Stokes' formula (e.g. Heiskanen and Moritz 1967):

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} S(\psi) \Delta g d\sigma \quad (1)$$

where: R is the mean Earth radius,  $\gamma$  is the normal gravity on the reference ellipsoid,  $\Delta g$  is the gravity anomaly,  $d\sigma$  is an infinitesimal surface element on the unit sphere  $\sigma$ , and  $S(\psi)$  is the Stokes' function which can be expressed as a series of Legendre polynomial  $P_n(\cos\psi)$  over the sphere:

$$S(\psi) = \sum_{n=2}^{\infty} (2n+1) \frac{1}{n-1} P_n(\cos\psi) \quad (2)$$

Stokes' formula (Eq. 1) need to be applied over the whole Earth, however in practice gravity datasets of the whole Earth are not available. Thus the gravimetric geoid modelling methods break up the gravity anomalies ( $\Delta g$ ) into three components:

$$\Delta g = +\Delta g_{REF} + \Delta g_F + \Delta g_h \quad (3)$$

where:  $\Delta g_F$  represents the free-air gravity anomalies,  $\Delta g_h$  is the effect of topography, and  $\Delta g_{REF}$  represents the gravity anomalies of a reference gravity field represented by a GGM.

Therefore, the full geoid undulation (N) is decomposed into three components too:

$$N = N_{REF} + N_{\Delta g} + N_h \quad (4)$$

where:  $N_{\Delta g}$  is the contribution of the reduced gravity anomalies computed by Stokes's integral,  $N_h$  is the contribution of the topography, and  $N_{REF}$  is the contribution of the reference gravity field.

Consequently, geoid modelling techniques require the utilization of a GGM to represent the global variations or long wavelengths of the Earth gravitational field, along with a DEM to depicts the topography of the local area and determine its effects on the developed geoid model. Hence, the precision of the utilized GGM and DEM influences the attained precision of the geoid models. Several research studies (e.g. Mahmoud 2012, Dawod 2008, Arabelos and Tscherning 2010, and Erol et al. 2009 ) have investigated the precision of several GGM and DEM as an initial procedure for precise geoid modelling. This paper aims to explore the accuracy of several most-recent GGM and DEM models in representing the gravity and topography fields over Egypt in order to select which models to be utilized in developing a precise national geoid model.

## 2. Global Geopotential Models (GGM)

The geoid undulations (N) may be computed using the following spherical harmonic expansion:

$$N = \left(\frac{GM}{r\gamma}\right) \sum_{n=2}^{n \max} (a/r)^n - \sum_{m=0}^n ((C_{nm}^- \cos m\lambda) + (S_{nm}^- \sin m\lambda)) P_{nm} \sin \phi \quad (5)$$

where: n is the degree of the GGM model, n max is the maximum degree of the GGM model, m is the maximum order of the model,  $\gamma$  is the normal gravity of the reference ellipsoid, r is the geocentric radial distance of the computation point projected on the ellipsoid, G is the Newtonian gravitational constant, M is the mass of the Earth, a is the semi-major axis,  $\phi$  is the geocentric latitude,  $\lambda$  is the geocentric longitude,  $C_{nm}^-$  and  $S_{nm}^-$  are the fully normalized harmonic coefficients, and  $P_{nm}$  is the fully normalized associated Legendre polynomial.

GGM models have been developed since the 1960s as an important tool for geoid modelling on a regional or local scales. So far, there are more than 150 GGM available at the website of the International Center for Global Earth Models (<http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html>). In developing GGM models, datasets from several sources might be utilized such as: satellite-based gravity data, terrestrial gravity data, satellite altimetry data, and terrestrial geodetic data. Therefore, the maximum degree and the precision of GGM models vary significantly.

