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OPTIMUM GEODETIC DATUM TRANSFORMATION TECHNIQUES FOR GPS SURVEYS IN EGYPT

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ABSTRACT

The issue of datum transformation of GPS output coordinates from the global geocentric WGS84 datum to the local national Egyptian non-geocentric coordinate system is a crucial dilemma in Egypt. Although the traditional similarity datum transformation models have been used for several decades, recent research studies show that this approach may not be the optimum technique to describe the mathematical relationship between geodetic datums.

In this paper, the available recent and accurate geodetic databases in Egypt are used to investigate the accuracy of traditional and unconventional approaches of datum transformations. Moreover, the famous models of the traditional similarity datum transformation technique have been examined in several scenarios with different number of unknown parameters. New regression formulas have been developed to transform the GPS coordinates from the global datum to the local one. The obtained results show that the overall accuracy of the traditional transformation technique is in the order of approximately 3 meters, while it is almost 0.5 m for the developed regression approach.

KEYWORDS
Geodetic Datums, GPS; Transformation Techniques

INTRODUCTION

Geodetic datum transformation is the determination of a mathematical relationship to be used in transforming a set of coordinates from one geodetic datum to another. In transforming coordinates from one datum to another, two items should be taken into consideration. The first item is the location of the geometric centers of each reference ellipsoid with respect to the center of gravity of the earth, or with respect to each other. The second point to be concerned is the differences is size and shape between the two ellipsoid.

There are several practical reasons necessitate the geodetic datum transformation. The most recent practical application of the geodetic datum transformation has been raised with the advent of satellite positioning techniques in the fifties. With the rapid growth of utilizing these space systems all over the world, the datum transformation determination has become a major practice in order to transform the coordinates from the global geodetic datum to several local datums for different countries [e.g. Moore and Smith 1998; Reit 1998]. In Egypt, the Global Positioning System (GPS) technique has been used since 1985 in establishing more precise geodetic control networks and for other surveying and mapping applications. Hence, the establishment of a precise mathematical relationship between the GPS-based global geodetic datum, known as the World Geodetic System (WGS84), and the Old Egyptian Datum (OED) becomes a crucial task for the Egyptian geodetic community. This problem has been investigated by several researchers [e.g. Alnaggar 1990; Rashwan 1993; Abd-Elmotaal and El-Tokhy 1997; and Finnmap 1988]. However, the data sets used did not posses a national coverage of the country and their satellite-coordinates were not of a high quality, especially the GPS data of the Finnmap project [Saad 1998, Dawod 1998]. More accurate GPS data are utilized in this research study to define several models of transformation between the WGS84 and OED geodetic datums.

DATUM TRANSFORMATION TECHNIQUES

Traditionally, the similarity datum transformation models have been introduced and utilized in geodetic applications since the sixties. This type of datum transformation has gained a great focus since it has a geometrical interpretation of the determined parameters. However, other mathematical models could be used to describe the relationship between different geodetic datums in a simpler manner that is easier in practice.

Traditional Similarity Transformation Models

A similarity transformation model is based on 7 parameters: three translation parameters (dx, dy, and dz), three rotation parameters (ω_x, ω_y, and ω_z); and a parameter (s) for the scale difference between the two coordinate systems. There are many similarity transformation models. Bursa-Wolf transformation model was introduced by Bursa and Wolf in 1963. Molodensky-Badekas model was introduced in 1962 by Molodensky and the more common interpretation of the model's differential transformation equations was given by Badekas in 1969. Hence, the model is commonly refereed as Molodensky-Badekas. Although these two models are the most famous transformation models used in geodetic applications, there are several other developed similarity transformation models. Veis proposed his transformation model in 1960. Vanicek and Wells developed another transformation model in 1974. Details concerning these models are found in various literatures [e.g. Rapp 1989; Nassar 1984; Lieck and van Gelder 1975].

Unconventional Transformation Techniques

The traditional similarity datum transformation models assume that the relationship between two coordinate systems could be modeled by a selected number of parameters, which is usually seven. However, in reality the actual parameterization is not as simple as implied in those models. For example, the old geodetic datum, that has been built up over a period of time, does not have a uniform accuracy. Also, there exist distortions in the old datums as new precise data are fitted into these geodetic frames. Hence, the development of more precise relationships between geodetic datums should be considered.

