THEORETICAL – EXPERIMENTAL INVESTIGATION OF SAW – TOOTH FOLDED PLATE STRUCTURES

MSc. THESIS

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SYNOPSIS

Both theoretical and experimental investigations on the structural behaviour of the saw-tooth folded plate are presented in this thesis. Two models are used to check the theoretical results. The first model is an aluminum model. The strains have been measured by S.R.4 strain gages at different points. The second model is reinforced concrete one. Its proportions are six times that of the aluminum model. It is subjected to both symmetrical and unsymmetrical loadings.

Investigations show that Gaafar's method which is recommended in 1961 by the ASCE Committe on folded plate structures agrees well with the experimental results. Also it is found that for some special loadings where the resultant of loads coinside with the shear center of the cross section; Gaafar and the beam methods give the same results .

A more general solution which is recommended when solving a certain problem for different cases of loading is given . The solution is based on Gaafar's method, with a proposed simplification in the calculations .

The finite element method is also used, and a general computer program is published. The maximum core size of the available computer being small, it is not sufficient to obtain more steady results for space structures specially when these structures are unsymmetrical. The method is clearly explained and the aluminum model is taken as an example. The data as well as the results associated to the example are printed to facilitate the analytical procedure.

NOTATIONS

Aab, A_{ab}	=	Cross sectional area of plate "ab"
a, b, c.....	=	Subscripts indicating the name of the fold lines
l	=	Plate depth
E	=	Modulus of elasticity
H	=	The algebraic difference between the edge stresses at a joint.
K	=	Stress distribution factor.
L	=	Longitudinal span.
M_T	=	Transverse Bending Moment.
$\bar{P}$	=	Harmonic joint load due to relative joint displacement.
t	=	Thickness of plate
I	=	Moment of inertia.
f	=	Normal stress.
δ_{ab}	=	Component of displacement of plate ab in its plane.
Δ	=	Relative displacement between two sucessive joints
m	=	slab moment in the transvere direction.
W	=	Total Jack load.
Mo	=	Free edge plate moment at the mid span section
θ	=	Angle of inclination of the principal axes.

Other notations are given where they are used.

CHAPTER

I

FOLDED PLATE STRUCTURES

OF

HOMOGENOUS MATERIALS

I - INTRODUCTION

I

A "hipped plate" structure or "folded plate" structure is a three dimensional assembly of plates . These plates are arranged as to produce a stable structure capable of carrying loads. The essential feature of such structure is that the individual component element is flat, not curved, so being of considerable simple framework, it has been widely used in various types of constructions such as, long span roofs, saw tooth roofs, bridges, towers, channels, silos, bunkers, etc.

According to their external shape, hipped plate structures may be distinguished as being prismatic, pyramidal, prismoidal and curved on plane. The prismatic hipped plate structures are composed of rectangular plates connected along their edges, and having diphragms perpendicular to the longitudinal axis at supports. Pyramidal hipped plate structures occur as pavilion roofs. Prismoidal hipped plate structures are an intermediate form of construction between prismatic and pyremidal roofs .Also, if only two plates intersected at a junction, the structure is termed a "simple" hipped plate. And if more than two plates intersected at a junction, the structure is termed a "multiple" hipped plate.

Considerable saving are obtained by this method of construction as compared with standard beam-and-girder design. In addition to the very definite saving of materials, the smooth surface, uninterrupted by beams, ribs and other protections, make for simple framework and erection and convenience in use. In conventional construction, beams would be provided at all junctions of adjoining slabs. It is simple to show that such beams are not only superfluous but also ineffective provided the angle between adjacent plates is not too close to 180°.

The simplest form of folded plates consists of the inclined plates in V-shape. However, this cross section has a disadvantage, in that the area of concrete to resist the compressive flexural force or to permit placement of the reinforcing steel may be inadequate. In Addition, at the end span, the legs of the V act as long cantilevers in transverse bending. A more general form is developed by adding horizontal plates at the top and bottom junctions of the inclined plates.

Floded plate structuresare well suited to the application of prestressing. Prestressed folded plate structures ay the same advantages that linear structures do, ely, reduced possibility of cracking, smaller deflections and smaller hight to span ratios. Also the precasting of folded plates offer the possibility of vast new field of applications and the prestressing can be done on the precasting bed with the possibility of additional economic advantages.

A very useful application of the folded plate is to north-light or saw-tooth roof construction, the window area being incorporated into the effective slab to provide greater height to span ratio. This system suits well the big covered halls where a uniform distribution of natural light, that is not possible form windows in the outside walls, is required. Any reinforced concrete saw-tooth roof lies under one of the following two headings:

1 - Slabs(solid or hollow), beams and girder constructions
2 - Beamless roofs hipped as conoids, straight as folded plates.

Comparing between the curved saw-tooth slabs of the shell form and the folded saw-tooth slab, it can be concluded that the curved slab is more convenient for the better distribution of stresses as the compression zone is bigger and this increases the stability of such roofs by reducing the compressive stresses. While in the folded slab the compressive stresses concentrate at the upper and lower corners of the fold. From the constructional point of view, the curved slab usually needs high standard of supervision beside higher cost for the shuttering.

2- ASSUMPTIONS

The following general assumptions are oftenly used in analyzing a folded plate structure:

1- The material is homogeneous, uncracked and elastic

2- Longitudinal edge joints are fully monolithic and contineous.

3- The principle of super-position holds, ie. the structure may be analyzed for the effects of its redundants as well as for the various external loadings. The results in both cases will then be super posed.

4- Individual plates posses negligible tersional resistance

5- The function of the supporting diaphragms is to supply the end reactions for the plate action. They are assumed incapable of providing restraint against rotation of the plate ends in their own planes.

6- The longitudinal strain, due to plate action varies linearely across the width of each plate.

3- PRINCIPLES OF ANALYSIS

Loads applied to the structure are resolved into their components perpendicular and parallel to the respective surfaces on which they act. The perpendicular components result in slab action whereas the parallel components are resisted by plate action of the respective members. with respect to the slab action, the structure is analyzed as a contineous, one way slab supported at the edges. The deformation of the edges of plate structure, where the plates are joined, is possible only if the plates are deformed in their planes. These edges can therefore be considered to be stiff and immobile supports for the transfer of loads in the cross direction.

The reactions of the contineous slab at each edge support are resolved into components which lie in the planes of the neibouring plates. These components represents slab loads which subject the plates to bending. The bending moments and shear forces in each plate are determined as for deep beams. Thus the same structural members are first designed as slabsfor bending out their planes, and then as paltes for bending in their planes The monolithic action along the common joints of adjacent plates prevent relative displacements there. Shear forces "T" are bound to act along edges to achieve the continuity.

Hince, each of the plates, in addition to its normal load P., causing bending moments M., is acted upon along its two edges, by shear forces "T". For the same reason of continuity, the stresses in two neighbouring plates must be equal along their common edge, e.g: $f_{2b} = f_{1b}$ $f_{2b} = f_{1b}$

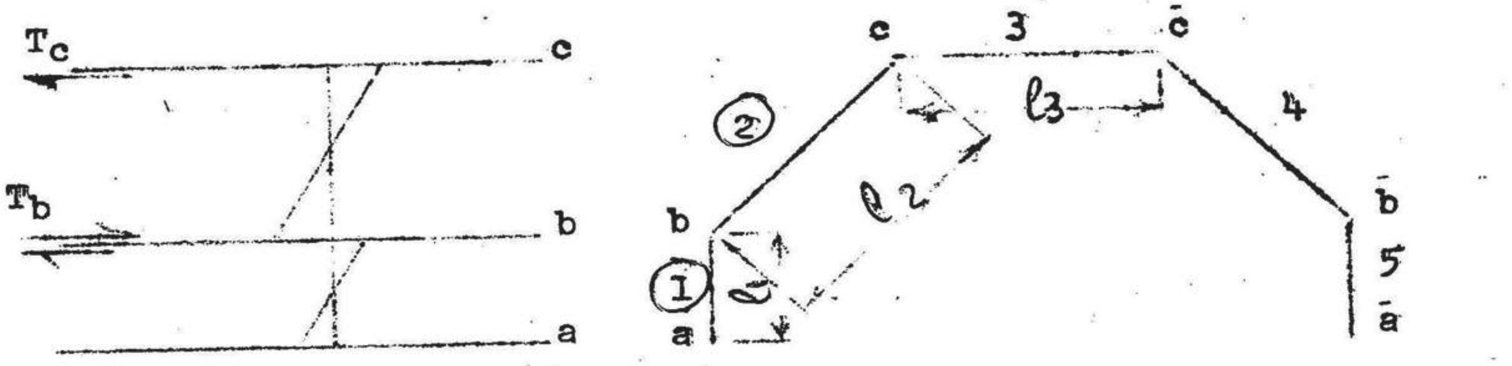


Fig. (1.1)

$$f_{2b} = \frac{6M_{o2}}{t_2 \ell_2^{\,2}} - \left(\frac{4T_b + 2T_c}{t_2 \ell_2} \right) \quad \ldots\ldots\ldots \quad (1)$$

$$f_{1b} = - \frac{6M_{o1}}{t_1 \ell_1^{\,2}} + \frac{4\ T_b}{t_1\ \ell_1} \quad \ldots\ldots\ldots \quad (2)$$

from (1) & (2) we have

$$\frac{1}{2} \left(\frac{M_{o2}}{z_2} + \frac{M_{o1}}{z_1} \right) = 2\ T_b \left(\frac{1}{A_2} + \frac{1}{A_1} \right) + T_c \left(\frac{1}{A_2} \right) \quad (3)$$

Equation (3) is similar to three moment theorem for contineous beams. Such an equation can be derived for each edge, and the shear forces "T" are determined from the total set of equations. When the moments M. and the shear forces Ta, Tb, etc., are determined for all plates of the structure, the first part of the analysis is considered to be completed. The second part of the analysis is concerned with the effect of the relative displacements of the joints, this relative displacements creates secondary loads which the structure should be analysed for it again. The consideration of the relative displacements of the joints may in many cases affect the results seriously.

4- STRESS DISTRIBUTION METHOD

Equation (3) is similar to the three moment equation. Winter & Pie had modified it to a stress distribution method which is similar to the moment distribution process developed by Hardy cross.

In the moment distribution method, an immaginary locking is assumed at each suport, which when released the balancing moment is distributed over the adjacent spans; similarly in the stress distribution process, imm-aginary non-sliding joints are supplied at the common edges of each plate element. The correction of such case is obtained by the application of a balancing edge shearing force T, the stresses of which are distributed over the adjacant plates .

The condition of distribution of the balancing moment in the Hardy Cross method is that the angles of deflections for the two adjacent moment should be the same over the common support. Again the condition of stress distribution in the Winter & Pie method is that the unit strains should be the same for the two adjacent plates along their common edge.

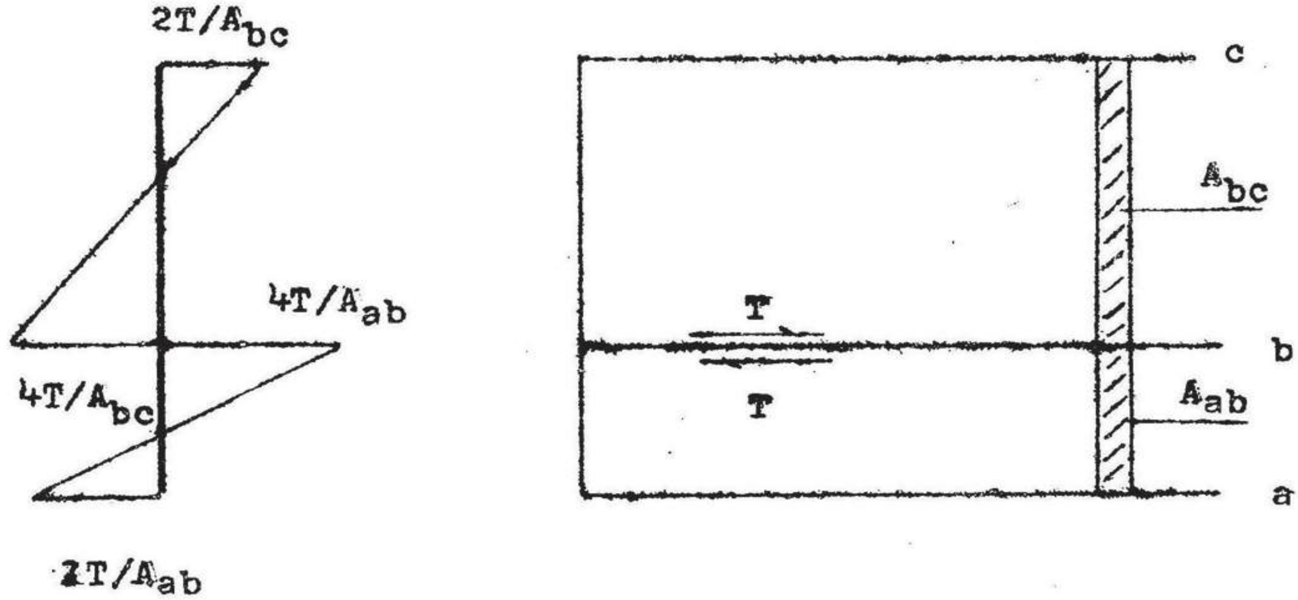


Fig. (1.2)

Fig.(1-2) shows the effect of two equal and opposite forces T on the adjacent plates. If the algebraic difference between the unequal free edge stresses at any joint "b" say, is equal to "H", therefore the balancing shear applied to remove the sliding joint is equal to "-H", and the part of this value transmitted to plate a-b is equal to:

$$f_{ab} = \frac{4T}{A_{ab}} = + \frac{(4T/A_{bc})}{(4T/A_{ab} + 4T/A_{bc})} \times (4T/A_{ab} + 4T/A_{bc})$$

$$= \frac{A_{bc}}{A_{bc} + A_{ab}} \times (-H) \qquad = K_{ab} \times (-H)$$

Similarly, the stress transmitted to plate b-c is equal to :

$$f_{abc} = \frac{A_{ab}}{A_{ab} + A_{bc}} \times (-H) \qquad = K_{bc} \times (-H)$$

where K is called the "stress distribution factor"

Again it is clear from the same fig. that the development of a stress f_{ab} at edge "b" of the plate a-b creates at the other far edge "a" a corresponding stress equals to $-\frac{1}{2} f_{ab}$ for rectangular cross sections.

calling the value $-\frac{1}{2}$ as the carry over factor, the steps adopted for the stress distribution process are carried out as follows

1- Calculate the free-edge stresses due to the plate loads alone as equal to $\pm$ Mo/Z.

2- Calculate the balancing stress "-H" at each joint, and distribute this stress over the two adjacent plates according to the distribution factor K .

3- Transfer the carry-over stresses to the far edges of the plates .

4- Repeat the above process using the transferred carry-over stresses as inetial free edge stresses untill satisfactory convergence is obtained .

5- HISTORICAL REVIEW

The principle of hipped plate construction was first developed by G. Ehlers in Germany in 1924. He wrote the first technical paper on this subject in 1930. In his method of analysis he considered the various plate elements as beams supporting at the cross and end diaphragms. Along the longitudinal edges, the plates were assumed to be connected by hinged joints that do not slide longitudinally and that are considered capable of transferring only edge shears T, between the continuous plate elements. Such connections neglect entirely the connecting moments transmitted between the plates due to the rigidity of the joints. The uniform loads on the plates were transformed to the line loads "P" acting at the joints. These loads "P" were resolved into two components, $P\bar{c}$ and $P\bar{b}$, parallel to the two adjacent plates. The plates, acting as beams between the diaphragms, carried the loads P. At the same time, the shear stresses T were created to maintain equal longitudinal strains along the common edges. This strain condition at each joint was used to determine the magnitude and distribution of the edge shear stress T.

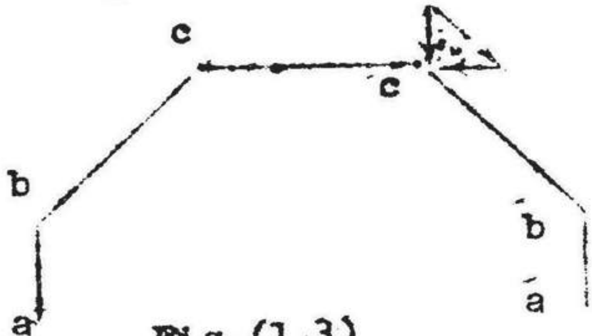


Fig.(1.3)

In 1932, E. Gruber published a paper in which he considered the effect of the rigiditly of the joints, the connecting moments acting along the common edges of the plates, and the effect of the relative displacements between the joints. As a first approximation, the hipped roof was assumed to be hinged along the joints. By Using this assumed hinged structure as a basic system, he developed his solution in the form of simultaneous differential equations of the fourth order, which could be solved by rapidly converging series. For a hipped roof of r+1 plates, r being the number of joints, the total number of unknowns is 7r+2, for a roof of five plates, this will involve thirty unknowns. The solution is complicated even if trigonometric series are used. In this solution, Mr. Gruber showed that the maximum longitudinal stresses on a cross section and the maximum deflections for a roof with hinged plates were about twice as great as those for the rigidly connected plates. Consequently he concluded that the influence of the rigid connections ought not to be neglected as it had been according to the usual practice, at that time.

Vlassow in 1936 determined the values of the stresses at the critical section by the solution of a set simultaneous linear algebraic equations that were established on the basis of equilibrium at the critical section and on the basis of continuity of joints transversely. Longitudinal variation of applied loads of transverse moments were approximated on the basis of a fourier series. The inaccuracies in the results, especially at sections other than the critical section depend on the number of terms employed in the fourier series.

The number of simultaneous equations involved were, in general 2n-2 in which n is the number of plates.

As many as eight unknowns may be present in each equation and a fair degree of computational effort is required for solution.

In 1947, Messrs Winter and Pei of Cornell University at Ithaca, N.Y., U.S.A. published paper on hipped plate construction in which they transformed the algebraic solution into a stress distribution method, which has the advantage of numerical simplicity over the other procedures. However, they also made the same simplifying assumption of neglecting the effect of the relative deflections of the joints.

In 1948, Mr. Pei presented a method of analysis considering joint displacements. The method requires the solution of 6n+1 simultaneous algebraic equations in which n is the number of plates. For a roof of five plates, the number of equations is thirty-one.

Non of the mathematical investigations previously mentioned gives any experimental evidence to substantiate the assumptions and verify the analytical procedures that were used.

In 1949, Prof. Gaafar of the Cairo University, Cairo, Egypt presented his thesis" The Analysis of Hipped Plate

Str. considering the Relative Displacements of Joints". He proved analytically, by his method, and experimentally that the relative displacements of joints strongly affect the normal stresses. He concluded from his work that:

1. When the relative transverse displacements of the joints are not taken into consideration, the stress values are so different from the experimental values (if the stresses are computed by Winter and Pei method).
2. On the other hand, when the relative displacements of the joints are considered, the values of the longitudinal stresses agree much better with the experimental results.
3. The Winter and Pie method may not even predict the right sence of the longitudinal normal stresses.
4. The structure under loading does not behave as a unit like that of the beam theory.

Prof. Gaafar has chosen the relative transverse displacement $\triangle$ between any two consecutive joints, as an unknown, and is treated as an additional load on the roof. These displacements will affect the slab moments, shears, and consequently, the plate loads P, and the plate deflections, δ. For a loading that is symmetrical about the middle section of the roof, the relative displacements are maximum at the middle of the span and are zero at the end diaphragms. The general form of the curve is known but its exact equation is not known. He considered a sine curve elastic line as an approximation for the actual one, and proved that the error in that assumption does not exceed 2%. Such use of the sine curve greatly simplifies the analytical treatments. In the case of multispan roofs, or roofs on which the loads are far from being symmetrical about the middle of the span, this sine wave treatment can not accurately be used, and the elastic line of the corresponding loaded beam would have to be adopted.

The steps used by Gaafar are as follows:

1. The first step in the analysis is the computation of the forces and of the transverse and longitudinal stresses acting at the edges of each plate element, neglecting the effect of the relative displacement of the joints. The analytical procedure used by Messrs. Winter and Pei provides a convenient solution for this problem. In this procedure the roof in the transverse direction is considered as a continuous one-way slab supported on rigid supports at the joints, and thus the shear forces Q are readily obtained.

The Q-forces at each joint are resolved into two components P-forces parallel to the continuous plates. The plates, acting as beams between the end diaphragms, carry the P-loads (plate action). At the same time, edge shear stresses (T) are created along the edges to maintain equal longitudinal strains along the common edges. The longitudinal plate stresses at sections of the roof, caused by the P-forces only, are corrected by the stress-distribution method to include the effect of the edge shear stresses T.

2. The second step in the analysis is to provide for the effect of the relative transverse displacement of the joints on the transverse and longitudinal stresses. This operation is most easily accomplished by choosing the relative transvrse displacement (Δ) between each pair of consecutive joints as unknown, determining the corresponding transverse (slab) fixed-end moments in terms of the Δ values, and correcting for rotation at the ends of the slab strips by the moment-distribution method. After the end moments have been determined, the Q-forces and P-forces are computed in the same manner as in step 1. This operation must be repeated for each different value of Δ.

3. After the Q-forces are found, the corresponding longitudinal (plate) stresses are computed in the same manner as in step 1.