This paper utilizes seven GGM models and evaluates their accuracy when compared against precise local geodetic datasets in Egypt. Those models are:

- EIGEN-6C4: A model released in 2014, that utilizes satellite tracking data (from both LAGEOS, GRACE, and GOCE missions) along with a global surface gravity anomaly grid. The model is up to 2190 degree. It was developed by both the Germany GFZ research center and the French CNES research center.
- GO\_CONS\_GCF\_DIR\_R5: A satellite-only GGM up to degree 300, developed by the European Space Agency (ESA), and released in 2014. It utilizes data from LAGEOS, GRACE, and GOCE satellite missions.
- GO\_CONS\_GCF\_TIM\_R5: Another satellite-only ESA model dated 2012 with a maximum degree of 250. It utilizes data from only GOCE satellite mission.
- DGM-1S: A model developed by Delft university, Netherlands, that was released in 2012. Also, it is a satellite-only GGM based on data from both GOCE and GRACE missions.
- EGM2008: An integrated GGM developed by the US National Geospatial-Intelligence Agency (NGA) up to 2190 degree. It was developed in 2008 based on satellite tracking data, terrestrial gravity data, and altimetry data. It was a millstone in GGM development, since its preceding model did not exceed 360 maximum degree.
- EIGEN-5C: A traditional integrated GGM (i.e. up to maximum degree of 360) developed by the Germany GFZ research center.
- EGM96: One of the most famous GGM that was developed, in 1996, by the US National Aeronautics and Space Administration (NASA) up to maximum degree of 360. It was based on a previous satellite-only GGM (EGM96S) combined with terrestrial and altimetry datasets.

Table 1 summarizes the main characteristics of those GGM models. From this table, it can be realized that the selection of such models depicts a variety in their nature in terms of development year, maximum model degree, and the types of data utilized in the development of each model.

**Table 1: Characteristics of Utilized GGM Models**

GGM Model	Year	Max Degree	Data Type
EIGEN-6C4	2014	2190	S (GOCO, GRACE, LAGEOS), G, A
GO_CONS_GCF_2_DIR_R5	2014	300	S (GOCO, GRACE, LAGEOS)
GO_CONS_GCF_2_TIM_R5	2014	280	S (GOCO)
DGM-1S	2012	250	S (GOCO , GRACE)
EGM2008	2008	2190	S ( GRACE ), G, A
EIGEN-5C	2008	360	S ( GRACE , CHAMP), G, A
EGM96	1996	360	EGM96S, G, A
where: S = Satellite tracking data, G = Terrestrial gravity data, A = Altimetry data, GOCE, GRACE, and LAGEOS are gravity satellite missions.			

The International Center for Global Earth Models (ICGEM) continuously analyzes the performance of every new released GGM model over several GPS/Levelling check points worldwide. Table 2 presents the statistics of evaluation of the selected seven GGM models in terms of the Root Mean Square error (RMS) of the undulation differences. It can be seen that the overall accuracy of EGM2008 over the entire 12036 check points indicates that it is the most precise GGM.

**Table 2: RMS of Differences of GGM-Based Geoid Undulations over GPS/Levelling Check Points (m)**

Test Area	USA	Canada	Europe	Australia	Japan	Brazil	ALL
No. of Check Points	6169	2691	1047	201	816	1112	12036
GGM Model							
EIGEN-6C4	0.247	0.126	0.121	0.212	0.079	0.446	0.236
GO_CONS_GCF_2_DIR_R5	0.405	0.299	0.345	0.327	0.447	0.507	0.392
GO_CONS_GCF_2_TIM_R5	0.398	0.310	0.343	0.336	0.450	0.505	0.390
DGM-1S	0.441	0.348	0.413	0.366	0.513	0.517	0.432
EGM2008	0.248	0.128	0.125	0.217	0.083	0.460	0.240
EIGEN-5C	0.341	0.278	0.266	0.244	0.339	0.524	0.342
EGM96	0.379	0.353	0.493	0.298	0.364	0.730	0.427