The stepwise multiple regression technique is recently utilized for datum transformation determination for two basic reasons: the need for better accuracy than could be achieved through the similarity datum transformation formulas; and the need for a technique more manageable for field use. This technique is based on modeling the differences ($\Delta\phi$, and $\Delta\lambda$, and ΔH) between the geodetic coordinates of two systems by three polynomials of sufficient terms in order to represent the differences over the network, to a given degree of accuracy. The stepwise multiple regression process starts by fitting a linear function, that is a constant and a variable. The procedure then sequentially adds one variable at a time to the equation, and its significance is tested in order to decide whether this variable is kept or removed from the model. This stepwise addition or removal of variables ensures that only significant variables are retained in the final equation. The procedure continues until the precision desired for the equation is obtained. This unconventional datum transformation technique has been used to describe the relationships between the WGS84 and 83 local geodetic datums all over the world [DMA 1987]. A similar approach has been used in Egypt based on the concept of modeling variable datum shifts (ΔX, ΔY, and ΔZ) at available stations not just one set of datum shifts for the entire region [Rashwan 1993]. However, a limited number of common points between WGS84 and OED systems (concentrated only in the Eastern Dessert) has been used, which enforces the polynomial degree not exceed 2.

The number of unknowns in the stepwise multiple regression equations is determined based on several factors: the available coordinates number and distribution; and the use of statistical measures to determine the significance of the models. Of the different available statistical measures, two indicators are of great importance: the standard error of estimation; and the coefficient of determination. The standard error estimation is an indicator of the precision of the developed regression equation, and is computed by (for $\Delta\phi$ as an example):

$$\sigma\ (\delta\Delta\phi) = \sqrt{\ (\textstyle\sum^{n}_{i=1} [\ (\Delta\phi_i \text{ known}) - (\Delta\phi_i \text{ computed})\ ]^2 / n\ )} \qquad (1)$$

The coefficient of determination is the indicator of how much of the variability in the dependant variable is explained by the independent variables used in the regression formula. For example, the coefficient of determination for the latitude differences can be computed by:

$$r^2\ (\delta\Delta\phi) = \textstyle\sum^{n}_{i=1} [\ (\Delta\phi_i \text{ computed}) - (\Delta\phi_{mean})\ ]^2 /\ \sum^{n}_{i=1} [\ (\Delta\phi_i \text{ known}) - (\Delta\phi\text{mean})\ ]^2 \qquad (2)$$

DATA USED AND DEVELOPED FORMULAS

Precise geodetic coordinates of 19 first-order geodetic stations known in both the WGS84 and OED geodetic datums has been used in this investigation (Figure 1). These GPS coordinates are the most accurate database available in Egypt. The coordinates come basically from two sources: the High Accuracy Reference Network (HARN95) established by the Egyptian Survey Authority and tied to some International Geodetic Stations (IGS); and the remaining stations have been observed by the Survey Research Institute as part of the Egyptian National Standardization Gravity Network (ENGSN97). Regarding the development of the stepwise multiple regression, a set of formulas has been tried out for each of the latitude differences and the longitude differences. The first formula is the linear equation, and then one variable is sequentially being added at a time to the equation. The following formulas are used:

For $\Delta\phi$:

$$\Delta\phi = c_o + c_1\ \phi \qquad (3)$$

$$\Delta\phi = c_o + c_1\ \phi + c_2\ \lambda \qquad (4)$$

$$\Delta\phi = c_o + c_1\,\phi + c_2\,\lambda + c_3\,\phi^2 \qquad (5)$$

$$\Delta\phi = c_o + c_1\,\phi + c_2\,\lambda + c_3\,\phi^2 + c_4\,\phi^3 \qquad (6)$$

$$\Delta\phi = c_o + c_1\,\phi + c_2\,\lambda + c_3\,\phi^2 + c_4\,\phi^3 + c_5\,\lambda^2 \qquad (7)$$

$$\Delta\phi = c_o + c_1\,\phi + c_2\,\lambda + c_3\,\phi^2 + c_4\,\phi^3 + c_5\,\lambda^2 + c_6\,\lambda^3 \qquad (8)$$

$$\Delta\phi = c_o + c_1\,\phi + c_2\,\lambda + c_3\,\phi^2 + c_4\,\phi^3 + c_5\,\lambda^2 + c_6\,\lambda^3 + c_7\,\phi^4 \qquad (9)$$

$$\Delta\phi = c_o + c_1\,\phi + c_2\,\lambda + c_3\,\phi^2 + c_4\,\phi^3 + c_5\,\lambda^2 + c_6\,\lambda^3 + c_7\,\phi^4 + c_8\,\lambda^4 \qquad (10)$$