4. From the values of the longitudinal stresses yielded in steps I and 3, the plate deflections δ_1, and δ_2, --- are computed. Therefore, the sum of δ, is in terms of the applied loads and the relative transverse deflection Δ. It will be found that there is only one set of Δ-values that will satisfy the algebraic relations between the Δ-values and the δ-values impased by the geometerical requirements of the corss section and the equilibrium conditions. The values of Δ can be computed from the geometrical relations between the Δ-values and δ-values because the latter already have been expressed in terms of the former by the computations in steps 2 and 3.

5. After the Δ-values have been computed, the slab moments and shears, and consequently longitudinal (plate) stresses producted by these transverse displacements, are known because they have been previously (steps 2 &3) expressed in terms of the Δ-values. These values are added algebraically to the corresponding values in step 1 to give the final results.

Later Mr. Brielmaier published a paper about the analysis of folded plate structures, he suggested a solution for such roofs by successive approximations. He concluded that "The change in the slab mometnts due to the plates" deflections may be large and should always be considered. This large change in the slab moments may result in an appraciable change in the slab reactions and hence in the plates stresses which in turn will change the plates deflections.

The computations for the longitudinal stresses and the transverse deformations, may be carried out by successive approximations starting in each step with the stresses and deformations yielded in the preceding one. In many cases; according to Brielmaie, the first correction for the plates deflections is sufficient. In some other cases, Several corrections are required, and in some the corrections are large and oscillating or converging slowly. He stated that when the change is not evaluated adequately by one or two corrections, it might be advisable to correct by using equations, which permit the direct solutions for final edge moments.

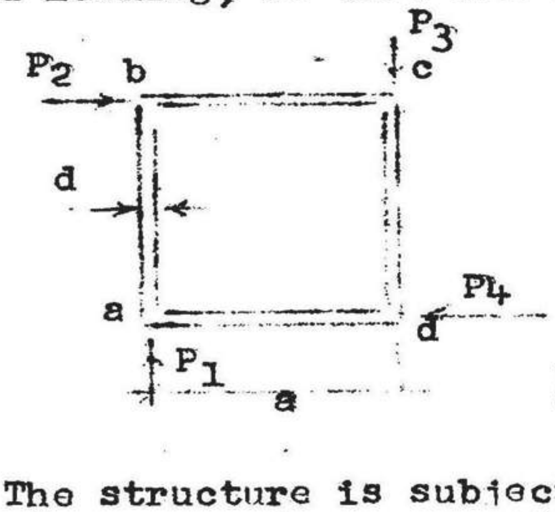
In the last few years, there has been some published papers about the use of the electronic digital computer to facilitate the analysis of the folded plate structure. The computer is used to solve numerous simple equations which are handeled by the matrix algebra. The digital computer method makes the designer tied and dependent on the computing center .

6- SPECIAL TOPICS

A-Theory of Hipped plate Structures and Torsion

The theory of hipped-plate structures provides a mean of taking various problems associated with the torsion of hollow sections("box"sections), L-sections, T-sections, etc. A few relevant examples will be given.

Cosider the prismatic hipped-plate structure represented in cross-section in Fig.1.4. Its paltes are assumed to be similar in respect of their conditions of support and loading, so that all the Mo diagrams



$\frac{T}{A} + 2T(\frac{1}{A}+\frac{1}{A}) + \frac{T}{A} = \frac{Mo}{Z}$

$\frac{6T}{A} = \frac{Mo}{Z}$

$T = \frac{Mo}{a}$

Fig.(1.4)

are affine. The structure is subjected to four point loads $P_1 = P_2 = P_3 = P_4 = P$ which produce a torsional moment $M_D = 2P.a$. The section modulus Z is assumed to be the same for all the plates . The shears T acting at the junctions A,B,C and D will then all be equal in respect of magnitade and direction. They can be computed from the equation:

$$6T = \frac{A}{Z} \times \frac{M_O}{2} = \frac{6}{a} M_o$$

The relieving moment developed by the shear forces T, which acts in opossition to the moment Mo, will then be :

$$M_T = Ta = M_O$$

and the final moment will be

$$M = M_O - M_T = 0$$

Hence in this case the structure is subjected to pure torsion, the stresses that occur in the component plates of the structure are shear stresses, not flextural stresses.

Another loading case is represented in Fig.1.5.The forces P_2 and P_4 acting on the structure will likewise produce a torsional moment, equal P.a

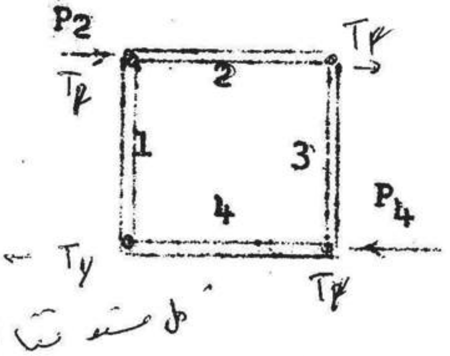


Fig.(1.5)

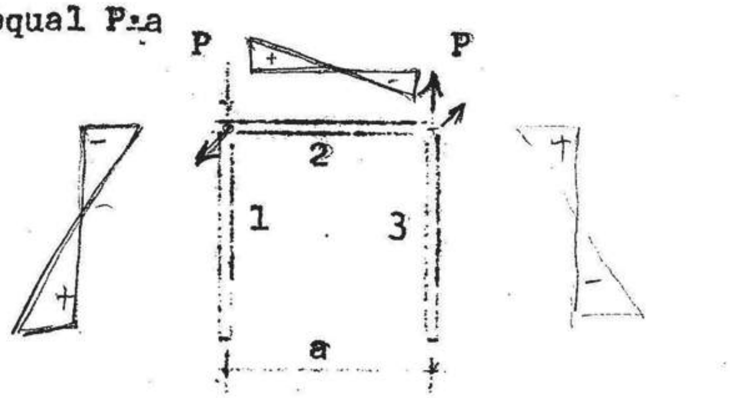


Fig. (1.6)

$\frac{6T}{A} = \frac{1}{2}\frac{M_o}{Z}$

The equation of three shears gives in this case

$$6T = \frac{A}{Z} \cdot \frac{M_o}{2} = \frac{M_o}{2K} = \frac{M_o x 6}{2 \times a}$$

$$\therefore \quad T = \frac{M_o}{2\ a}$$

The relieving moment is:

$$M_T = T.a = \frac{M_o}{2}$$

This moment is only half as large as the applied moment Mo. So the structure is subjected to doubly antisymmetrical state of stress.

The next example is represented by Fig.1.6. The junction shear is

$\frac{5T}{A} = \frac{1}{2}\frac{M_o\ 6}{Z\ 6\ A}$

$$5\ T = \frac{A}{Z} \times \frac{M_o}{2} \qquad \therefore T = \frac{6}{10}\ \frac{M_o}{a}$$

therefore

$$M_{T2} = \frac{6}{10}\ M_o \qquad \therefore M_2 = -\frac{6}{10}\ M_o$$

$$M_{T1,3} = \frac{3}{10}\ M_o \qquad \therefore M_{1,3} = \frac{7}{10}\ M_o$$

Again flextural stresses occur in addition to shear stresses.

The magnitude of the shear stresses can also be determined using theory of the hipped plate structure. Before doing this it will however, be interested to determine them with the aid of torsional theory for hallow bodies.

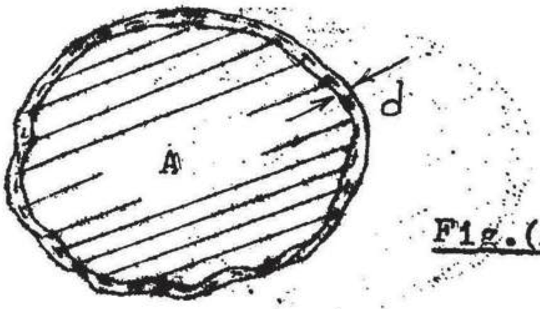


Fig. (1.7)

Applying Bredt's theorem on the shown Fig. we get:

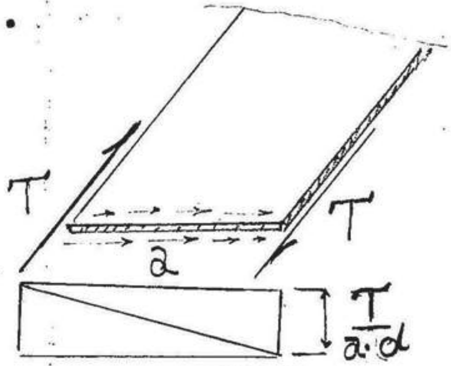
$$q.d = \frac{M_d}{2A}$$

where "Md" is the applied torque

"A" is the area inclused by the center line of the section of the wall of the hallow body.

For the case of ex. I;

$$\therefore \quad q = \frac{M_d}{2 x d x a^2}$$



Applying hipped plate theory :

$$T = \frac{M_d}{2.a}$$

$$\therefore \quad q = \frac{M_d}{2d.a^2}$$

ssuming this force to be uniformly distributed over the length "a" then :

$$q = \frac{Md}{2da^2} \quad \text{ie the same as obtained before}$$

Also it can be proved that the longitudinal shear stresses have the same magnitude of the transverse shears. As shown in Fig.(1,8)

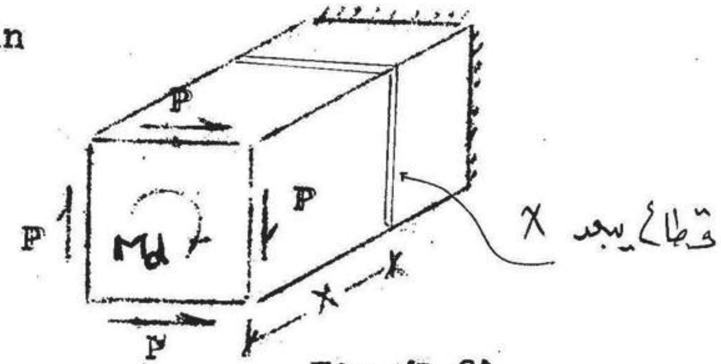


Fig.(1.8)

$$P = \frac{Md}{2a}$$

P.X

$$M_o = P_x = \boxed{\frac{Md}{2a}\,x}$$

$$T = \frac{Mo}{a} = \boxed{\frac{Md}{2a^2}\,x} \quad \text{(from Eq. of 3 shears)}$$

nt of

But $\boxed{dTn = d.qn.dx}$ therefore $\frac{dT_n}{dx} = d.\, q_n$

$$\therefore\ q_n = \frac{1}{d}\,\frac{dT_n}{dx}$$

Thus we obtain $\boxed{q_n = \frac{M_d}{2d.a^2}}$ the same as transverse shear stresses.

ie. the longitudinal shear stresses are of the same magnitude as the transverse shear stresses.

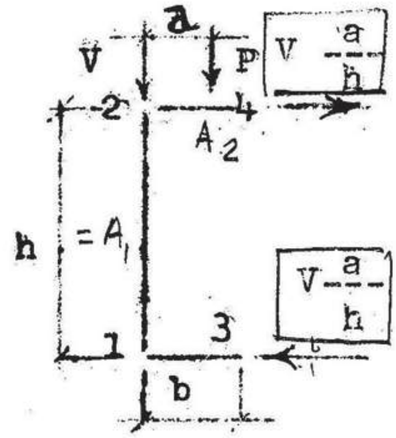


Fig.(1.9)

Consider the channel-shaped hipped -plate structure shown in Fig(1.9) loaded by a vertical load P which produces the lateral forces:

$$V = P,\quad H = P \times \frac{a}{h} \quad \text{and} \quad \bar{H} = -P\,\frac{a}{h}$$

$$T = \frac{M}{h}\;\frac{1 - \frac{a}{b} \times \frac{A_1}{A_2}}{1 + \frac{2}{3} \times \frac{A_1}{A_2}} \qquad \text{where } M = V \times \text{constant}$$

$$\therefore\; f_1 = -f_2 = \frac{M}{h}\;\frac{2 + 3\,\frac{a}{b}}{\frac{A_1}{3} + \frac{A_2}{2}}$$

$$f_3 = -f_4 = -\frac{M}{h}\;\frac{1 + \frac{3a}{b} + \frac{a}{b}\,\frac{A_1}{A_2}}{\frac{A_1}{3} + \frac{A_2}{2}}$$

For $a = 0$

$$f_1 = -f_2\;\frac{2M}{h\left(\frac{A_1}{3} + \frac{A_2}{2}\right)} \quad \text{and} \quad f_2 = -f_4 = -\frac{M}{h\left(\frac{A_1}{3} + \frac{A_2}{2}\right)}$$

This shows that the stresses are not proportion to the distance from the neutral axis.
To satisfy this assumption, the load P must be applied in such a way what we obtain:

$$f_1 = f_4 = - f_2 = - f_3$$

The corresponding distance "ao" is determined from the equation :

$$2 + 3 \frac{ao}{b} = - \left(1 + \frac{3\ ao}{b} + \frac{a_o A_1}{b A_2}\right)$$

$$\therefore \quad ao = - \frac{3b}{6 + \frac{A_1}{A_2}}$$

When the load passes through this point-which is located on the horizontal axis of symmetry and is called the shear center - the stresses are

$$f_1 = f_4 = - f_2 = - f_3 = \frac{M_1}{h\left(A_2 + \frac{A_1}{6}\right)}$$

o o o o o o

B. Stairs as Hipped-Plate Structures

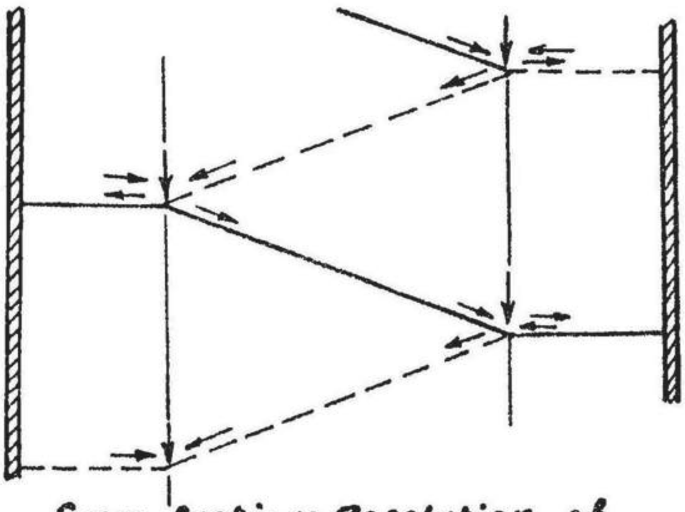
Design of stairs as hipped plate is characterised by the saving of materials. It is most advantageous in cases where the walls of the staircase are of reinforced concrete which is often adopted in industrial and tall buildings.

As shown in Fig(1.10) the loads are considered as being transmitted to the junctions of the plates, where they are resolved into forces directed in the planes of the plates intessecting there.

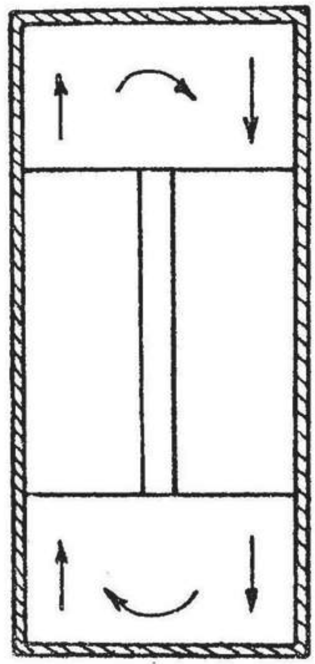
The horizontal shear forces arising from the upper and from the lower flight act upon the landing in opposite directions and consequently exercise a twisting moment upon it. In this connection the landing may be regarded as a plate supported on three sides and is capable of resisting this moment.

A very favourable arrangement is to support the flight of steps on the wall of the staircase or, better still, to build it into the latter. In this case only the loads corresponding to the shaded areas in Fig.(1.11) will be transmitted to the junctions of the hipped-plate structure. Consequently the horisontal shear in the landing will be smaller in magnitude than if the flight were not supported on the walls. The flight also will act as a plate supported on three sides. The force P acting in the plane of the flight is in equilibrium with the shear force $\bar{P}$ in the staircase wall. The moment developed by the couple formed by P and $\bar{P}$ is in turn resisted by the couple H-$\bar{H}$. These forces H and $\bar{H}$ produce a moment that acts upon the landing and reduces the above montioned twisting moment due to the horisontal shears.

The wall enclosing the staircase must be able to take up the shear stresses that occur. In case of the absence of this wall, the load P in the plane of the flight will be transmitted in equal portions to corner points A and $\bar{A}$ of the junction between the flight and the upper and lower landing respectively. Whence the forces are transmitted directly to the wall by the built-in landings. For structureal reasons it will then be necessary to provide special reinforcement at the corners A and $\bar{A}$ (especially shear reinforcement).

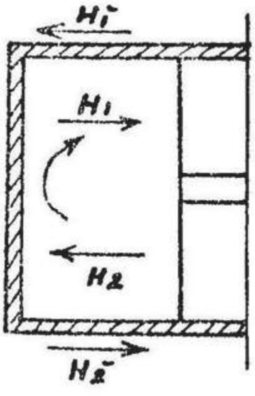


Cross Section: Resolution of forces at the junctions.
Fig: (1-10)



Shear forces in landing and flight

Plan



Landing as rigid plate
Fig.(1-12)

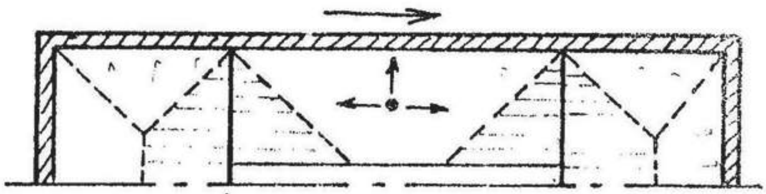
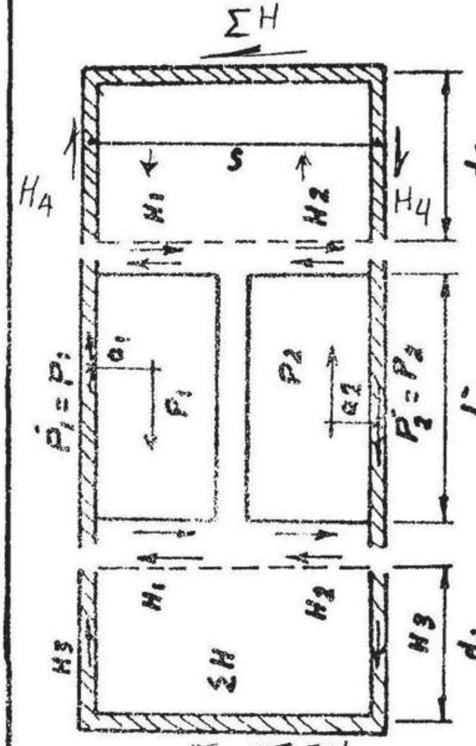


Fig. (1-11)
Shear forces in wall



Flight as rigid plate.

Fig.(1-13)

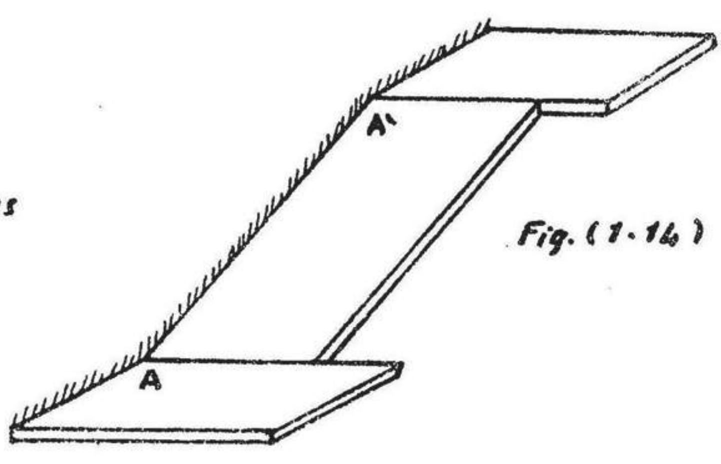


Fig. (1-14)

$$H_1 = \frac{P_1 a_1}{l}, \quad H_2 = \frac{P_2 a_2}{l}, \quad \Sigma H = H_1 + H_2$$

$$H_3 = \frac{\Sigma H d_1}{S}, \quad H_4 = \frac{\Sigma H d_2}{S}$$

Fig. (1-13)

C H A P T E R

II

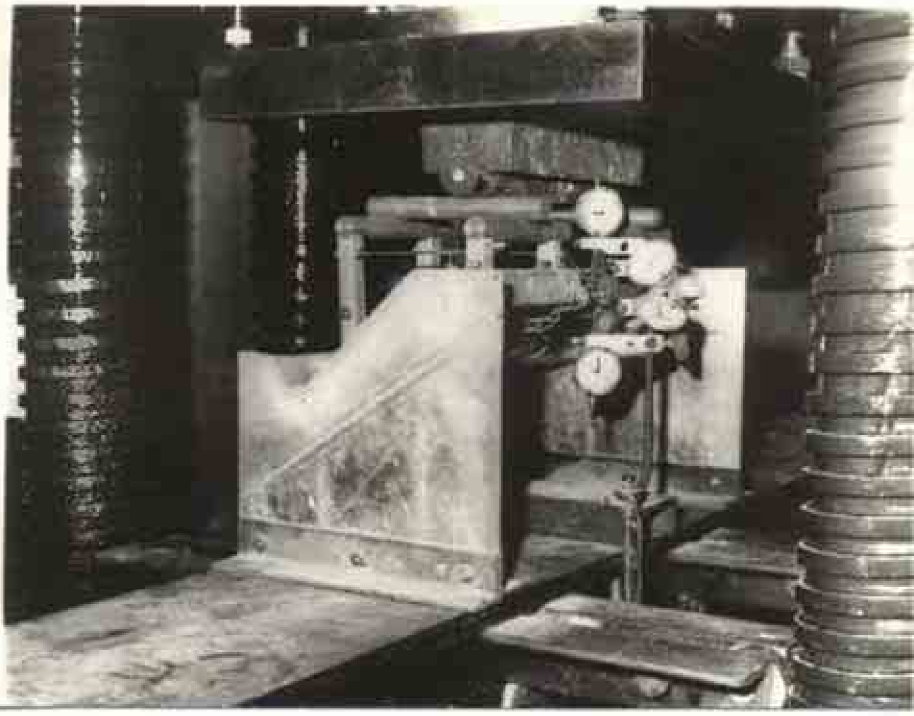
THE ALUMINUM MODEL

1- DESCRIPTIVE OF THE MODEL

An aluminum model is used for the experimental investigation of strains in a saw-tooth folded plate structure. The dimensions of the model are shown in Fig.(2-1) and Fig .(2-2) . The loads are applied at sixteen paints on the model Fig.(2-4) and Fig.(2-5). S .R.4 resistance gages are placed to measure the longitudinal strains at the middle section Fig.(2-4). These gages are placed on both sides of each plate. and connected together in series to give the average strain. The transverse strains are measured at one point on both sides by means of S.R.4 resistance gage. Also the vertical and horizontal displacements of joints are measured by means of Dial Gages Fig.(2-6).