source: <http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html>

### 3. Digital Elevation Models (DEM)

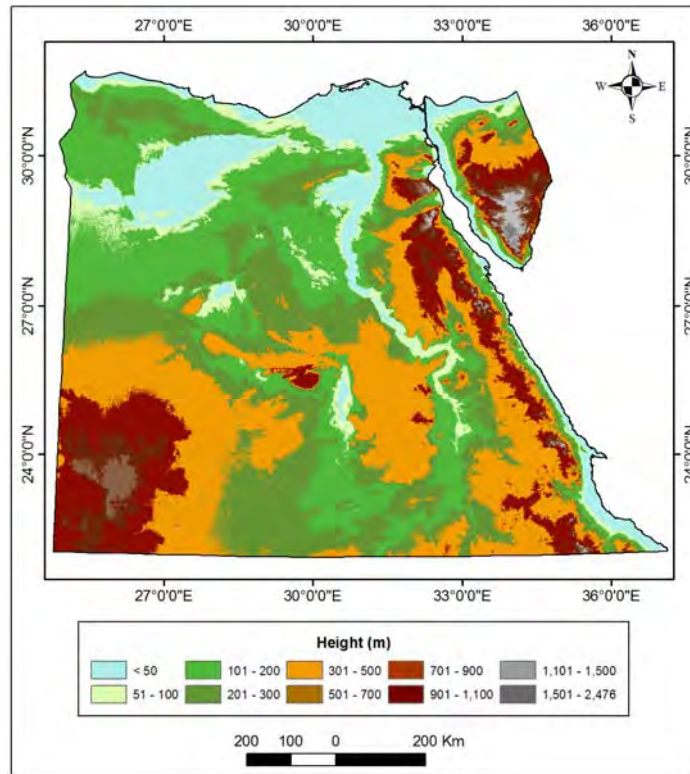
DEM models depict the topography of a spatial area in a digital format, and have been utilized for a wide range of applications including, for instance, topographic mapping, hydrologic modelling, water resources management, and hazards assessment. Several global DEM have been developed and released in the last two decades. Nevertheless, the spatial resolution and the precision of global DEM models vary significantly. In this research, three global DEM models have been tested in order to evaluate their accuracy in depicting the topography of Egypt. Those models are:

- Shuttle Radar Topography Mission (SRTM): SRTM is a joint project between the U.S. National Imagery and Mapping Agency (NIMA) and the National Aeronautics and Space Administration (NASA) (<http://www2.jpl.nasa.gov/strm/>). A 3-arc-second SRTM DEM (SRTM3) for many parts of the world has been compiled and released.
- GTOPO30: It is a global DEM, completed in late 1996, with a horizontal grid spacing of 30" (approximately 1 km). GTOPO30, developed over a three-year period through a collaborative effort led by the U.S. Geological Survey's EROS Data Center (EDC), and was derived from several raster and vector sources of topographic information.
- Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER): The Ministry of Economy, Trade, and Industry (METI) of Japan and the United States National Aeronautics and Space Administration (NASA) have jointly released Version 1 of the Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) Global DEM. ASTER horizontal resolution equals 1 arc second, that is 30 meters approximately, and it is available free of charge (<http://www.gdem.aster.ersdac.or.jp/>).