For $\Delta\lambda$:

$$\Delta\lambda = d_o + d_1\,\lambda \qquad (11)$$

$$\Delta\lambda = d_o + d_1\,\lambda + d_2\,\lambda^2 \qquad (12)$$

$$\Delta\lambda = d_o + d_1\,\lambda + d_2\,\lambda^2 + d_3\,\phi \qquad (13)$$

$$\Delta\lambda = d_o + d_1\,\lambda + d_2\,\lambda^2 + d_3\,\phi + d_4\,\lambda^3 \qquad (14)$$

$$\Delta\lambda = d_o + d_1\,\lambda + d_2\,\lambda^2 + d_3\,\phi + d_4\,\lambda^3 + d_5\,\lambda^4 \qquad (15)$$

$$\Delta\lambda = d_o + d_1\,\lambda + d_2\,\lambda^2 + d_3\,\phi + d_4\,\lambda^3 + d_5\,\lambda^4 + d_6\,\phi^2 \qquad (16)$$

$$\Delta\lambda = d_o + d_1\,\lambda + d_2\,\lambda^2 + d_3\,\phi + d_4\,\lambda^3 + d_5\,\lambda^4 + d_6\,\phi^2 + d_7\,\phi^3 \qquad (17)$$

$$\Delta\lambda = d_o + d_1\,\lambda + d_2\,\lambda^2 + d_3\,\phi + d_4\,\lambda^3 + d_5\,\lambda^4 + d_6\,\phi^2 + d_7\,\lambda^5 \qquad (18)$$

$$\Delta\lambda = d_o + d_1\,\lambda + d_2\,\lambda^2 + d_3\,\phi + d_4\,\lambda^3 + d_5\,\lambda^4 + d_6\,\phi^2 + d_7\,\lambda^5 + d_8\,\lambda^6 \qquad (19)$$

where $\Delta\phi$ and $\Delta\lambda$ are in arc of seconds, while ϕ and λ are WGS84 coordinates in degrees. It has been found that the differences in latitude range from 0.2104“ to 0.8654” with an average of 0.4678” and a standard deviation equal 0.2042”. The differences in longitude range from 5.8129“ to 6.0897” with an average of 5.9450” and a standard deviation equal 0.0873”. Figures 2 and 3 depict contour maps for these differences.

RESULTS AND ANALYSIS

The available data sets have been utilized to determine mathematical relationships between the WGS84 and OED coordinate systems. In order to test the obtained results, four stations have been considered as check points that have not been used in the processing stage. Another criterion has been chosen in order to increase the creditability of the results, which is the use of statistical tests to identify any outlier in the data. Coordinates of three stations have been flagged as erroneous observations, and hence, were removed. Identifying and removing outliers in precise geodetic networks increase the reliability of the obtained results [Alnaggar and Dawod 1995]. Both traditional similarity and unconventional multiple regression techniques have been evaluated.