Fig.(2-5) and Fig.(2-7) show that the model is provided by two end diaphragms welded to the superstructure. The diaphragms are made also from the same material of the roof construction and having the same thickness. Each diaphragm is connected to the tested machine during the test to add an extra mean of stability.

In order to have a truely simply supported condition one of the two diaphragms are provided with a notch parallel nearly to the expected neutral axis.



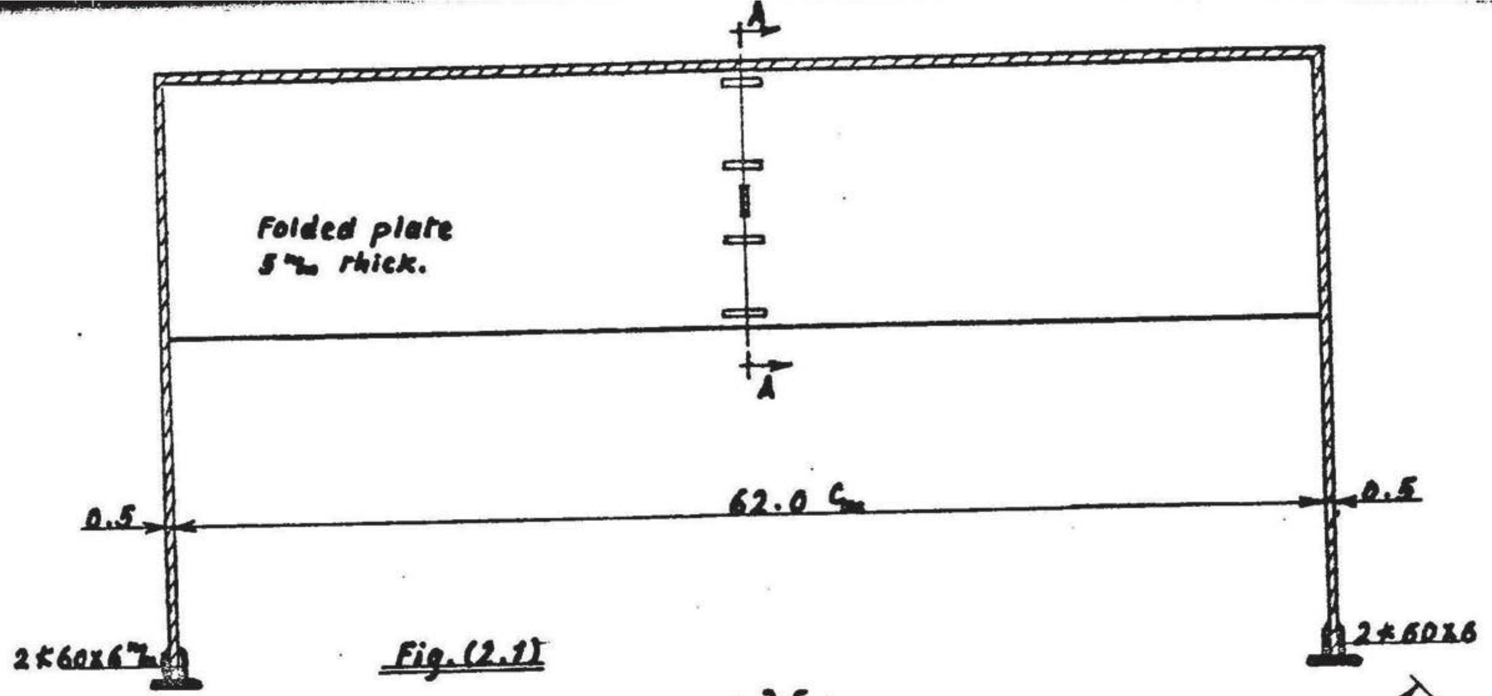


Fig. (2.1)

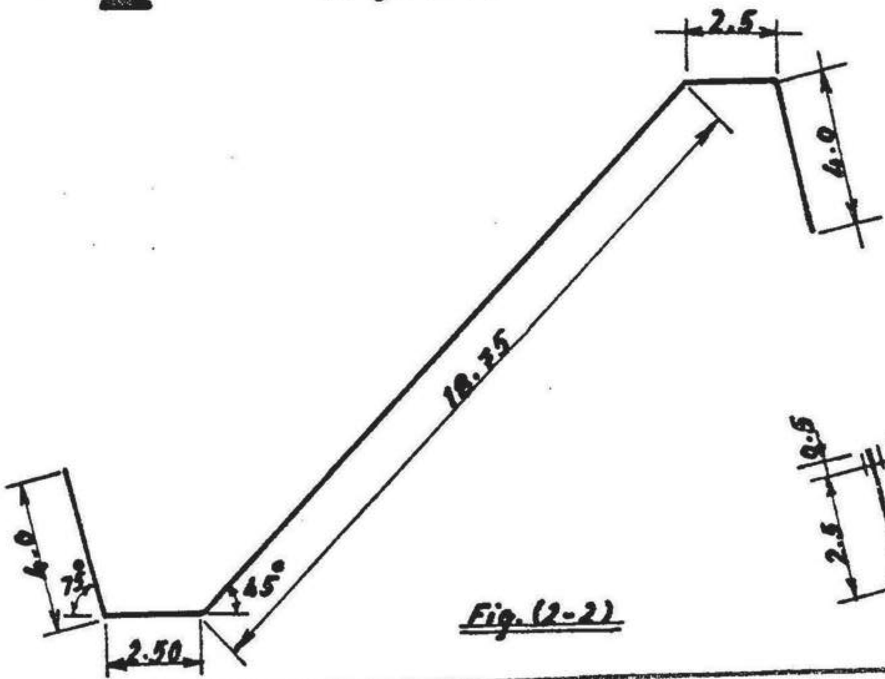
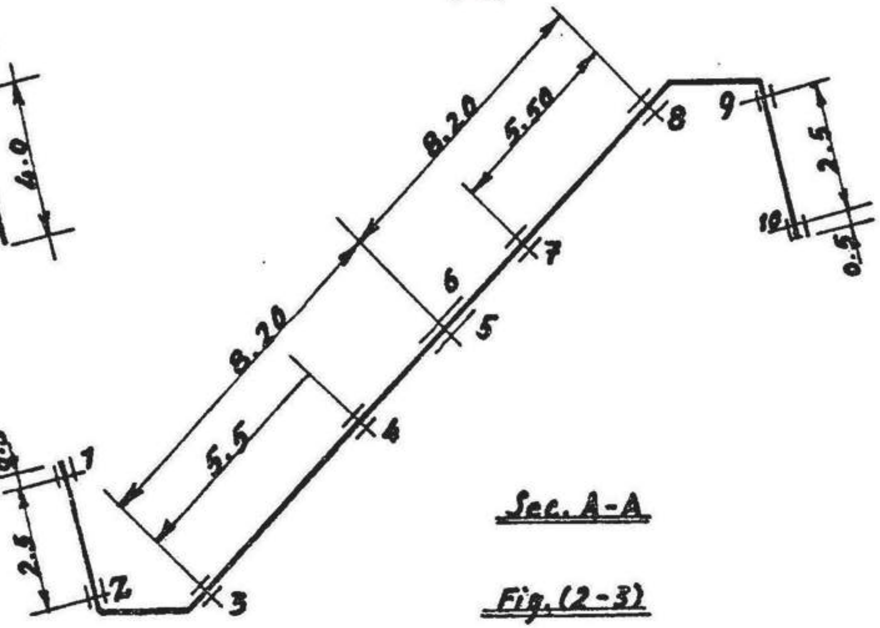


Fig. (2-2)



Sec. A-A

Fig. (2-3)

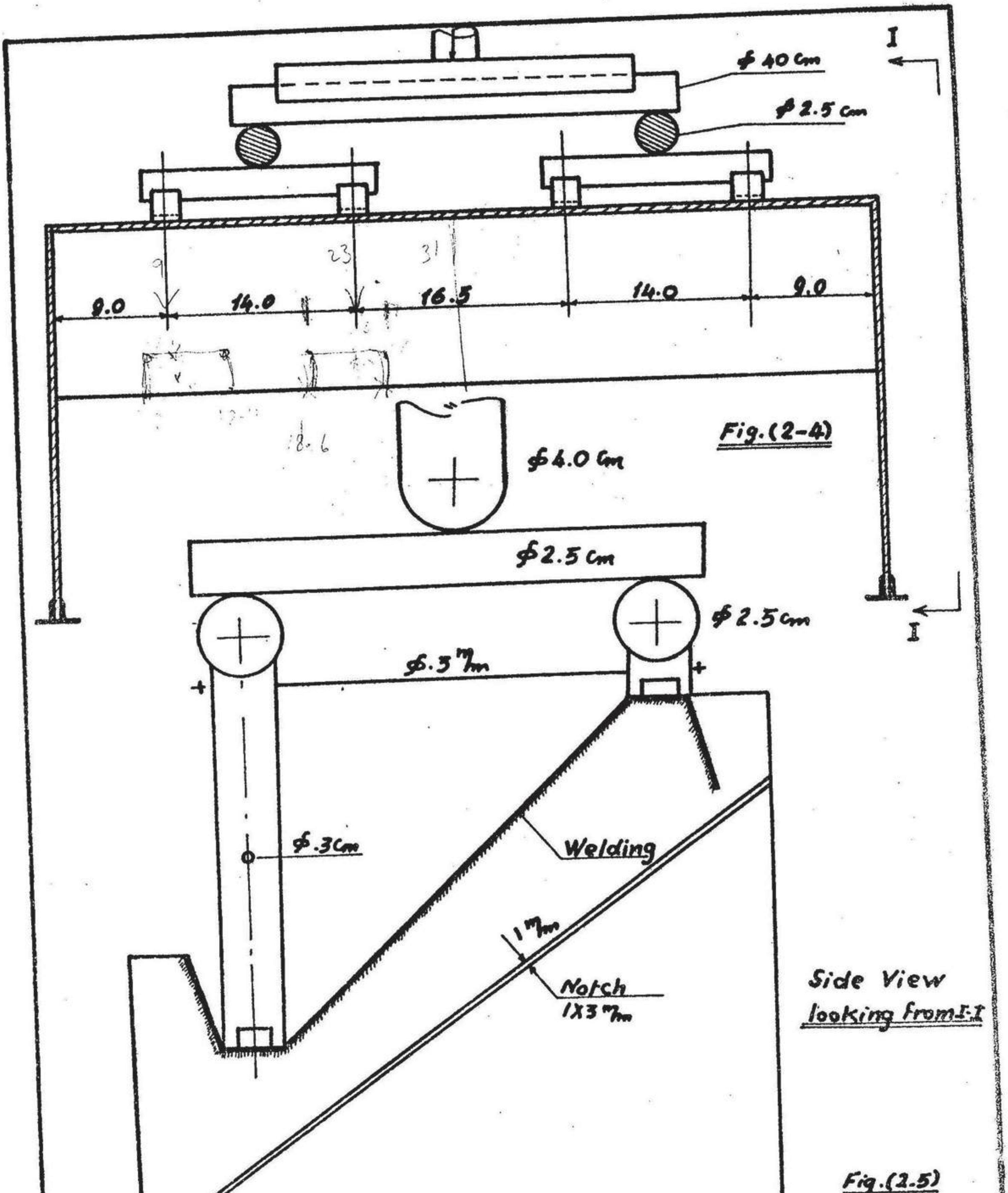
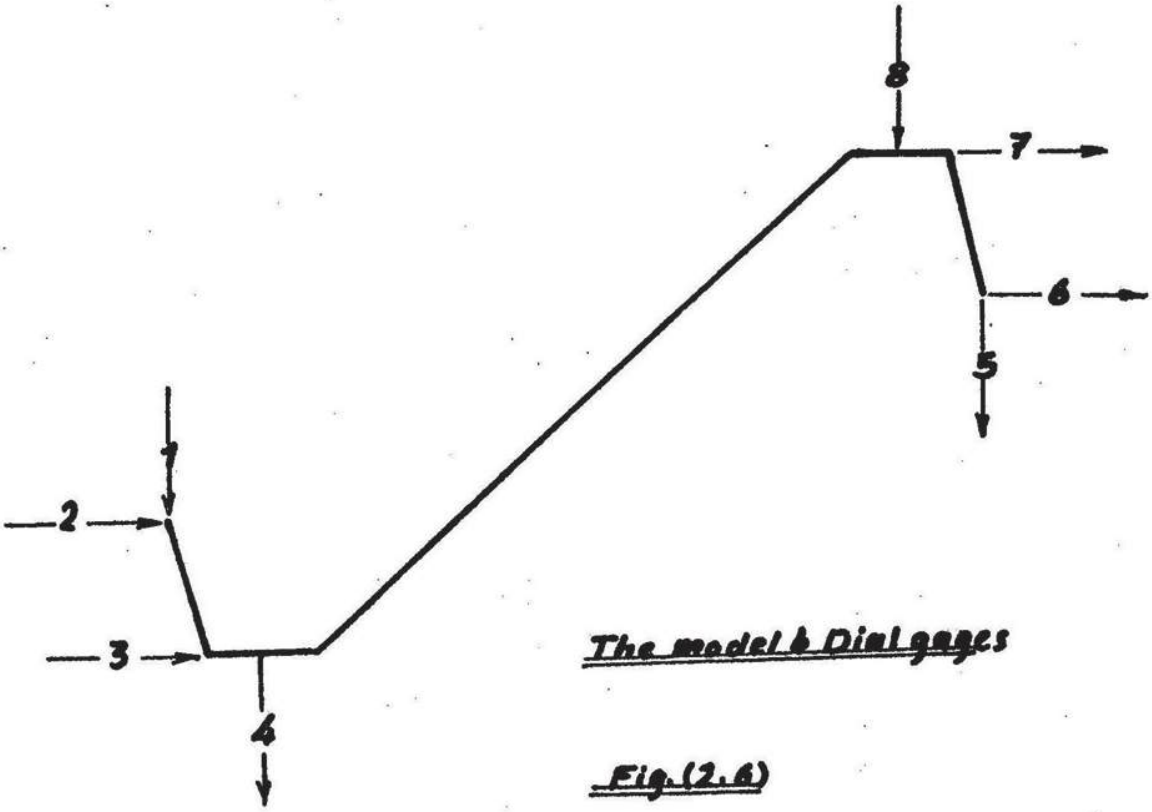

I
ϕ 40 cm
ϕ 2.5 cm
9.0
14.0
16.5
14.0
9.0
ϕ 4.0 cm
ϕ 2.5 cm
ϕ 2.5 cm
I
ϕ .3 mm
ϕ .3 cm
Welding
1 mm
Notch
1X3 mm
Side View
looking from I-I

Fig.(2-4)

Fig.(2.5)



The model & Dial gages

Fig.(2.6)

Other Diaphragm

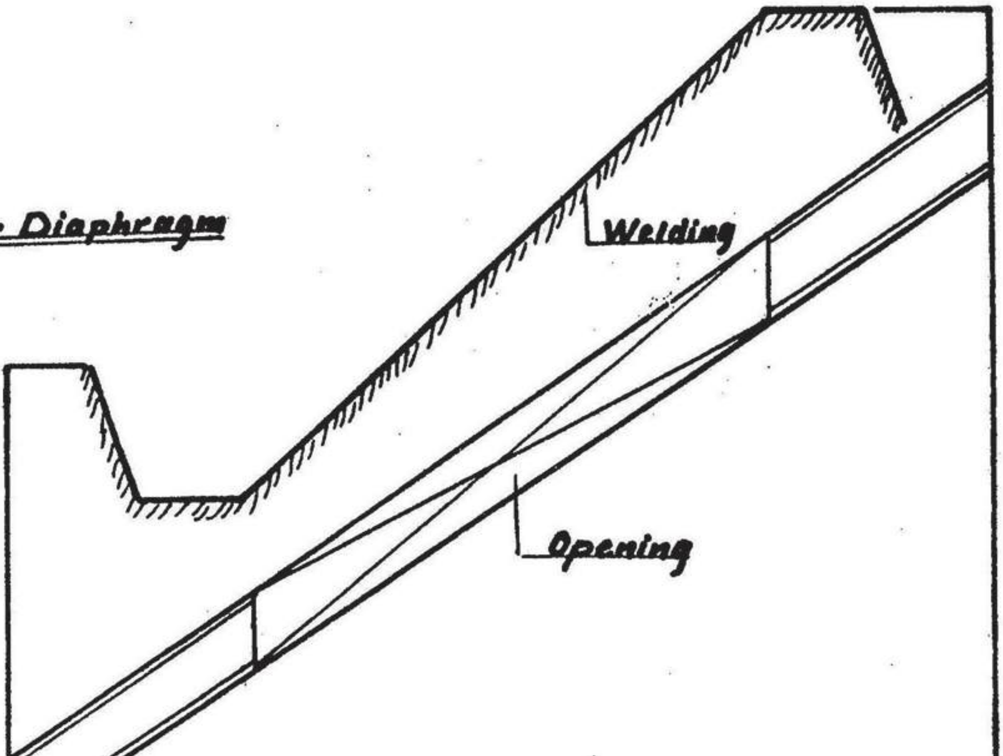
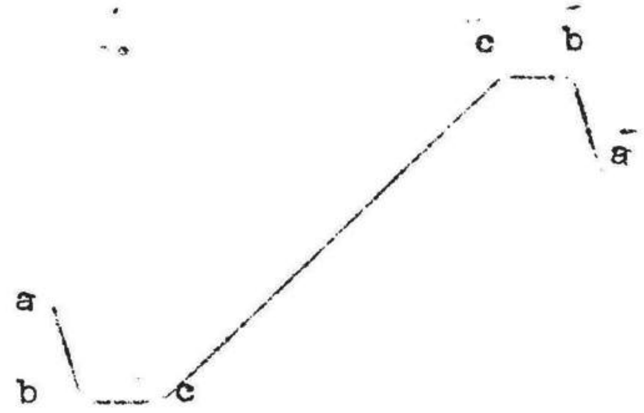


Fig.(2-7)

2- ANALYSIS OF THE MODEL(GAAFAR'S METHOD)

a. Elementary Analysis :

- Properties of the Different Plates

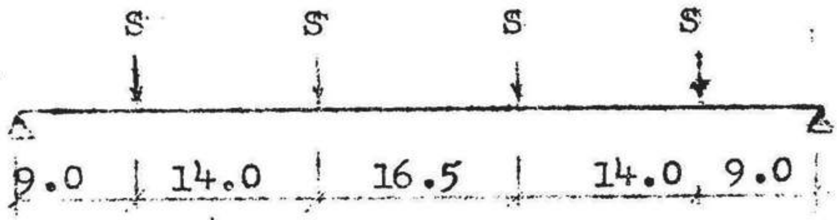


L = 62.5 cm

t = 0.50 cm

Plate	Dim. Cm. ℓ x t	Area Cm^2	$Z = \frac{\ell^2 \times t}{6}$ Cm^3
AB, $A\bar{B}$	4 x 0.5	2.0	1.33
BC, $\bar{B}\bar{C}$	2.5x.5	1.25	0.52
$C\bar{C}$	18.75x.5	9.375	29.2

Free edge Moments:



B.M = 2S x 23 - Sx14 = 32 S kg.cm

- Plate loads, bending moments and the corresponding stresses at middle section:

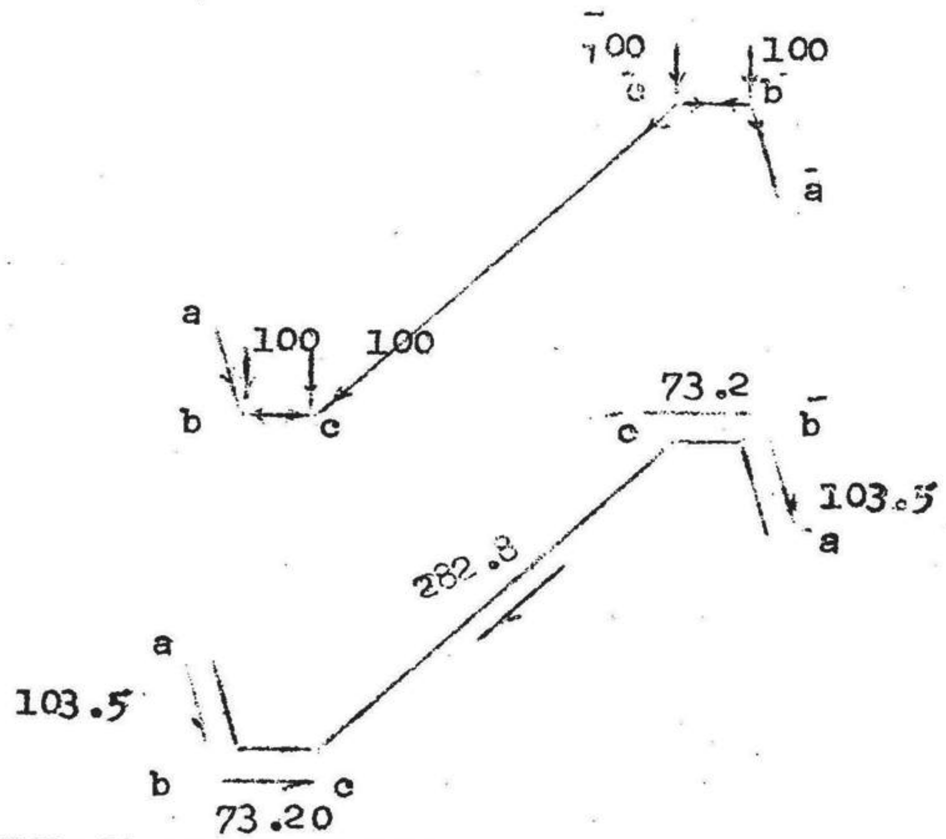


Plate	loads	B.M=32S	Z	$\sigma = \frac{M}{Z}$
AB, $\bar{AB}$	103.5	3340	1.33	2510
BC, $\bar{BC}$	73.2	2340	0.52	4500
$\bar{C}C$	282.8	9050	29.2	310

Stress Distribution Process

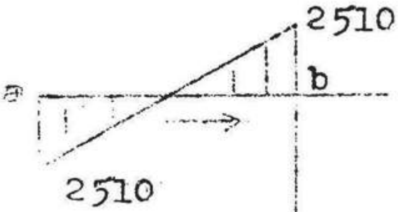
Distribution factors at point "B"

$$k_{ba} = \frac{A_{bc}}{A_{bc} + A_{ba}} = \frac{1.25}{1.25+2} = 0.385 \qquad k_{bc} = 0.615$$

at point "C"

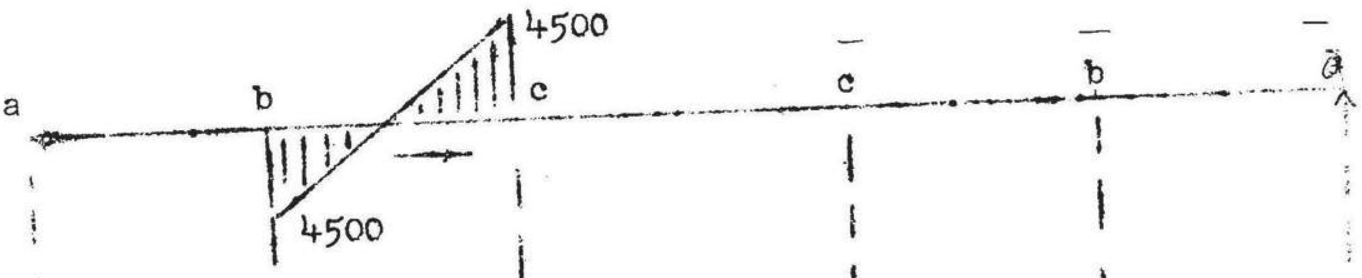
$$k_{cb} = \frac{9.375}{9.375+1.25} = 0.88 \qquad k_{c\bar{c}} = 0.12$$

A-B Loaded alone:



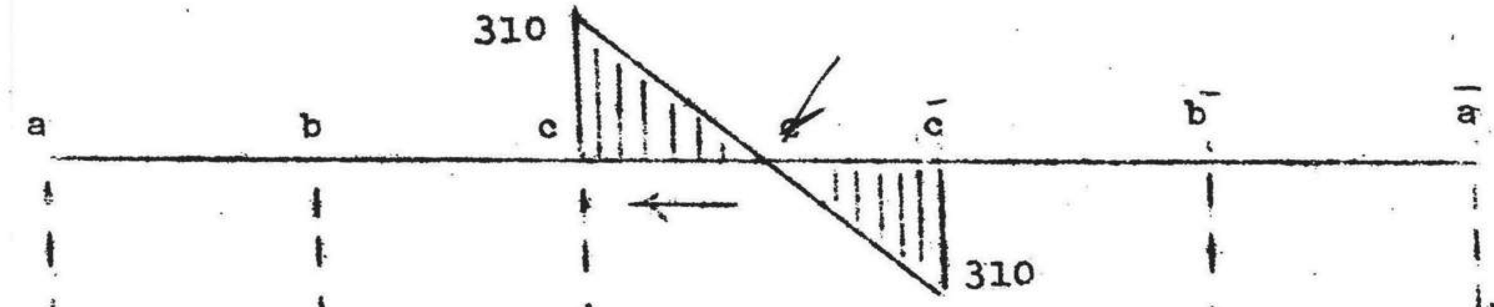
	a	b	b	c	c	c̄	c̄	b̄	b̄	d̄
D.F.		.385	.615	.88	.12	.12	.88	.615	.385	
F.E.M.	-2510	-2510								
Bal.		+970	+1540							
C.O.	+485			+770						
Bal.				-680	-90					
C.O.			-340			-45				
Bal.		+131	+209			+5.4	+39.6			
C.O.	+65			+105	+2.7			+19.8		
Bal.				-94.8	-12.9			-12.1	-7.7	
C.O.			-47.4			-6.5	-6.1			-3.9
Bal.		+18.2	+29.2			+1.6	+11.			
C.O.	+9.1			+14.6	+.8			+5.5		
Bal.				-13.6	-1.8			-3.4	-2.1	
	-1950.9	+1390.8		-105		+44.1		-9.8		+3.9

B-C **Loaded alone:**



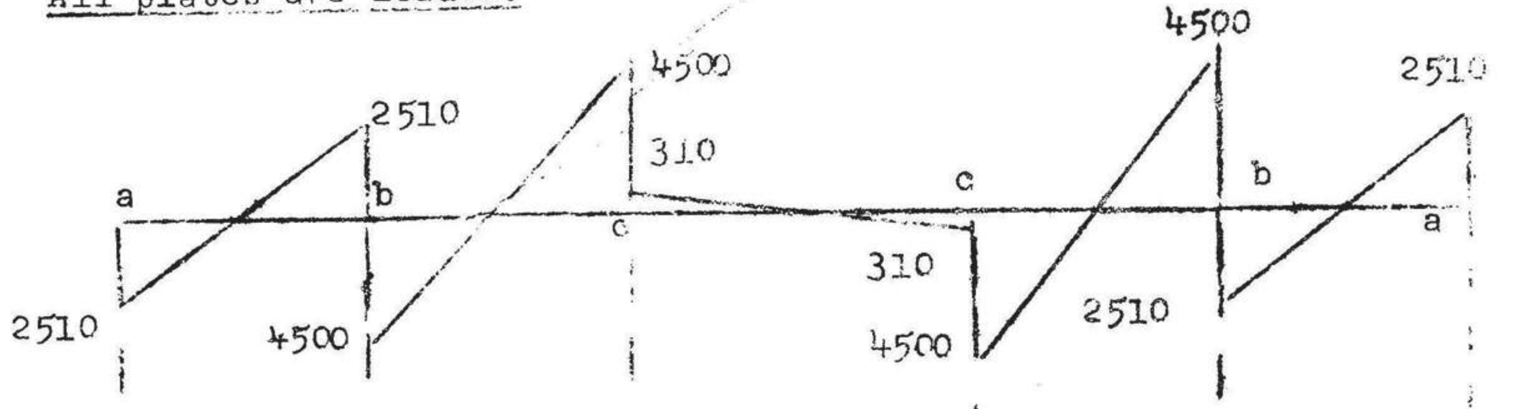
	a	b	b	c	c	c	c	b	b	a
D.F.		.385	.615	.88	.12	.12	.88	.615	.385	
F.E.M.			-4500	-4500						
Bal.		+1740	+2760	+3960	+540					
C.O.	+870		+1980	+1380		+270				
Bal.		-760	-1220	-1215	-165	-32.5	-237.5			
C.O.	-380		-608	-610	-16	-83		-119		
Bal.		+233	+375	+550	+76	+10	+73	73	+46	
C.O.	+117		+275	+187	+5	+38	+37	+37		+23
Bal.		-106	-169	-169	-23	-9	-66	-23	-14	
C.O.	-53		-85	-85	-5	-12	-12	-33		-7
Bal.		+33	+52	+79	+11	+3	+21	+20	+13	
C.O.	+17		+40	+26	+2	+6	+13	+11		+7
Bal.	-7	-15	-25	-24.5	-3.5	-2	-17	-7	-4	-2
	+564	-1125		+415		-185		+41		-21

Loaded alone:



		.385	.615	.88	.12	.12	.88	.615	.385	
1,					+310	+310				
c				-273	-37	-37	273			
.			-137		-19	-19		137		
.		+53	+84	+16.7	+2.3	+2.3	+16.7	+84	+53	
).	27		+8.4	+42	+1.2	+1.2	+42	+8.4		+27
-.		-3.2	-5.2	-38	-5.2	-5.2	-38	-5.2	-3.2	
).	-1.6		-19	-2.6	-2.6	-2.6	-2.6	-19		-1.6
1.		+7.2	+11.8	+4.6	+0.6	+.6	+4.6	+11.8	+7.2	
0.	+3.6		+2.3	+5.9			+5.9	+2.3		+3.6
1.		-1	-1.3	-5.2	-.7	-.6	-5.2	-1.3	-1.	
	+29	-56		+245		-245		+56		-29

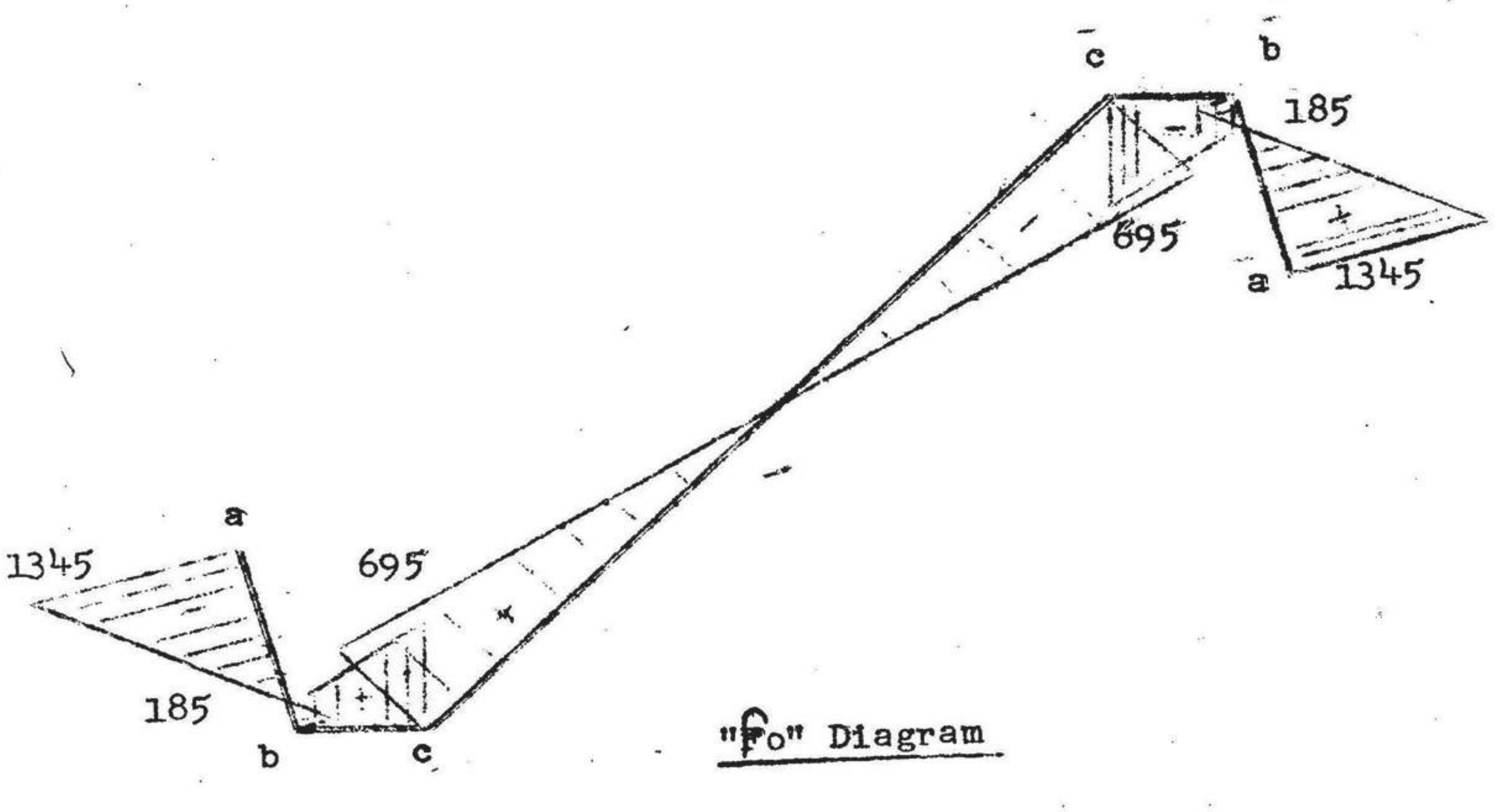
All plates are loaded:



D.F.		.385	.615	.88	.12	.12	.88	.615	.385	
F.E.M.	-2510	-2510	-4500	4500	310	+310	-4500	-4500	-2510	-2510
Bal.		+2700	+4310	+3685	+505	+505	+3685	+4310	+2690	
C.O.	+1350		+1842	+2155	+252	+252	+2155	+1842		+1345
Bal.		-710	-1132	-2117	-290	-290	-2117	-1132	-710	
C.O.	-355		-1060	-566	-145	-145	-566	-1060		-355
Bal.		+407	+653	+636	+75	+75	+636	+653	+407	
C.O.	+204		+318	+327	+37	+37	+327	+318		+204
Bal.		-125	-193	-320	-44	-44	-320	-193	-125	
C.O.	-63		-160	-97	-22	-22	-97	-160		-63
Bal.		+61.5	98.5	+104.8	+14.2	+14.2	+104.8	+98.5	+61.5	
C.O.	+31		+52	+49	+7.1	+7.1	+49	+52		+31
Bal.		-20	-32	-49.4	-6.7	-6.7	-49.4	-32	20	
C.O.	-10		-25	-16	-3.3	-3.3	16	-25		-10
Bal.	+5	+10	+15	+17	+2.3	+2.3	+17	+15	+10	+5
		1345	+185		695	695		-185		+1345

- Stresses after distribution neglecting

The effect of joint displacements

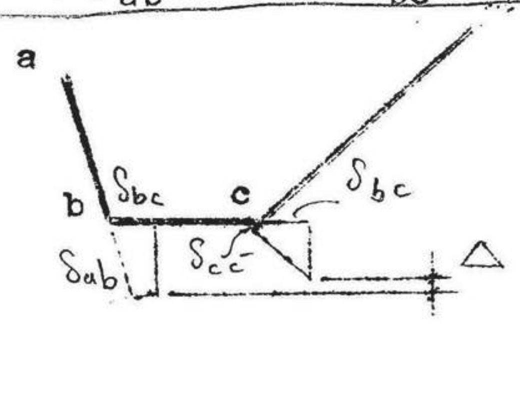


"Fo" Diagram

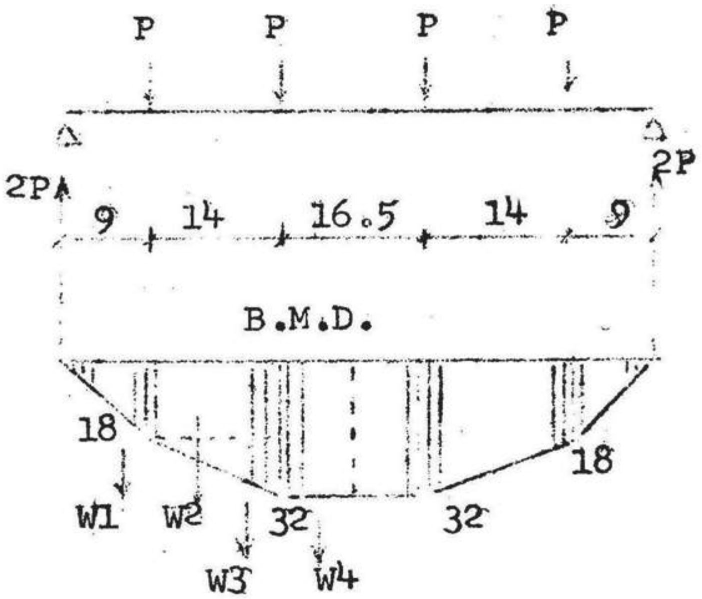
b . Correction Analysis:

- Geometrical Relation between △ and δ:

$$\triangle = 1.035\,\delta_{ab} - 1.268\,\delta_{bc} - 1.414\,\delta_{cc}$$



Displacement Parallel to plate element δ :



The elastic weights

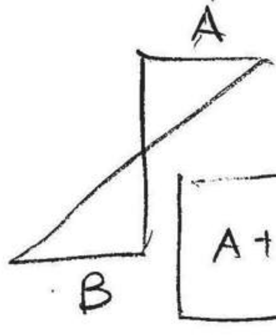
$W1 = \frac{18 \times 9}{2} = 81$

$W2 = 18 \times 14 = 252$

$W3 = \frac{14 \times 14}{2} = 98$

$W4 = 32 \times 8.25 = 264$

$\Sigma W = 695$



$A + B = H$

$\delta = 695 \times 31.25 - (\quad)$

$= 21700 - (264 \times \frac{8.25}{2} + 98(8.25 + \frac{14}{3})$

$+ 252 \times 15.2 + 81 \times 25.25)$

$21700 - (1090 + 1260 + 3850 + 2050) = 13450 \frac{P}{EI}$

H is the algebraic difference between [illegible]

$H = \frac{2M}{I} \times \frac{d}{2} = \frac{M}{I} d = \frac{32P}{I} d$

$H = \frac{M_1}{I_1}\frac{d_1}{2} + \frac{M_2}{I_2}\frac{d_2}{2}$?

$\therefore P = \frac{IH}{32d}$

$\therefore \delta = \frac{13450}{EI} \times \frac{IH}{32d} \times \frac{L^2}{62.5 \times 625} = \frac{HL^2}{9.3Ed}$

$$\delta = \frac{HL^2}{9.3\ Ed}$$

$A = \frac{\Delta P}{A} + \frac{M}{I} y$

$B = \frac{\Delta P}{A} - \frac{M}{I} y$

$|A - B| = H = \frac{2M}{I} y$

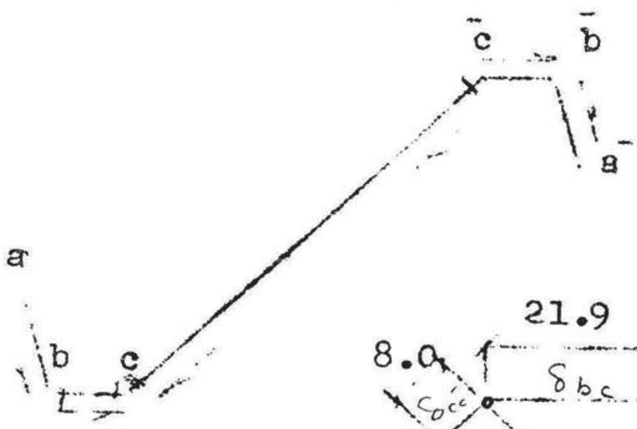
Where "d" is the plate depth .