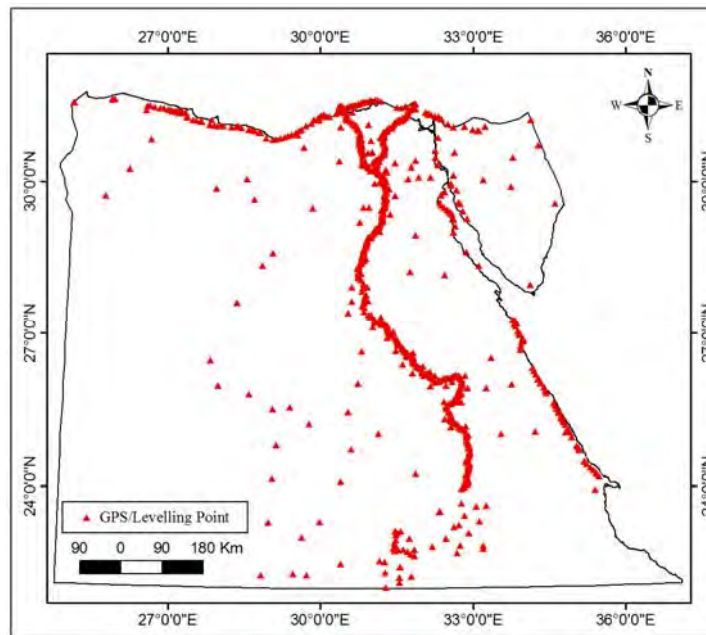
The window of each DEM that corresponds to the Egyptian territories has been downloaded and utilized in this research. For example, the SRTM3 DEM of Egypt (Figure 1) show that the elevations range from -134 m to 2476 m with a mean of 304 m. On the other hand, the GTOPO30 DEM show that the terrain elevations vary between -174 m and 2575 m with an average of 302 m. Moreover, the ASTER DEM show that the terrain elevations vary between -154 m and 2495 m with an average of 300 m.

### 4. Available Data

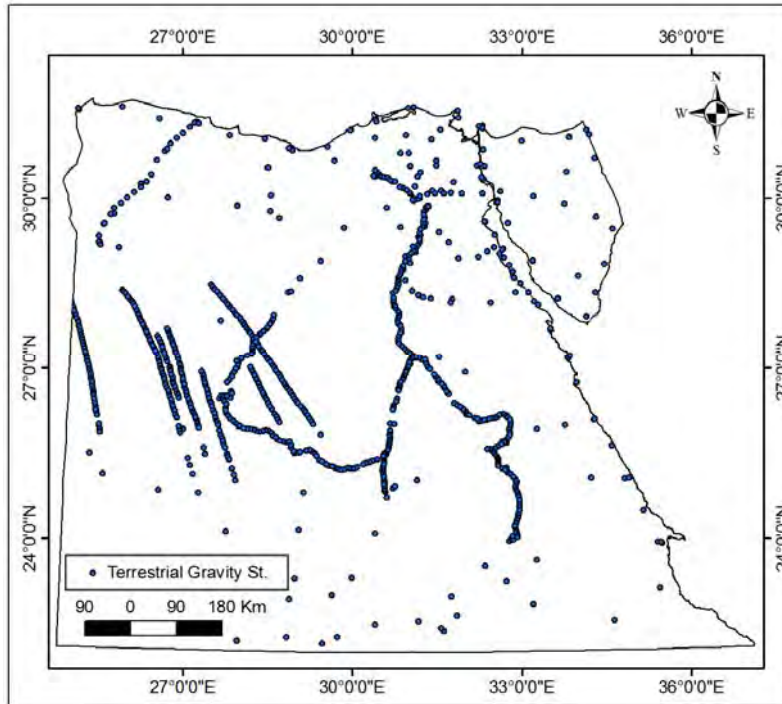
The available geodetic dataset contains 1074 GPS/Levelling stations, along with 941 observed terrestrial gravity points (Figures 2 and 3). The GPS/levelling stations have been observed by the Survey Research Institute (SRI) in several surveying projects mainly in the last five years. Their orthometric heights have been observed by precise levelling while their geodetic heights have been observed by dual-frequency GPS receivers. Thus, an observed geoid undulation is computed at each station. The terrestrial gravity dataset consists of the most-precise data of the Egyptian National Gravity Standardization Network of 1997 (ENGSN97) along with older gravity measurements carried out in 1960s. For the DEM evaluation, known elevations at 1227 points have been utilized (Figure 4).