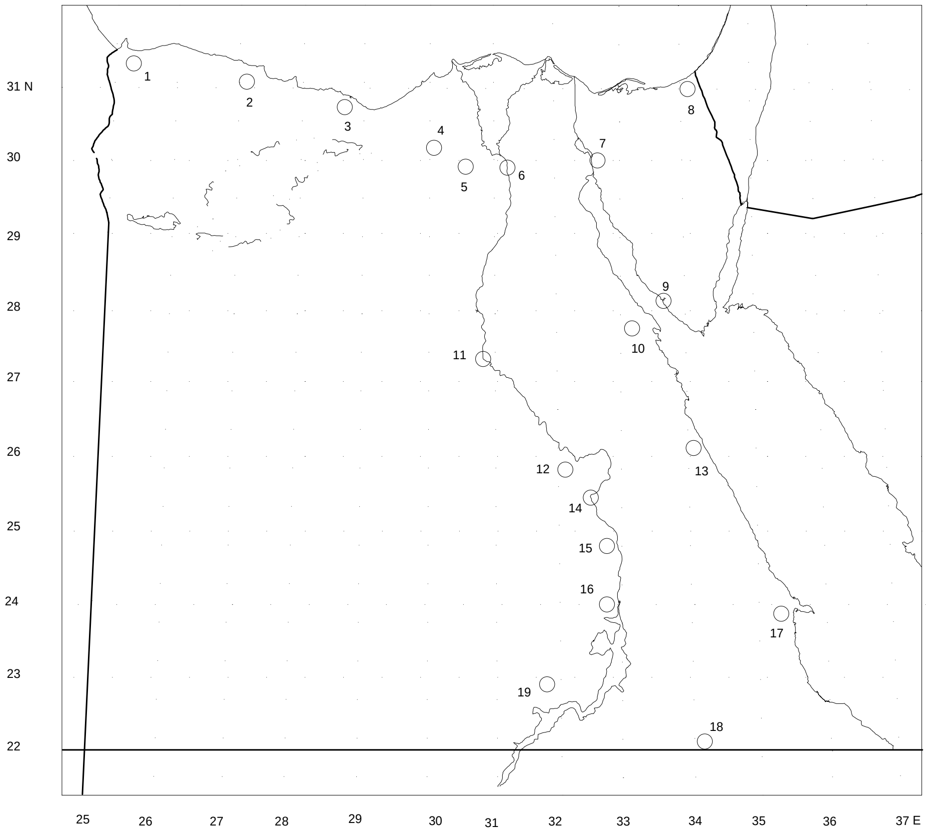
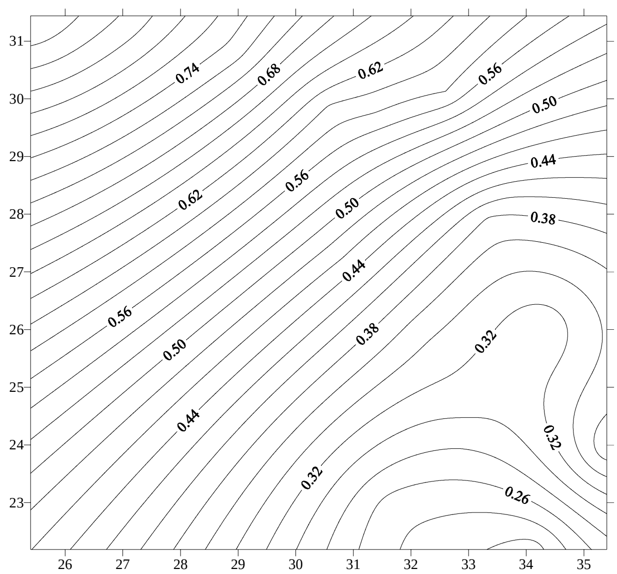


Figure 1: Available Geodetic Stations



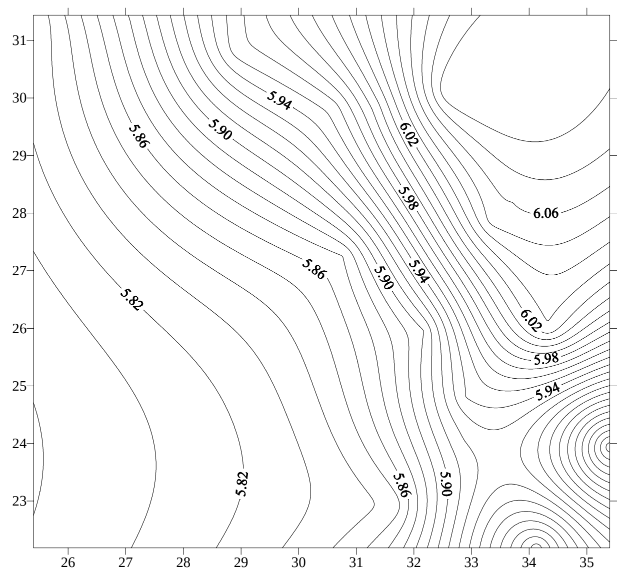


Figure 2
A Contour Map of Latitude Differences
(WGS84 – OED in seconds)

Figure 3
A Contour Map of Longitude Differences
(WGS84 – OED in seconds)

Results of similarity datum transformation

Transformation parameters using both Bursa-Wolf and Molodensky-Badekas datum transformation models have been determined. For each model, three, four, and seven-parameter cases are developed. The obtained results are summarized in Table 1. Carefully analyzing this table, the following concluding remarks may be drawn:

* Both models give identical results and precision estimates for the three-parameter case.
* When solving for the translations and scale parameters, the Molodensky-Badekas give more precise results regarding the three translation unknowns, although the value and precision of the scale factor is the same in both models.
* The Molodensky-Badekas model results in identical values for the translation parameters in the three cases, even though their precision estimates are different. That is because this model solves for mean datum shift parameters.
* Comparing the seven-parameter case for both models, it is clear that the Molodensky-Badekas model is superior to the Bursa-Wolf model.
* The Molodensky-Badekas seven-parameter model is proved to be the most precise similarity transformation model.