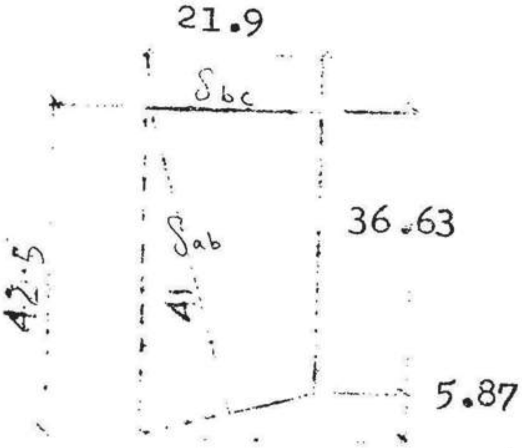
Relative displacement " Δ " between edges $\bar{B}$ & C or B & $\bar{C}$:

$$① \quad \delta_{ab} = \frac{1530}{9.3 \times 4} \frac{L^2}{E} = 41 \frac{L^2}{E}$$

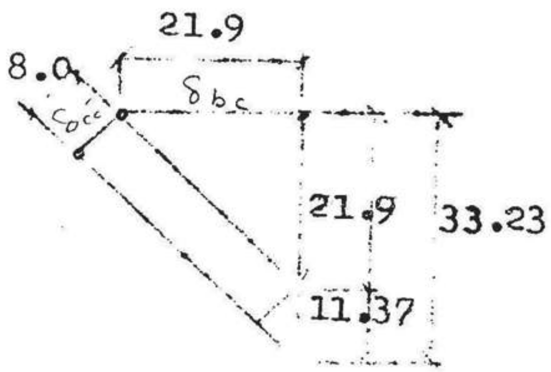
$$② \quad \delta_{bc} = \frac{510}{9.3 \times 2.5} = 21.9 \frac{L^2}{E}$$

$$③ \quad \delta_{cc} = \frac{1390}{9.3 \times 18.75} = 8.0 \frac{L^2}{E}$$





Point b



Point c

$$\Delta = 3.4 \frac{L^2}{E}$$

- Plate loads due to transverse deformations.

b $\bar{P}$ — c $\bar{P}$ — $\bar{c}$ $\bar{P}$ — $\bar{b}$ $\bar{P}$

	b – c	c – $\bar{c}$	$\bar{c}$ – $\bar{b}$
D.F	.915	.085	
F.E.M.	+M	o	
bal	.915M	0.085	
Final M	0.085M	0.085M	

- **Stiffnesses factors:**

$k_{cb} = \frac{3}{4} \times \frac{I}{2.5} = \frac{3}{10}$ $\qquad k_{c\bar{c}} = \frac{I}{18.75} \times \frac{1}{2}$ due to symmetry.

$$\therefore k_{cb} = \frac{\frac{3}{10}}{\frac{3}{10} + \frac{1}{37.5}} = \frac{37.5 \times 3}{37.5 \times 3 + 10} = .915$$

$$M = \frac{3EI}{l^2} \Delta \qquad \bar{P} = \frac{0.085M}{l}$$

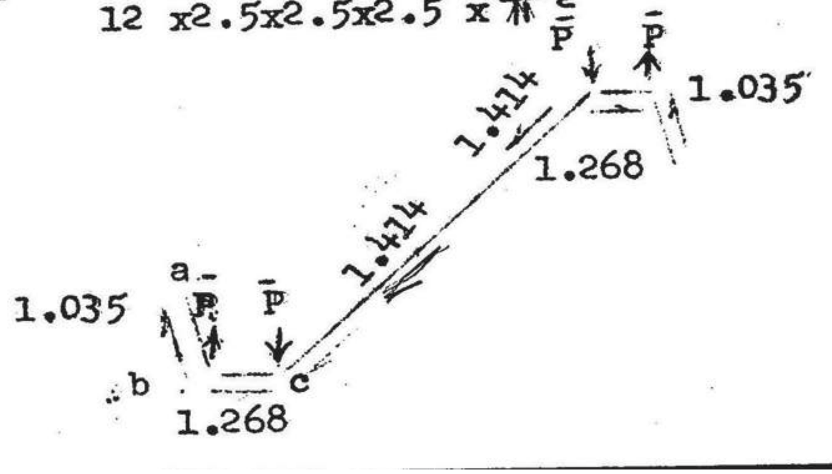
$$\therefore \bar{P} = \frac{0.085 \times 3EI}{l^3}$$

$$\text{i.e. } \bar{P} = \frac{0.085 \times 3 \times 1 \times .5 \times .5 \times .5}{12 \times 2.5 \times 2.5 \times 2.5} E \Delta \text{ kg/cm}$$

$$\text{If } \phi = \frac{\bar{P} L^2}{\pi^2} \times 10^{-6} \text{ kg.cm.}$$

$$\therefore \phi = \frac{0.085 \times 3 \times 1 \times .5 \times .5 \times .5 \times 62.5 \times 62.5}{12 \times 2.5 \times 2.5 \times 2.5 \times \pi^2} \times 10^{-6} = .0672 \times 10^{-6} \, E\Delta$$

$0.0672 \times 10^{-6} \, E\Delta$



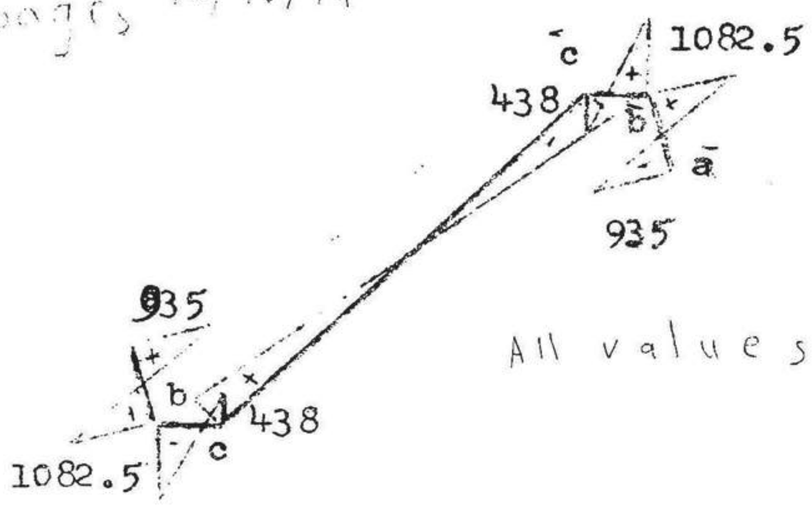
Bending moments & stresses due to transverse deformations:

Palte	Ld.	B. M.	Stresses
AB	1.035 $\bar{P}$	1.035 x10^6 ⊄	0.778 x10^6 ⊄
BC	1.268 $\vec{P}$	1.268 x10^6 ⊄	2.435x10^6 ⊄
C$\bar{C}$	2.8228$\bar{P}$	2.828x10^6 ⊄	0.097x10^6 ⊄

stresses by superpositions.

plate	a x10^{+3}	b x10^{+3}	c x10^{+3}	$\bar{c}$ x10^{+3}	$\bar{b}$ x10^{+3}	$\bar{a}$ x10^{+3}
AB Loaded	+608	−430	+31.5	−13.7	+3.05	−1.2
BC Loaded	+305	−610	+225	−100	+22	−11
C$\bar{C}$ Loaded	+9.1	−17.5	+76.5	−76.5	+17.5	−9.1
$\sum$	+935	−1082.5	+438	−438	+1082.5	−935

refere to pages 42, 43, 44



All values × 10^{+3} ⊄

- <u>Substituting the values of δ in the geometrical relation</u>

$$\delta ab = (41 - \frac{2017}{4\pi^2}\phi \times 10^3)\frac{L^2}{E} = (41 - 52\times 10^3 \phi)\frac{L^2}{E}$$

$$\delta bc = (21.9 + \frac{1520}{2.5\pi^2})\frac{L^2}{E} = (21.9 + 61.5\phi \times 10^3)\frac{L^2}{E}$$

$$\delta cc = (8 + \frac{2\times 438}{18.75\pi^2})\frac{L^2}{E} = (8 + 4.73\phi \times 10^3)\frac{L^2}{E}$$

but we have:

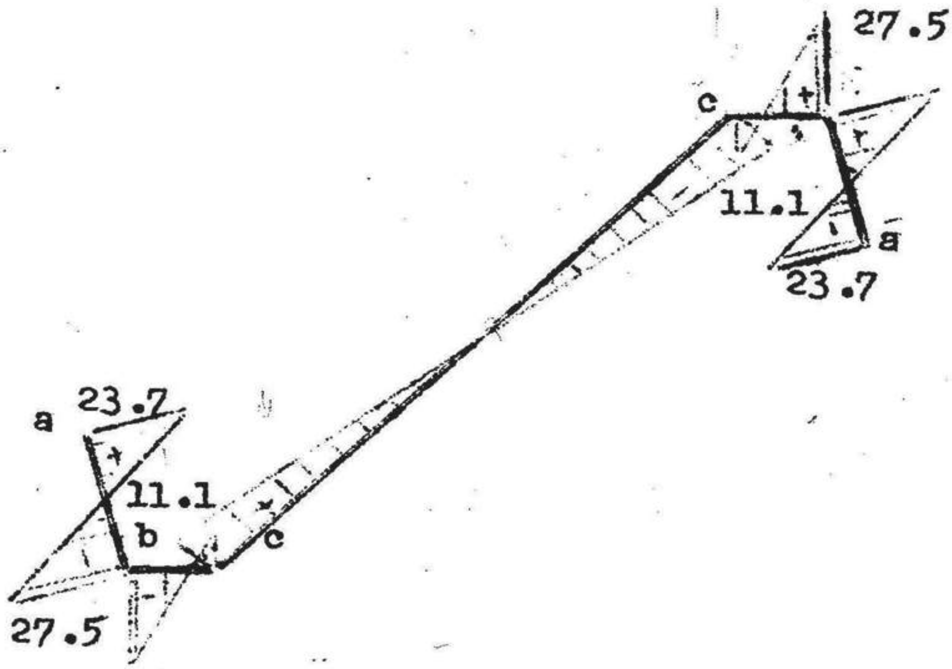
$$\Delta = (\delta ab/\cos 15 - \delta bc\ \tan 15) - (\delta bc + \sqrt{2}\ \delta cc)$$

$$\text{ie } \Delta = 1.035\,\delta ab - 1.2688\,\delta bc - 1.414\,\delta cc$$

$$= 3.6\frac{L^2}{E} - 138.0\times 10^3 \phi \frac{L^2}{E}$$

$$= 3.6\frac{L^2}{E} - 36.3\,\Delta \qquad \therefore \Delta = 0.0967\frac{L^2}{E}$$

$$\therefore \phi = 25.4 \times 10^{-6}$$

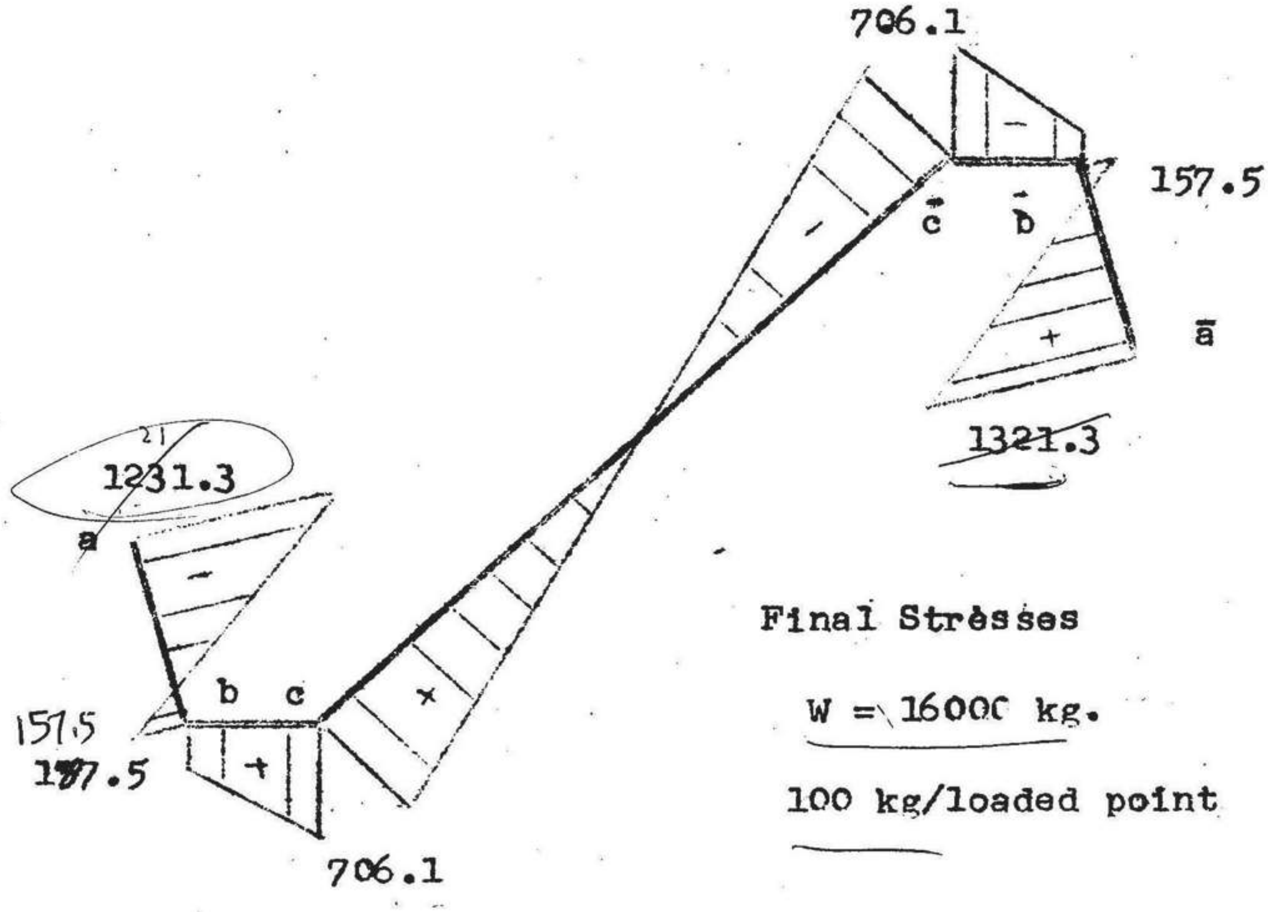


<u>Stresses due to relative displacement.</u>

c- Final Results

- Longitudinal Stresses.

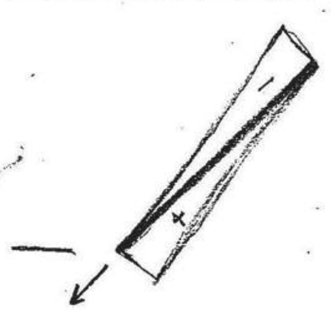
Stresses due to transverse defromations are added algebraically to the corresponding values obtained in the elementary analysis to get the final stresses.



Final Stresses

W = 16000 kg.

100 kg/loaded point



$N = \frac{100 \times 4}{62.5} \cong 8$ Kg

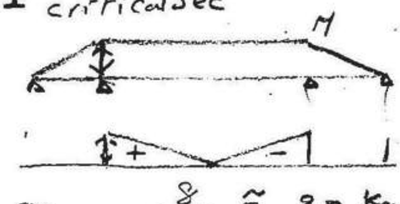
-Transferse Stresses

$$M = \frac{0.085 \times 3 \times .5 \times .5 \times .5}{12 \times 2.5 \times 2.5} \times \frac{62.5 \times 62.5}{1} \times 0.0967 = 0.16 \text{ Kg.cm/cm}$$

critical Sec

$N = 0$

$$F = \frac{6M}{bt^2} = \frac{6 \times 0.16}{0.5 \times 0.5} = 3.84 \text{ kg/cm}^2$$



$f_{2\,max} = 3.84 + \frac{8}{0.5} \cong 20$ Kg/cm² V. Small

- Displacements

$$\delta ab = \left(-\frac{1345+185}{9.3 \times 4}\right) \frac{L^2}{E} - \frac{(23.7+27.5)}{4 \times \pi^2} \times \frac{L^2}{E}$$

$= 0.232 \quad - .0070 \quad = 0.225$ cm

$$\delta bc = \frac{(695-185)}{9.3 \times 2.5} \frac{L^2}{E} + \frac{(27.5+11.1) \times}{2.5 \times \pi^2} \frac{L^2}{E}$$

$= 0.1250 \quad + 0.009 \quad = .134$ cm

$$\delta c\bar{c} = \frac{695 \times 2}{9.3 \times 18.75} \frac{L_2}{E} + \frac{11.1 \times 2}{18.75\,\pi^2} \times \frac{L^2}{E}$$

$= 0.04580 \quad + 0.00068 \quad = .04648$ cm

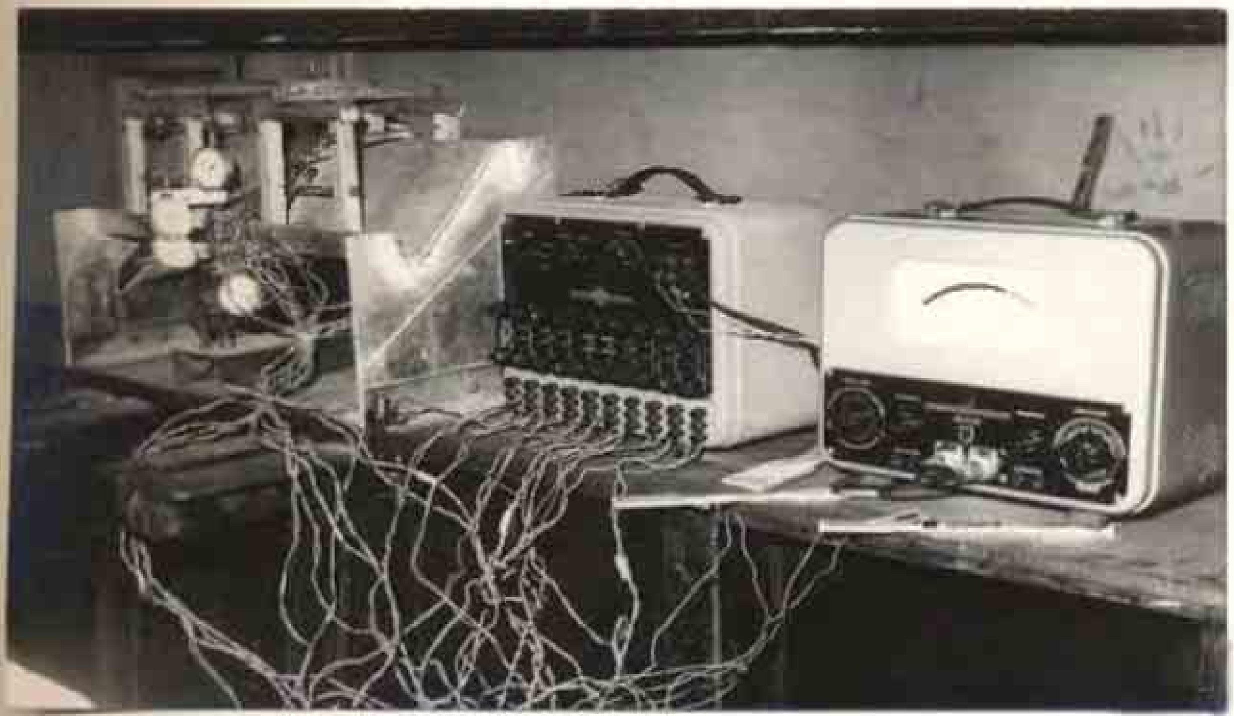
$\Delta_{bv} = \delta ab/\cos 15 - \delta bc \tan 15°$

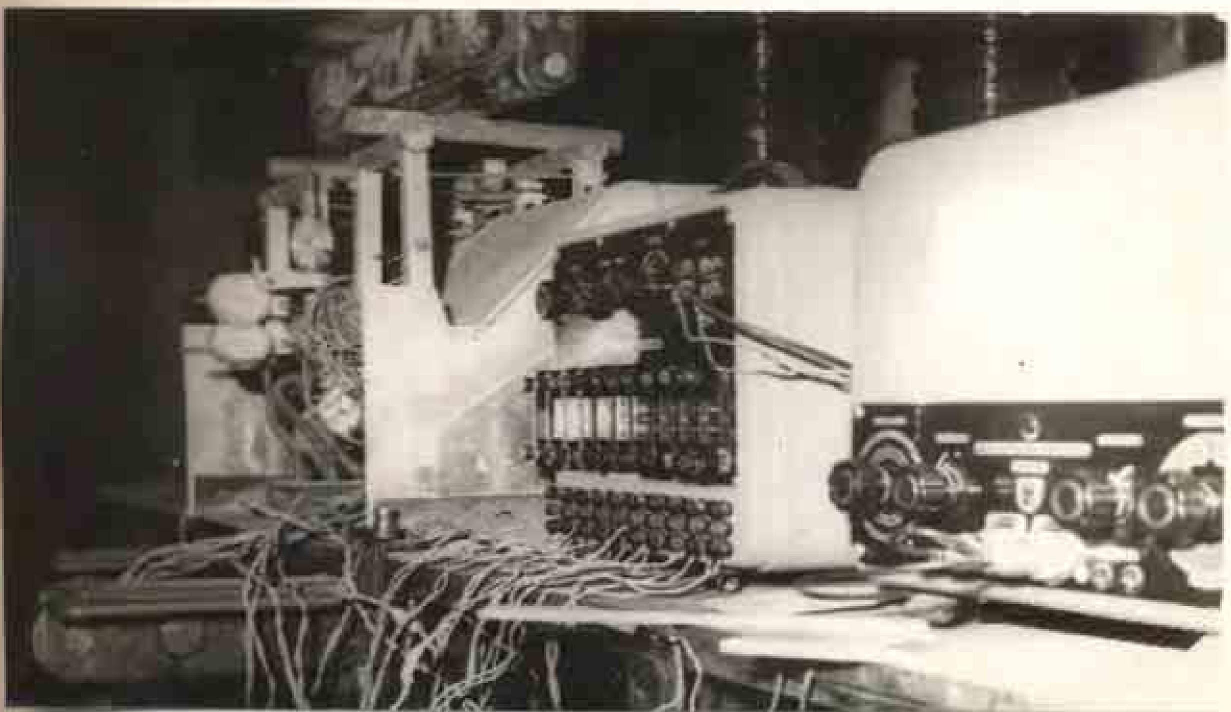
$= 0.2350 \quad - 0.0347 \quad = 0.2003$ cm