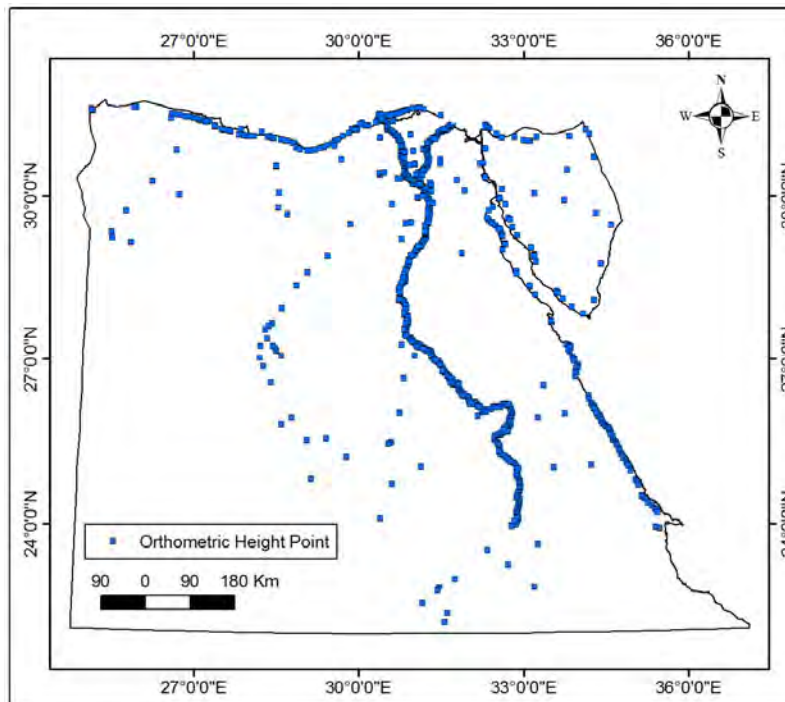
**Figure 1: SRTM3 DEM of Egypt**



**Figure 2: Available Known GPS/Levelling Points**



**Figure 3: Available Known Terrestrial Gravity Points**



**Figure 4: Available Known Elevation Points**

## 5. Data Processing and Attained Results

The performed data processing consists of two stages: (1) evaluation of GGM models, and (2) evaluation of DEM models. The attained results are discussed in the two following sub-sections.

### 5.1 GGM Evaluation Results

The performance analysis of the GGM seven models consists of two procedures: (1) comparing the observed N values against the estimated GGM-based N for each GGM model, and (2) comparing the observed free-air gravity anomaly values against the corresponding GGM-based for each GGM model. The GRAVSOF 2.9 package (Forsberg and Tscherning 2008) has been utilized in computing geoid undulations and free-air gravity anomalies of the selected GGM models at the 941 check points. The accomplished results are presented in Table 3. It can be realized, from this table, that the EGM2008 GGM produces the smallest differences (in terms of RMS values) compared to the observed terrestrial free-air gravity values. The EIGEN-6C4 GGM came in the second place with a close RMS value, while the GO\_CONS\_GCF\_2\_TIM\_R5 GGM give the largest differences. As seen in Table 1, both EGM2008 and EIGEN-6C4 have maximum degree of 2190, while GO\_CONS\_GCF\_2\_DIR\_R5 has maximum degree of only 300. That might explain the attained results, since the higher degree of a model, the smaller spatial resolution in representing the gravity field.

**Table 3: Performance of GGM Models in terms of Gravity Anomalies**

GGM Models	Differences between observed and GGM-Based Free-Air Gravity Anomalies (mGal)			
	Minimum	Maximum	Average	RMS
EIGEN-6C4	-55.2	91.9	23.8	25.5
GO_CONS_GCF_2_DIR_R5	-83.4	92.0	18.1	27.6
GO_CONS_GCF_2_TIM_R5	-82.9	91.7	18.4	28.1
DGM-1S	-50.6	84.2	17.7	27.9
EGM2008	-52.73	80.26	24.75	25.1
EIGEN-5C	-45.7	77.1	18.4	26.7
EGM96	-55.2	122.4	16.9	27.3