Results of multiple regression datum transformation

The second stage in the processing is the investigation of the stepwise multiple regression technique for geodetic datum transformation. The differences in both latitude and longitude between the WGS84 and OED systems have been computed and used as the independent variables in the regression equations. Formula (3) through (19) have been used to determine the unknown coefficients. The obtained results are presented in tables 2 and 3.

Table 1: Traditional Similarity Transformation Parameters (WGS84 to OED)

Model	B-W	M-B	B-W	M-B	B-W	M-B
ΔX (m)	123.842	123.842	143.587	123.842	70.281	123.842
$\sigma_{\Delta X}$	± 0.96	± 0.96	± 10.83	± 0.96	± 16.48	± 0.96
ΔY (m)	-114.878	-114.878	-102.540	-114.878	5.836	-114.878
$\sigma_{\Delta Y}$	± 0.96	± 0.96	± 6.74	± 0.96	± 15.12	± 0.96
ΔZ (m)	9.590	9.590	22.003	9.590	34.008	9.590
$\sigma_{\Delta Z}$	± 0.96	± 0.96	± 6.57	± 0.96	± 13.91	± 0.96
R_X (“)	---	---	---	---	-1.35314	-1.35314
σ_{RX}					± 0.17	± 0.17
R_Y (“)	---	---	---	---	-1.67408	-1.67408
σ_{RY}					± 0.35	± 0.35
R_Z (“)	---	---	---	---	5.24269	5.24269
σ_{RZ}					± 0.30	± 0.30
S (ppm)	---	---	-5.466	-5.466	-5.466	-5.466
σ_S			± 0.78	± 0.78	± 0.78	± 0.78

Table 2: Stepwise Multiple Regression for Differences in Latitude (WGS84 to OED)

Formula	No. of Coefficients	Degree of Freedom	Standard Error of Estimation σ ($\delta\Delta\phi$)	Coefficient of Determination r^2 ($\delta\Delta\phi$)
3	2	10	0.0879	0.83
4	3	9	0.0645	0.91
5	4	8	0.0565	0.93
6	5	7	0.0468	0.95
7	6	6	0.0477	0.95
8	7	5	0.0403	0.96
9	**7**	**5**	**0.0360**	**0.97**
10	8	4	0.0469	0.95

Table 3: Stepwise Multiple Regression for Differences in Longitude (WGS84 to OED)

Formula	No. of Coefficients	Degree of Freedom	Standard Error of Estimation σ ($\delta\Delta\lambda$)	Coefficient of Determination r^2 ($\delta\Delta\lambda$)
11	2	10	0.0974	0.19
12	3	9	0.0915	0.05
13	4	8	0.0720	0.41
14	5	7	0.0611	0.58
15	6	6	0.0386	0.83
16	7	5	0.0316	0.88
17	8	4	0.0258	0.92
18	**8**	**4**	**0.0110**	**0.98**
19	9	3	0.0227	0.95

From tables 2 and 3, it can be seen that the regression formula (9) and (18) are the most precise equations to represent the differences in latitude and longitude between the WGS84 and OED systems respectively. The values of the coefficients of determinations prove that these two formulas almost describe these differences perfectly. The corresponding standard errors of estimation prove that this technique is of a reasonable level of precision for GPS surveys. The final multiple regression formulas, for transforming coordinates from the WGS84 to the OED systems, recommended to be used in GPS surveys in Egypt are:

$$\Delta\phi'' = -320.474 + 30.6751\,\phi_{84} + 3.0402\,\lambda_{84} - 1.7380\,\phi^2_{84} + 0.0436\,\phi^3_{84} - 0.0004\,\phi^4_{84} - 0.1056\,\lambda^2_{84} + 0.0012\,\lambda^3_{84} \quad (20)$$

$$\Delta\lambda'' = 4357.7294 - 734.6377\,\lambda_{84} + 49.4639\,\lambda^2_{84} - 0.1705\,\phi_{84} - 1.6600\,\lambda^3_{84} + 0.0278\,\lambda^4_{84} + 0.0037\,\phi^2_{84} - 0.0002\,\lambda^5_{84} \quad (21)$$

where: $\Delta\phi$ and $\Delta\lambda$ are obtained in arc of seconds, while ϕ_{84} and λ_{84} are the WGS84 coordinates in degrees.