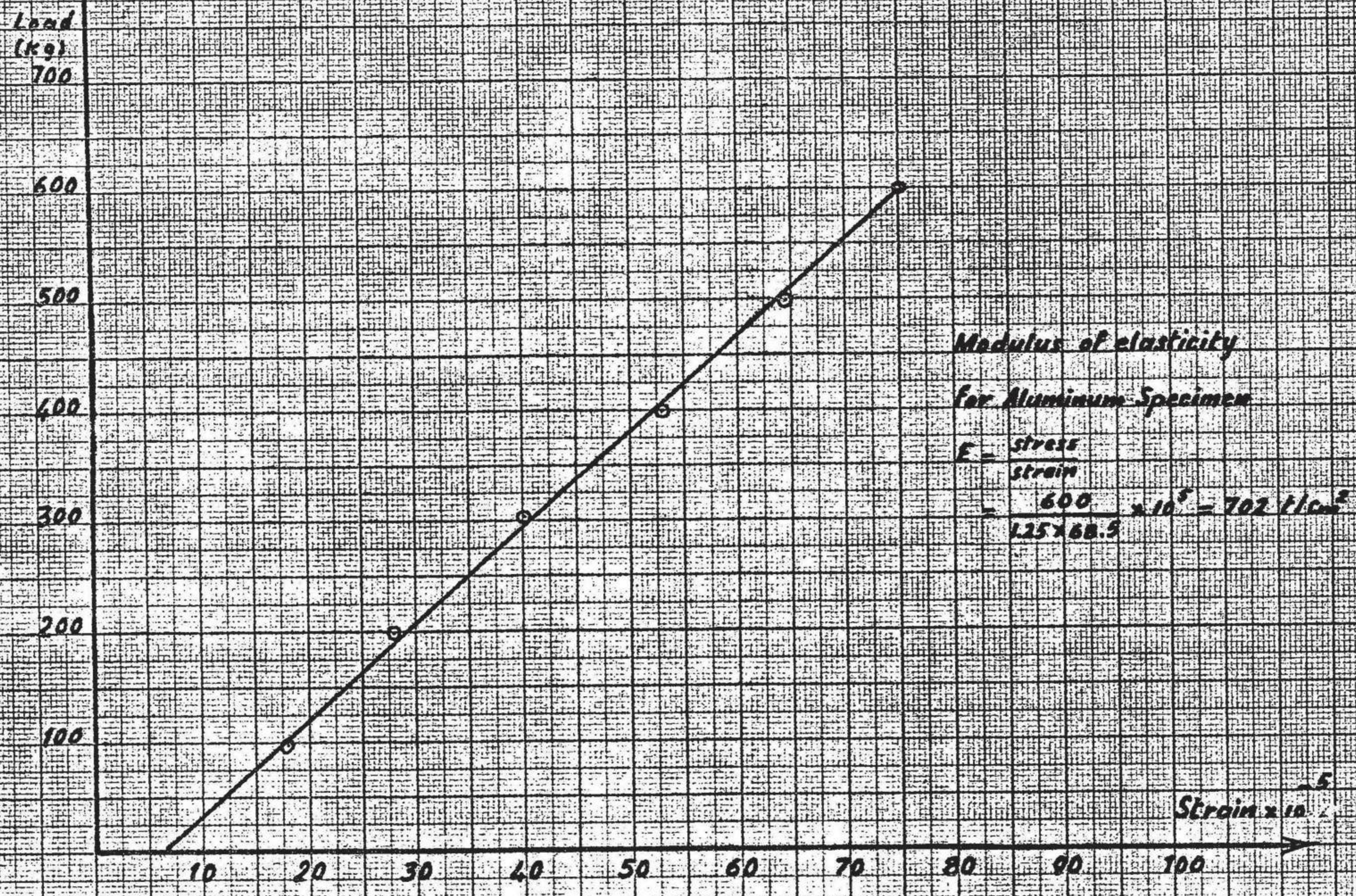
$\Delta_{cv} = \delta bc \tan 45° + \delta c\bar{c}/\cos 45°$

$= 0.1340 \quad + 0.0656 \quad = 0.1996$ cm

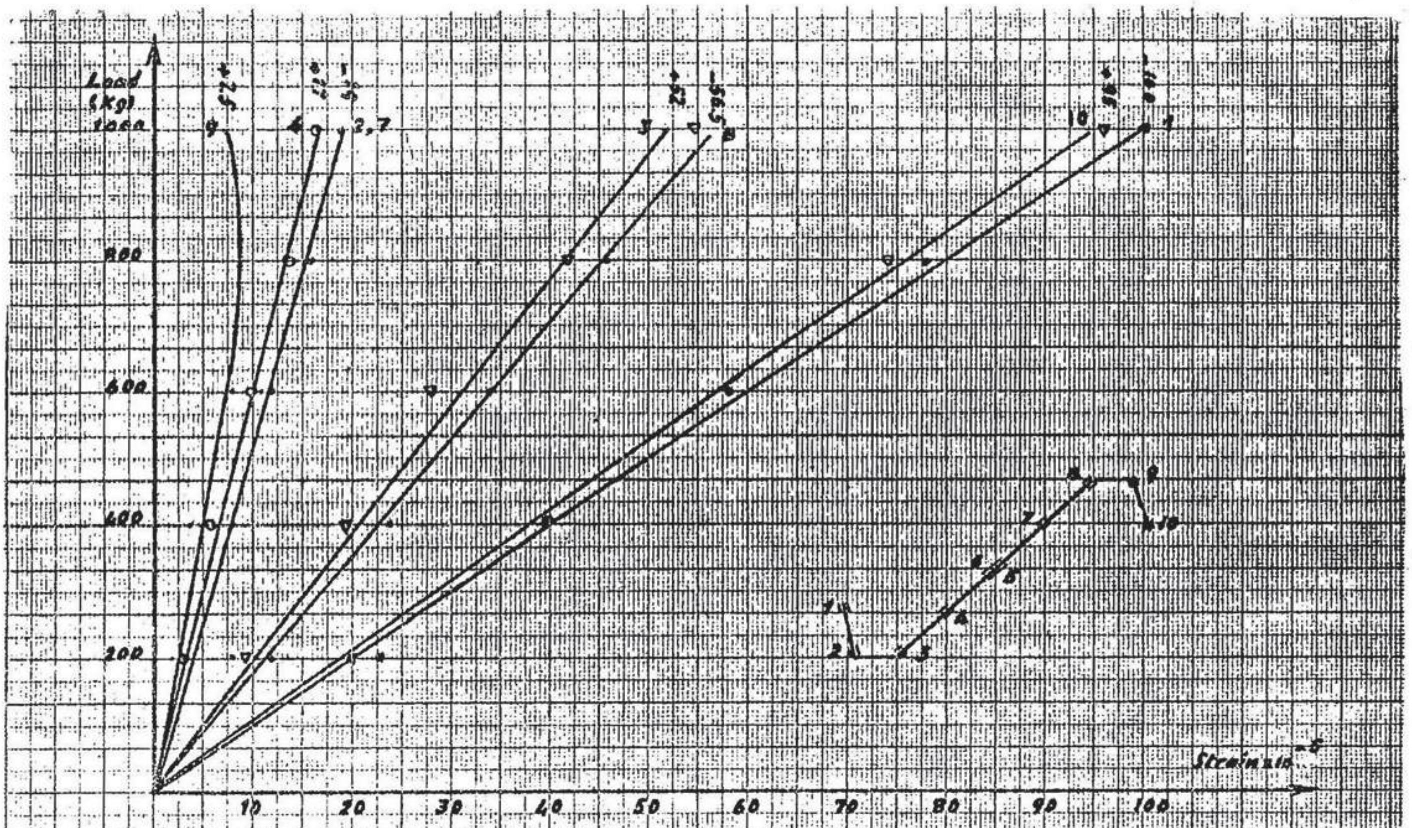
3 - EXPERIMENTAL RESULTS



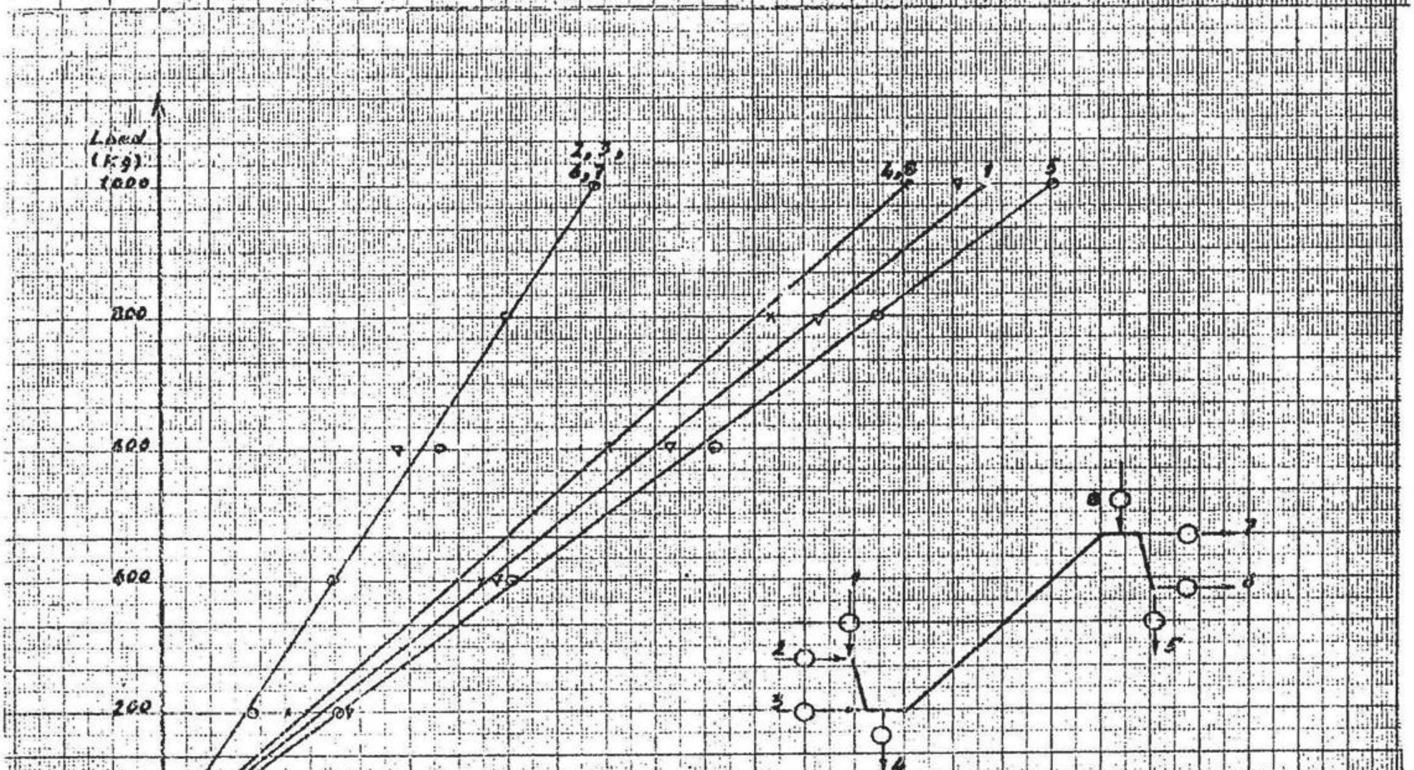




Load _ Strain Curve for Aluminum.



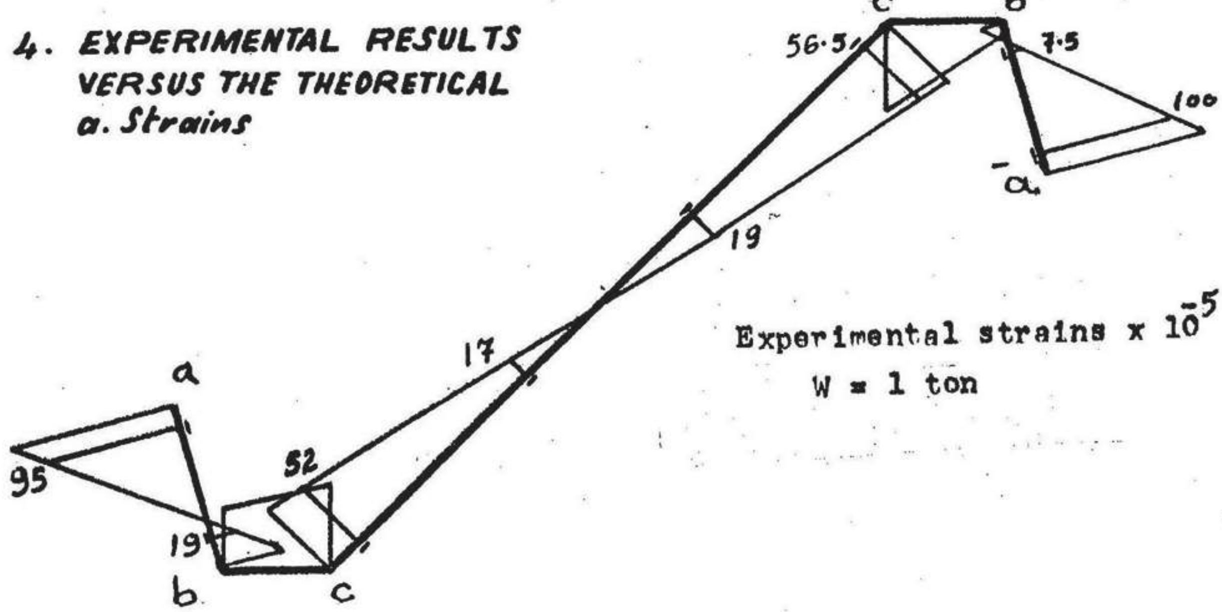
Aluminum Model-Load Versus Strain.



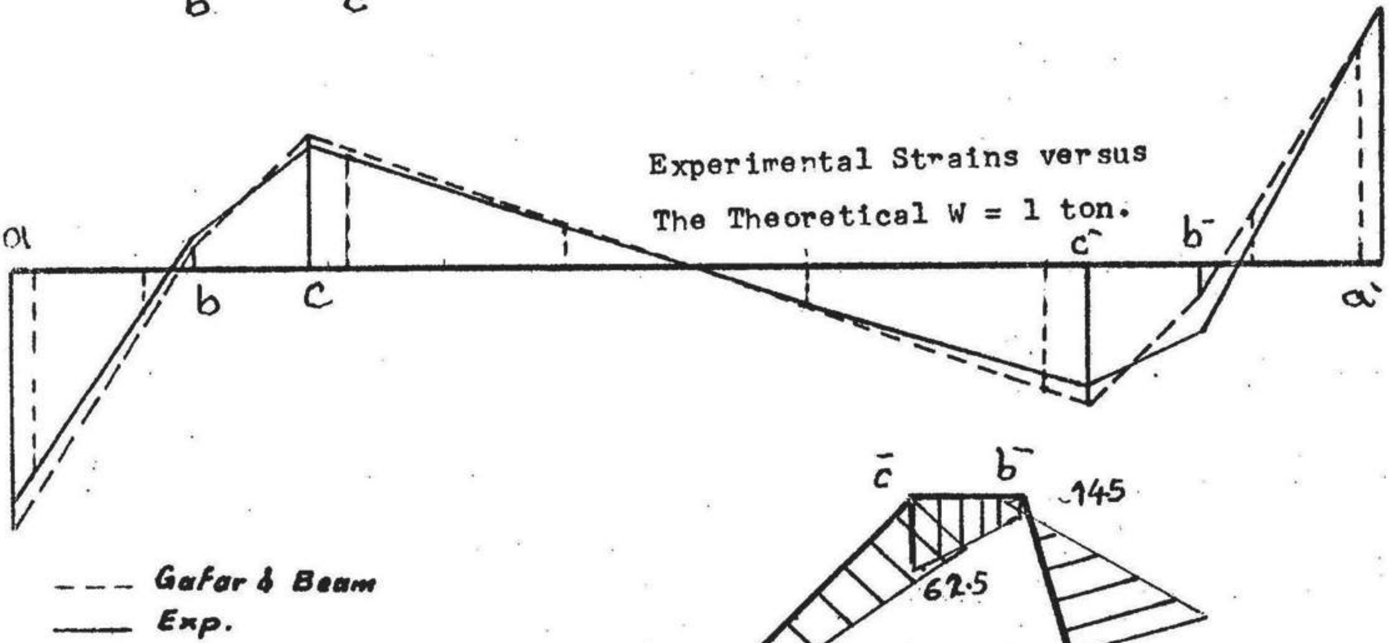
Aluminum Model-Load Versus Displacement.

4. EXPERIMENTAL RESULTS VERSUS THE THEORETICAL

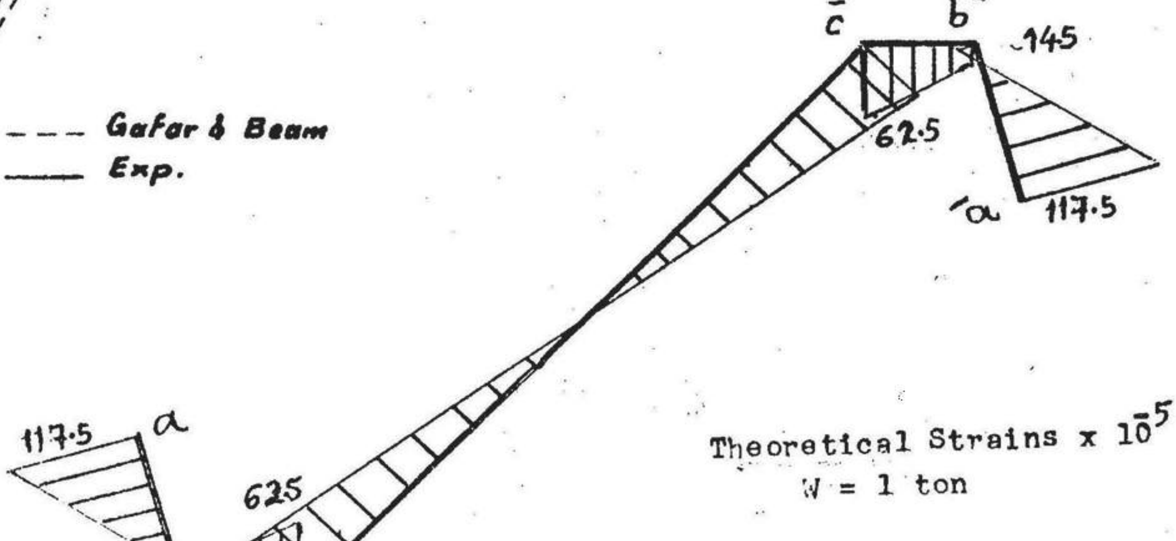
a. Strains



Experimental strains x 10^{-5}
W = 1 ton

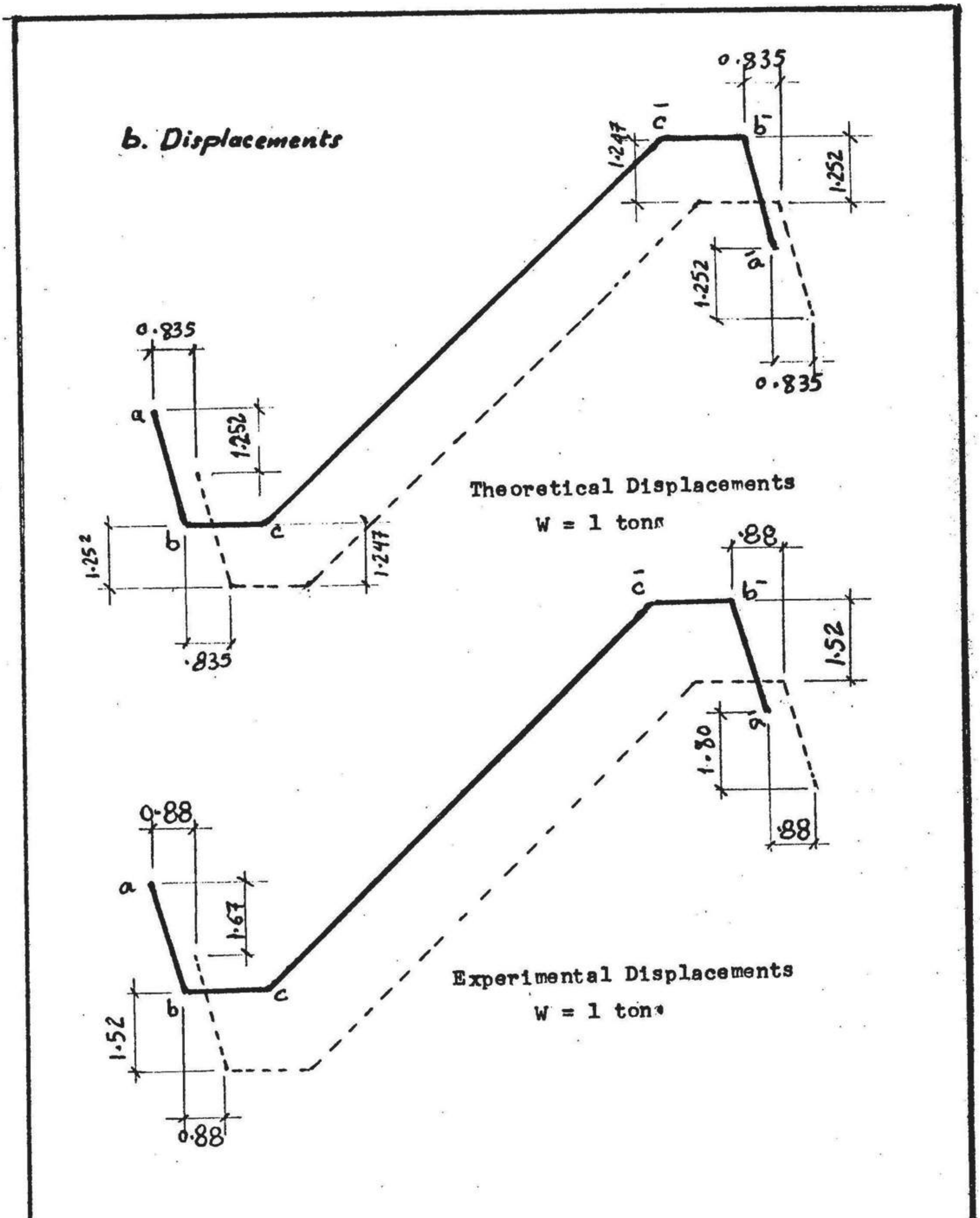


Experimental Strains versus
The Theoretical W = 1 ton.