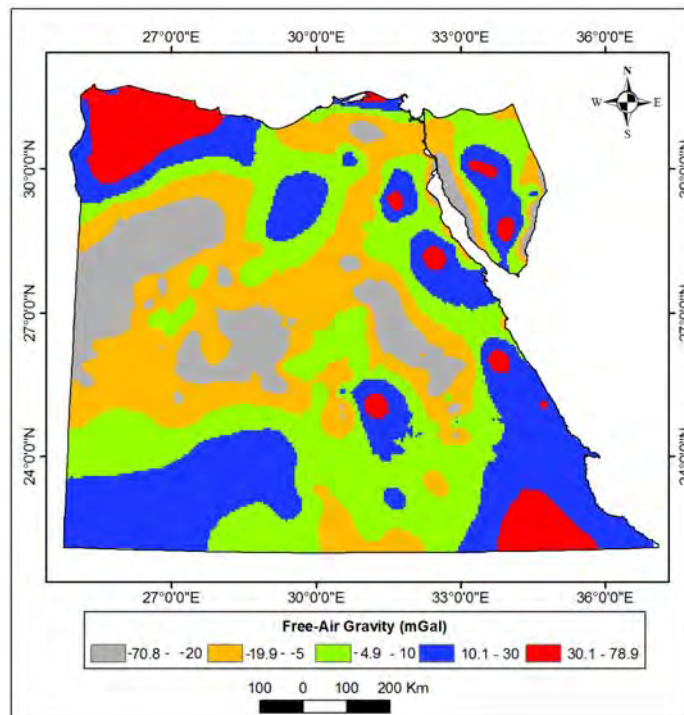
Table 4 presents the geoidal undulations comparison between the observed or known undulations and the GGM-based corresponding values at the 1074 GPS/Levelling check points. It can be seen that the EGM2008 GGM, again, produces the smallest RMS for the differences with a value of  $\pm 0.23$  m. The EIGEN-5C came in the second place with an RMS of  $\pm 0.32$  m. In the last place, the EGM96 GGM came with a value of  $\pm 0.52$  m. The higher precision of EGM2008 might be attributed to the amount and quality of the terrestrial dataset utilized in the model development.



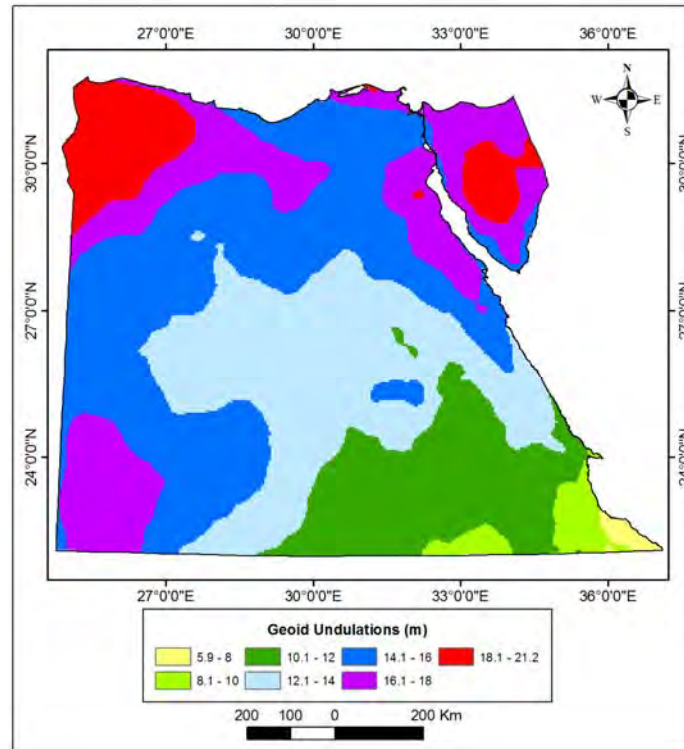
**Table 4: Performance of GGM Models in terms of Geoid Undulations**