Therefore, the transformed coordinates on the OED coordinate system are given by:

$$\phi_{OED} = \phi_{84} + \Delta\phi \quad (22)$$

$$\lambda_{OED} = \lambda_{84} + \Delta\lambda \qquad (23)$$

Comparison of both techniques

In order to compare the validity of results obtained from both transformation techniques, four check points have been utilized. The coordinates of those stations have been computed through the final precise model for each technique that are the Molodensky-Badekas seven-parameter model as long as equations 9 and 18 for the multiple regression approach. Then, the transformed coordinates are compared with the corresponding known coordinates. The obtained results are given in table 4 and 5.

Table 4: Accuracy of similarity transformation over check points

St.	φ" (obs)	φ" (comp)	Δφ " (obs-comp)	λ" (obs)	λ " (comp)	Δλ " (obs-comp)
1	48.292	48.25155	0.04045	36.3080	36.34625	-0.03825
2	31.8621	31.89153	- 0.02943	01.1468	01.05376	0.0934
3	48.8467	48.80847	0.03933	45.0296	45.12296	-0.09336
4	33.1216	33.14792	- 0.02632	31.2929	31.21780	0.0751
Mean			**0.00601"**			**0.00913 "**

Table 5: Accuracy of regression transformation over check points

St.	φ" (obs)	φ" (comp)	Δφ " (obs-comp)	λ" (obs)	λ " (comp)	Δλ " (obs-comp)
1	48.2920	48.26612	0.02588 "	36.3080	36.32019	-0.01219
2	31.8621	31.85634	0.00576 "	01.1468	01.12932	0.01748
3	48.8467	48.86464	-0.01794 "	45.0296	45.03211	-0.00251
4	33.1216	33.13322	-0.01162 "	31.2929	31.29999	-0.00709
Mean			**0.00052 "**			**-0.00051 "**

From tables 4 and 5, it can be seen that the overall accuracy of the traditional transformation technique is in the order of approximately 3 meters, while it is almost 0.5 m for the utilized regression approach. Hence, it may be concluded that the developed multiple regression datum transformation is one order more accurate than the traditional similarity datum transformation technique. The reason for that is the disability of the similarity transformation models to represent the distortions existing in the old local geodetic networks. The same situation happens for several national and regional coordinate systems all over the world [DMA 1987].

CONCLUSIONS

The issue of datum transformation of GPS coordinates from the WGS84 to the OED coordinate systems is an important dilemma in Egypt. The famous models of the traditional similarity datum transformation technique (namely Bursa-Wolf and Molodensky-Badekas models) have been investigated. It has been found that both models give identical results and precision estimates for the three-parameter case. When solving for the translations and scale parameters, the Molodensky-Badekas give more precise results regarding the three translation unknowns. Additionally, it has been concluded that the Molodensky-Badekas model results in identical values for the translation parameters in the three cases. Moreover, the comparison of the seven-parameter case for both models reveals that the Molodensky-Badekas model is superior to the Bursa-Wolf model. Hence, it is concluded that this model is the most precise similarity transformation pattern. On the other hand, the traditional similarity datum transformation approach seems not to be the best technique to describe the mathematical relationship between the global and national datums. Consequently, the development of more precise relationships between WGS84 and OED geodetic datums has been considered. The stepwise multiple regression technique has been utilized for datum transformation determination. New regression formulas have been developed to transform the GPS coordinates from the global WGS84 datum to the local national Egyptian coordinate system. The obtained results show that the overall accuracy of the traditional transformation technique is in the order of approximately 3 meters, while it is almost 0.5 m for the utilized regression approach. Hence, it may be concluded that the developed multiple regression datum transformation approach is one order more accurate than the traditional similarity datum transformation technique. This approach is proved to be more simple, manageable, practical; and accurate for different types of GPS surveys in Egypt. Consequently, it is recommended to be applied using all available geodetic stations, with a national coverage, to come up with an accurate set of regression equations to transform GPS coordinates into the Egyptian national coordinates system.

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