Theoretical Strains x 10^{-5}
W = 1 ton

b. Displacements



C H A P T E R

III

THE REINFORCED CONCRETE MODEL

1- DESCRIPTION OF THE MODEL

The second model is of the reinforced concrete. It is a single span, 5-plates, saw-tooth folded plate and its general dimensions are shown in Fig.(3-1) and Fig.(3-2). It is clear that its proportions are the same as the aluminum model.

The mix was made according to the following ratio per cubic meter of finished concrete :

Cement	Sand	Gravel	W/C
400 kg	580 kg	1160 kg	0.5

The course aggergate has a maximum nominal size of 6.0mm and the fine aggregate is the Giza aggregate. Also the cement used was of the rapid hardening portland cement of Helwan. The curing was made by covering the roof with a layer of sand witted twice a day for the first two weeks and then left in the room temperature till date of test which was after four weeks. The properties of the hardened concrete and the steel used are shown in tables (3-1) & (3-2).

Fig.(3-7) Shows the loading system where the loads from the jack are transmitted to a sixteen loaded points through a system of group of simple beams similar to that of the aluminum model. Also the position of the longitudinal and transverse strain gages as well as the dial gages are shown in Fig.(3-8) and Fig.(3-9).

The model is supported at two end diaphragms having a thicknesses of 6.0 cm and of concrete dimensions and details of reinforcement as shown in Fig.(3-6).

This model was subjected to three cases of loading namely:

a - Symmetrical loading.

b- Unsymmetrical loading where loads were applied at the upper joints.

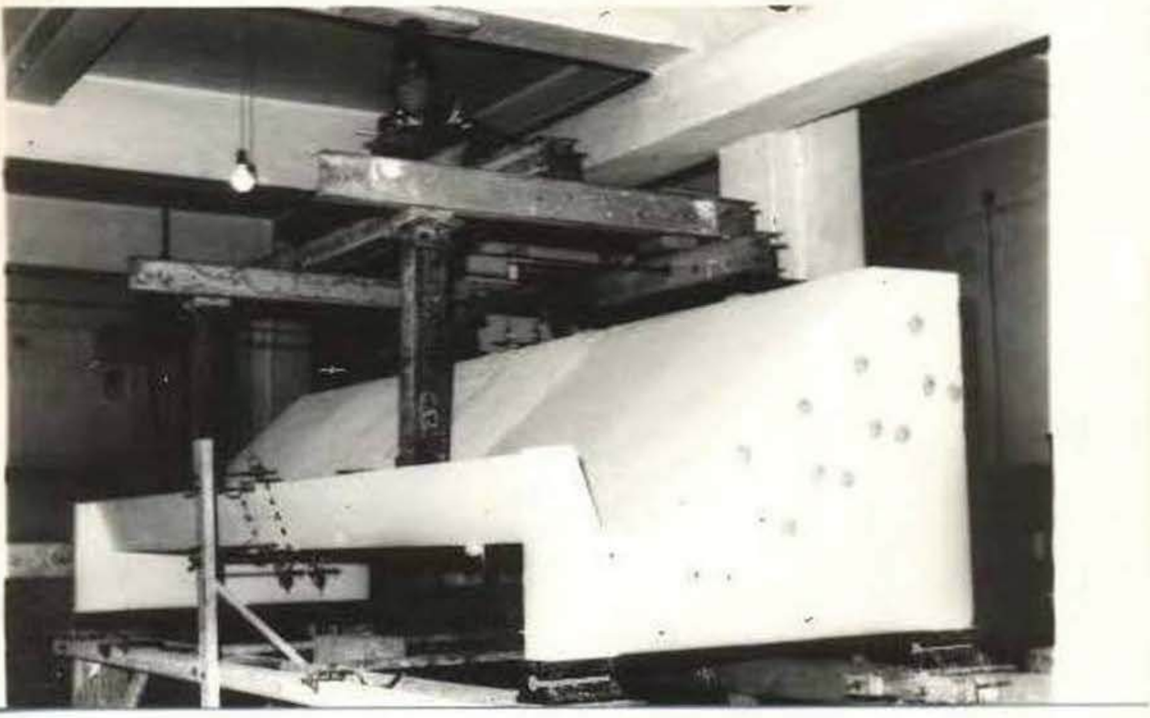
c - Unsymmetrical loading where loads were applied at the lower joints.

Table (2-1): Properties of Hardened Concrete

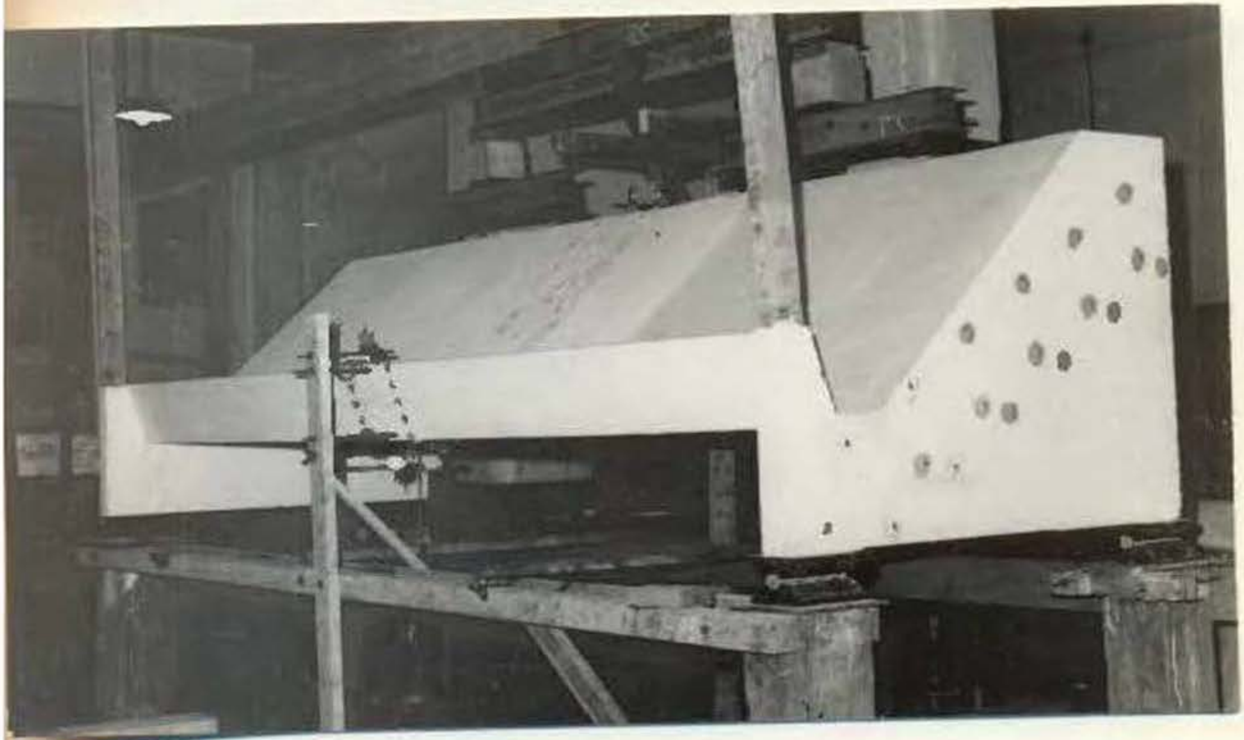
Piece No	Strength Kg / Cm²			E For plain Concrete Prism
	Cube	Prism	Cylinder	
1	312	280	240	-----
2	300	280	244	------
3	320	288	252	----
Average	311	283	245	300×10^3 kg/cm²

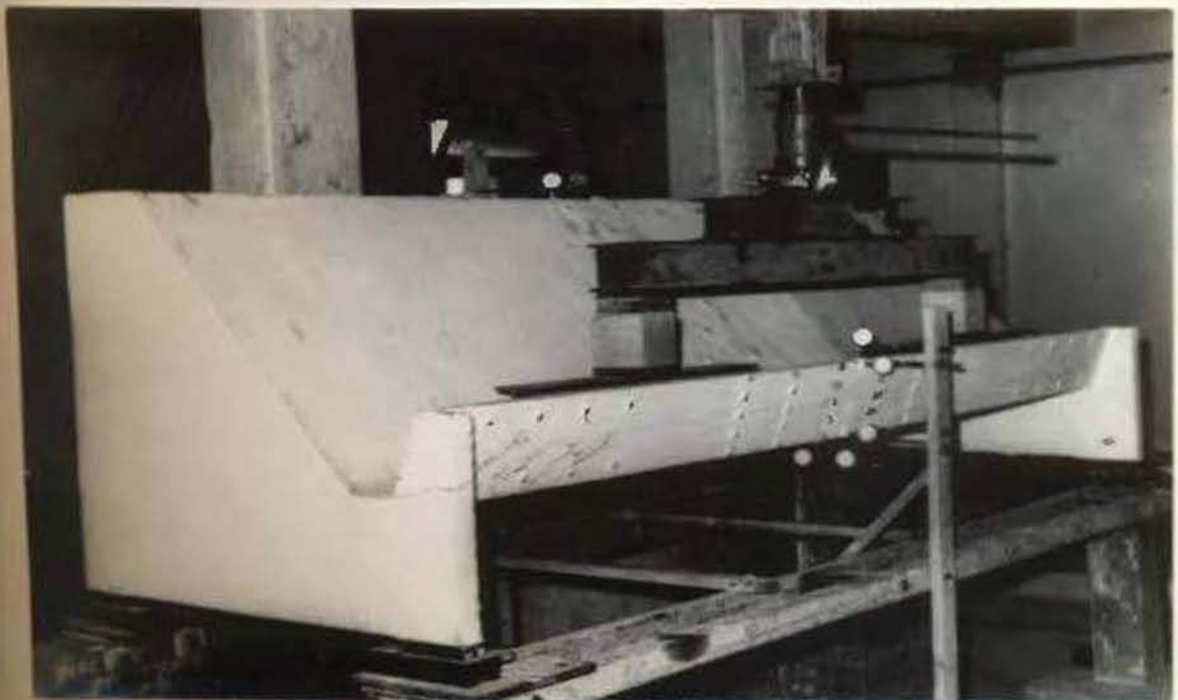
Tagle (3-2) Properties of the Steel Barsused

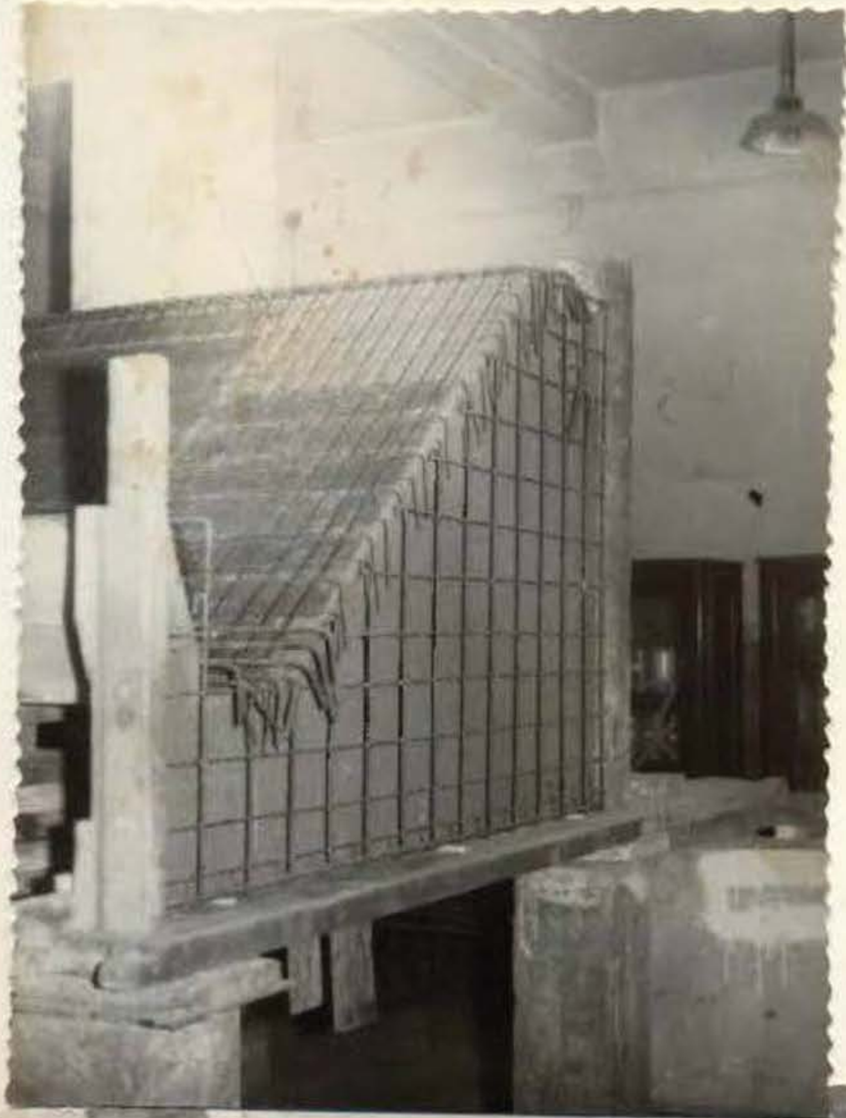
Properties	ø 2mm.	ø 6mm.	ø 13 mm.
Area of C.S cm²	0.0314	0.279	1.33
Yield load kg.	83	750	3750
Ult. load kg.	109	1050	4900
Yield stress kg/cm²	2640	2690	2810
Ult. stress kg/cm²	3470	3760	3670



V.







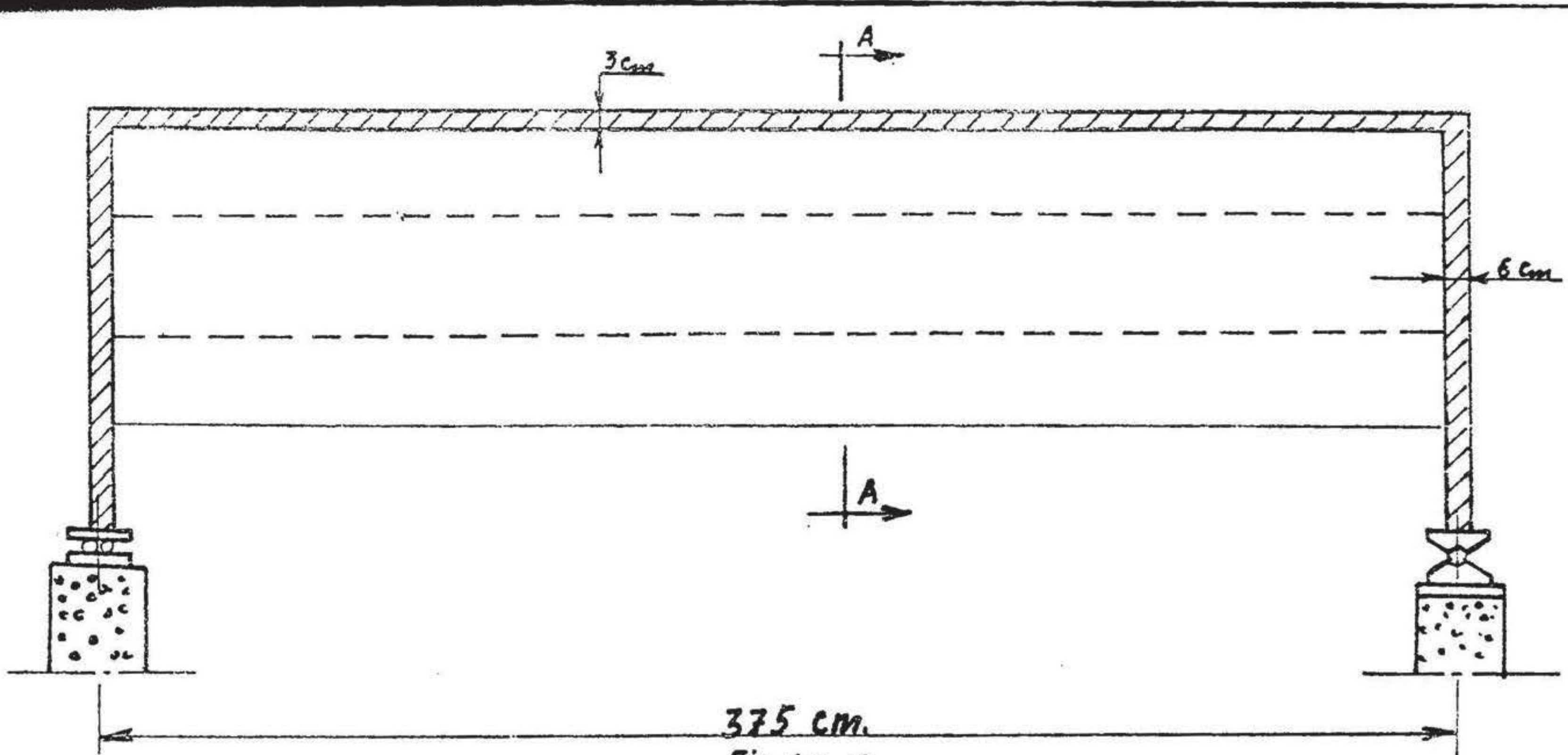


Fig.(3-1)

CONCRETE DIMENSIONS

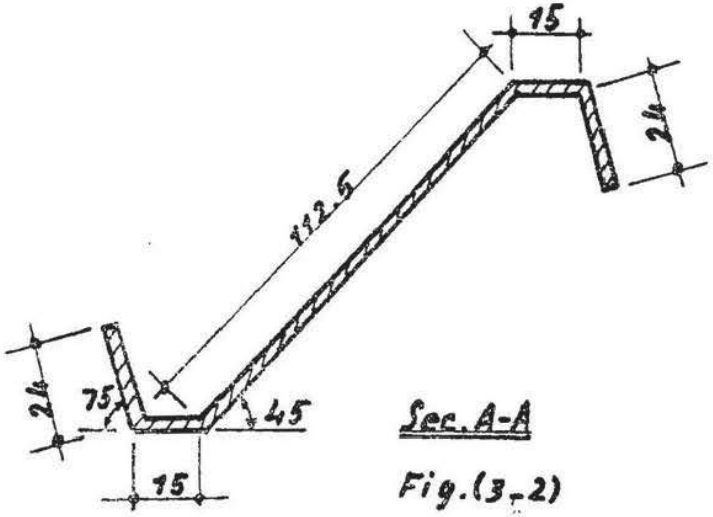


Fig.(3-2)