GGM Models	Differences between observed and GGM-Based Geoidal Undulations (m)			
	Minimum	Maximum	Average	RMS
EIGEN-6C4	-1.69	0.78	-0.67	0.36
GO_CONS_GCF_2_DIR_R5	-1.51	1.44	-0.29	0.38
GO_CONS_GCF_2_TIM_R5	-1.50	1.49	-0.27	0.38
DGM-1S	-1.93	0.70	-0.71	0.37
EGM2008	-0.59	0.31	-0.22	0.23
EIGEN-5C	-1.50	0.56	-0.68	0.32
EGM96	-1.77	1.77	-0.81	0.52

Based on the attained results, it can be concluded that the EGM2008 is the most precise GGM model in representing the gravity field of Egypt in terms of free-air gravity anomalies and geoidal undulations too. Figures 5 and 6 depicts the long-wavelength contribution of the gravity field over the entire Egyptian territories based on the EGM2008 GGM. Such spatial variations of both geoidal undulations and gravity anomalies should be considered in developing a national precise geoid model.



**Figure 5: EGM2008 Free-Air Gravity Anomalies of Egypt**



**Figure 6: EGM2008 Geoid Undulations of Egypt**

## 5.2 DEM Evaluation Results

For the DEM evaluation, the ArcGIS 10.2 package has been utilized to interpolate the levels of the 1227 available data points based on their geodetic coordinates, and then compare them to the observed orthometric heights. The statistics of the estimated height differences are presented in Table 5. Clearly, it can be realized that the SRTM DEM produces the smallest differences (both in average and RMS values), while the GTOPO30 gave the biggest differences. Similar results have been reported in different countries, such as Nigeria (Amans et al., 2013) and Saudi Arabia (Mirza et al., 2011). Hence, it can be concluded that SRTM is the most precise global DEM model in representing the topography of Egypt.

**Table 5: Performance of DEM Models in terms of Orthometric Heights**

DEM Models	Differences between observed and DEM-Based Orthometric Heights (m)			
	Minimum	Maximum	Average	RMS
SRTM	-9.0	14.8	1.1	4.3
ASTER	-25.8	25.4	2.0	8.1
GTOPO30	-163.9	205.6	4.9	25.2

## 6. Conclusions

A geoid model is fundamental for converting the GPS-based ellipsoidal heights into MSL-based orthometric heights usually used in geomatics, GIS, surveying, geodetic, and mapping applications. So far, there is no available published precise national geoid model that covers the entire territories of Egypt. Thus, this research study has compared seven of the most-recent released GGM (namely EIGEN-6C4, GO\_CONS\_GCF\_DIR\_R5, GO\_CONS\_GCF\_TIM\_R5, DGM-1S, EGM2008, EIGEN-5C, and EGM96) over most-recent precise GPS/levelling points and terrestrial gravity stations, in order to decide the most precise one that precisely represent the Earth's gravitational field over Egypt. On the other hand, DEM is needed in geoid development in order to compute the topographic effects. Hence, three global DEM (SRTM, ASTER, and GTOPO30) have been investigated, too, over a precise geodetic dataset of Egypt.

The accomplished results show that the EGM2008 GGM produces the smallest differences (in terms of RMS values) compared to the observed terrestrial free-air gravity values. The EIGEN-6C4 GGM came in the second place with a close RMS value, while the GO\_CONS\_GCF\_2 TIM\_R5\_2014 GGM give the largest differences. Regarding the geoid undulations, it has been found that the EGM2008 GGM, again, produces the smallest RMS for the differences, and the EIGEN-5C came in the second place. Consequently, it can be concluded that the EGM2008 is the most precise GGM model in representing the gravity field of Egypt in terms of free-air gravity anomalies and geoidal undulations too. Concerning the global DEM evaluation, the attained findings show that the SRTM produces the smallest differences (both in average and RMS values), while the GTOPO30 gave the largest differences. As a result, it is recommended that the EGM2008 GGM and the SRTM DEM should be considered in the undergoing developing of a precise national Egyptian geoid model.

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