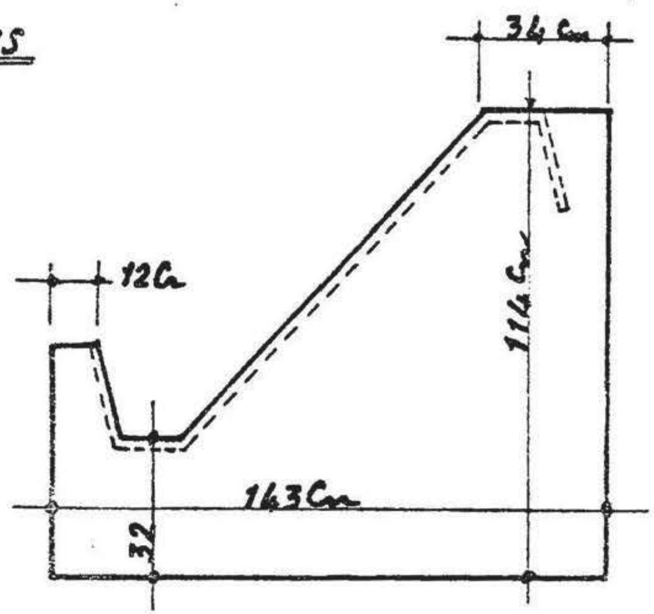
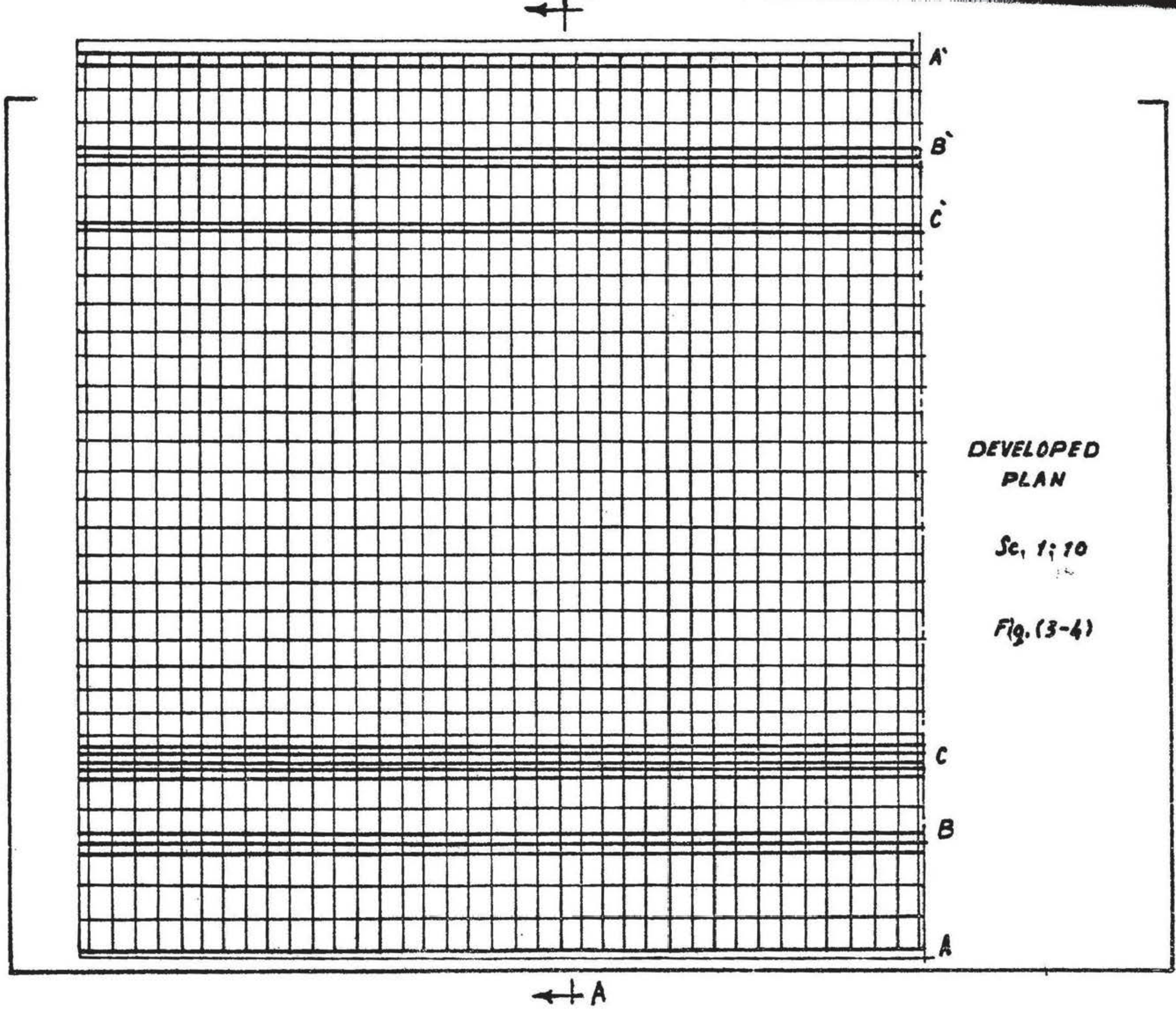


Fig.(3-3)



DEVELOPED
PLAN

Sc, 1; 10

Fig. (3-4)

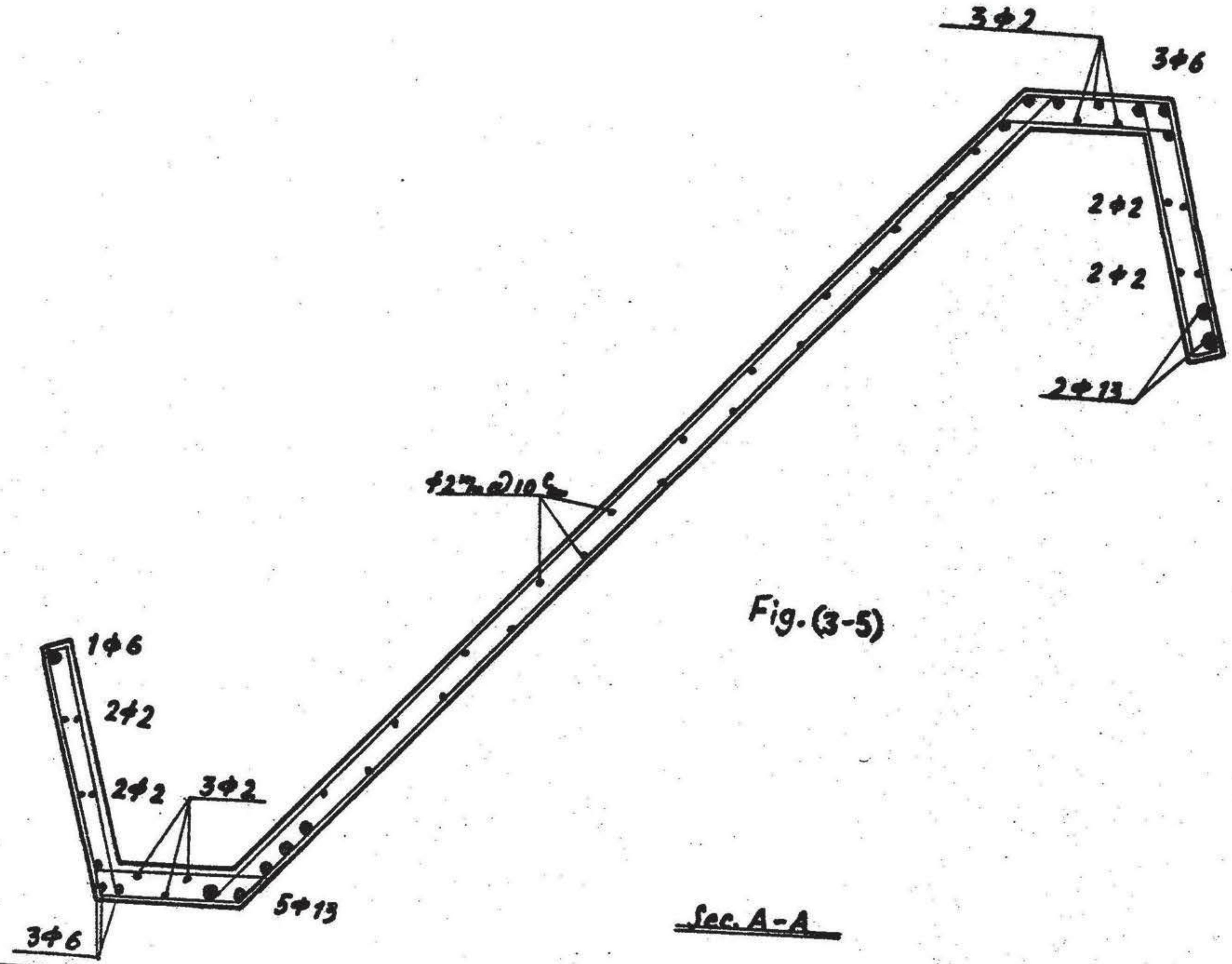
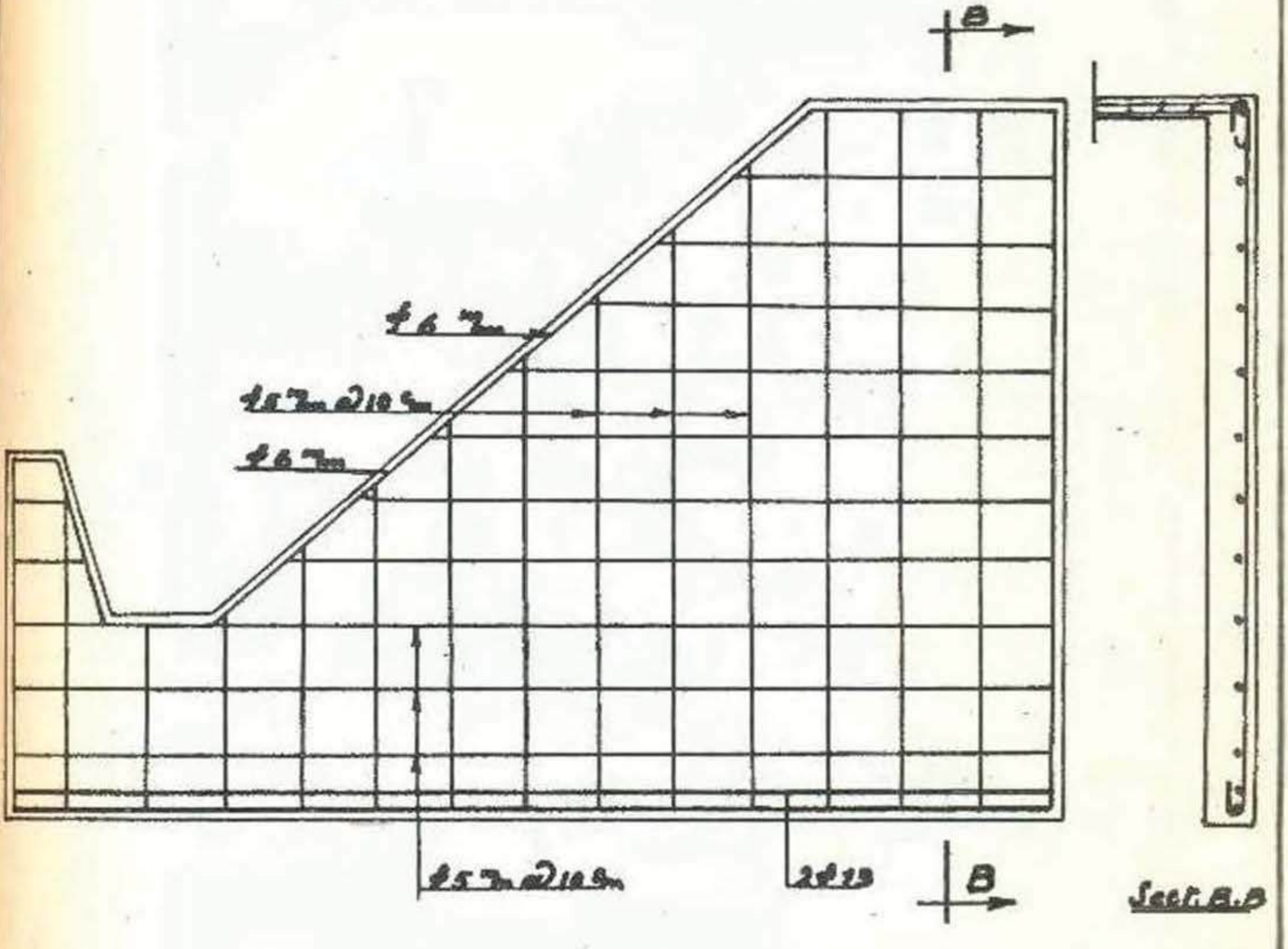


Fig.(3-5)

Sec. A-A

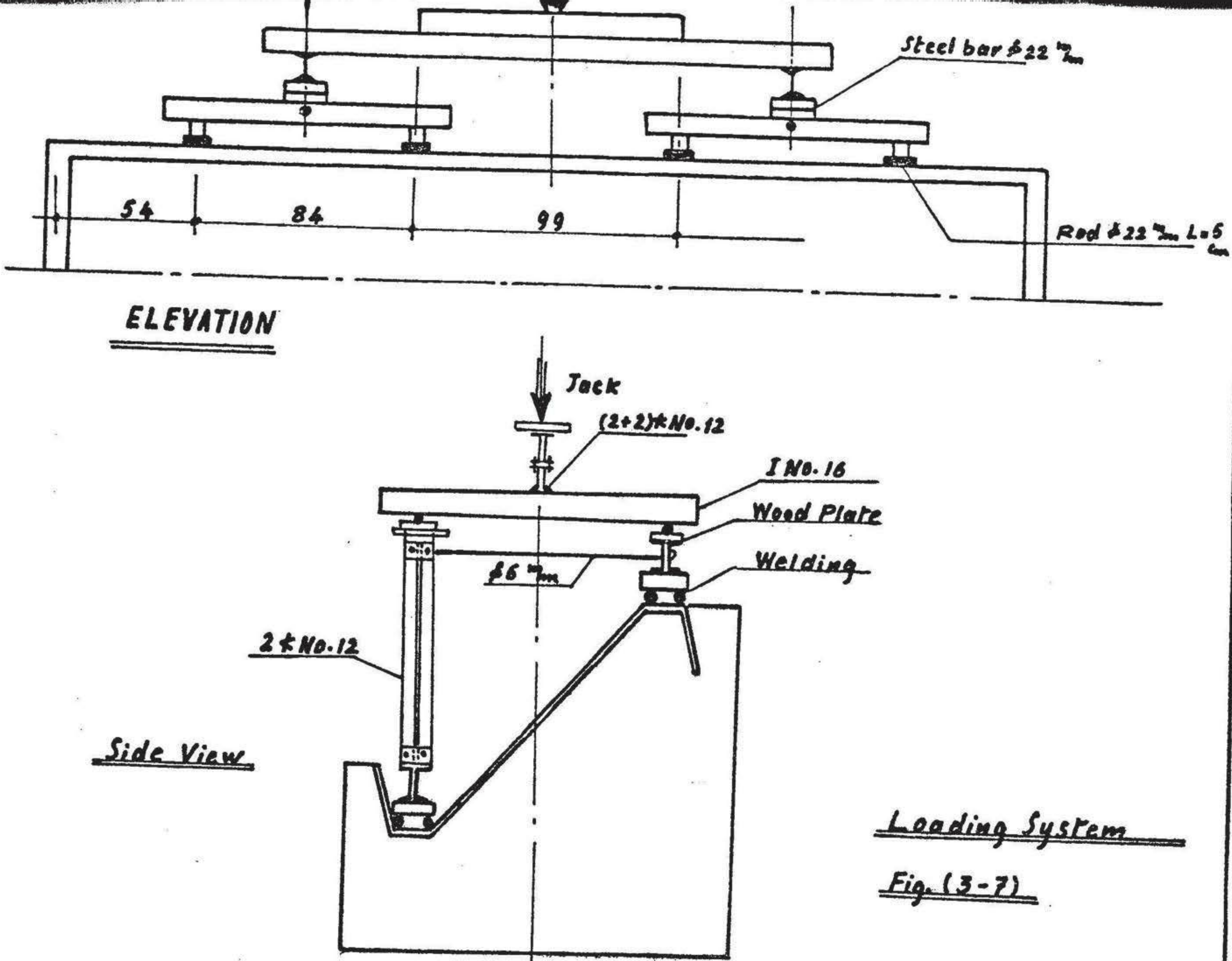


Sectional elevation

Sec. 1.10

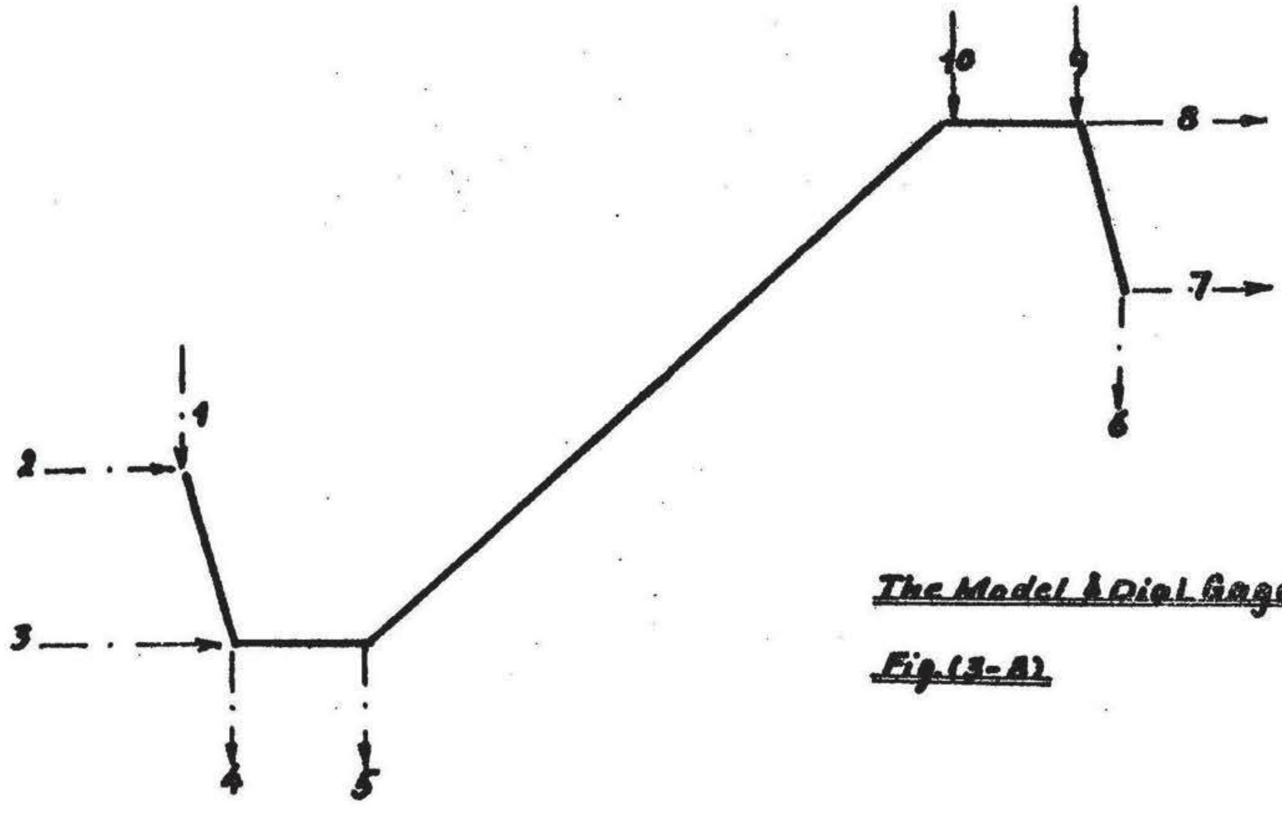
DETAILS OF THE END DIAPHRAGM

Fig. (3-6)



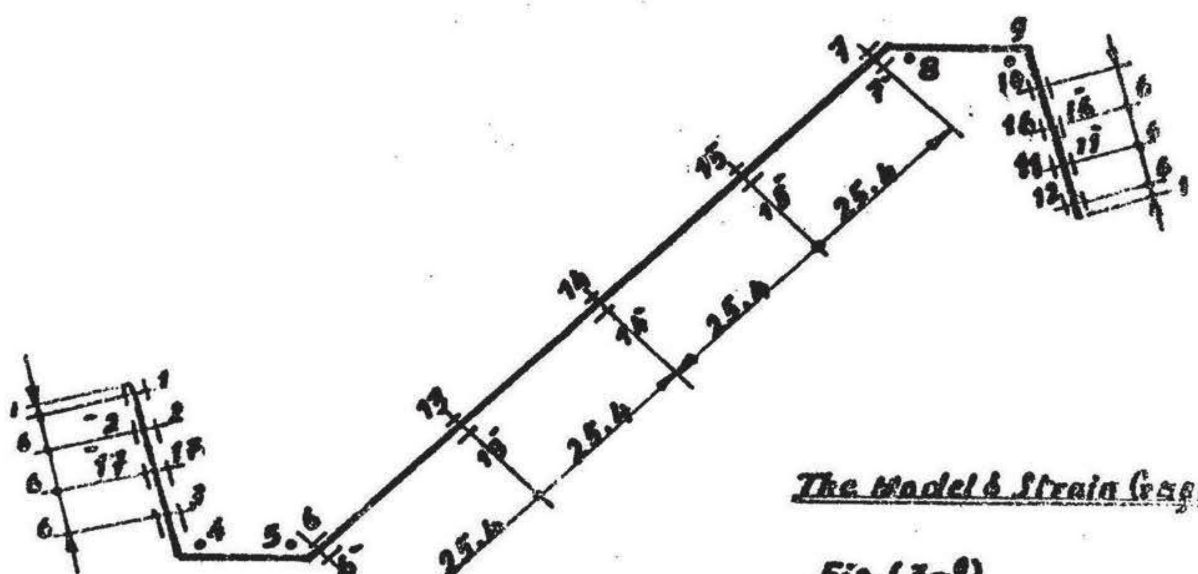
Loading System

Fig. (3-7)



The Model & Dial Gages

Fig.(3-A)



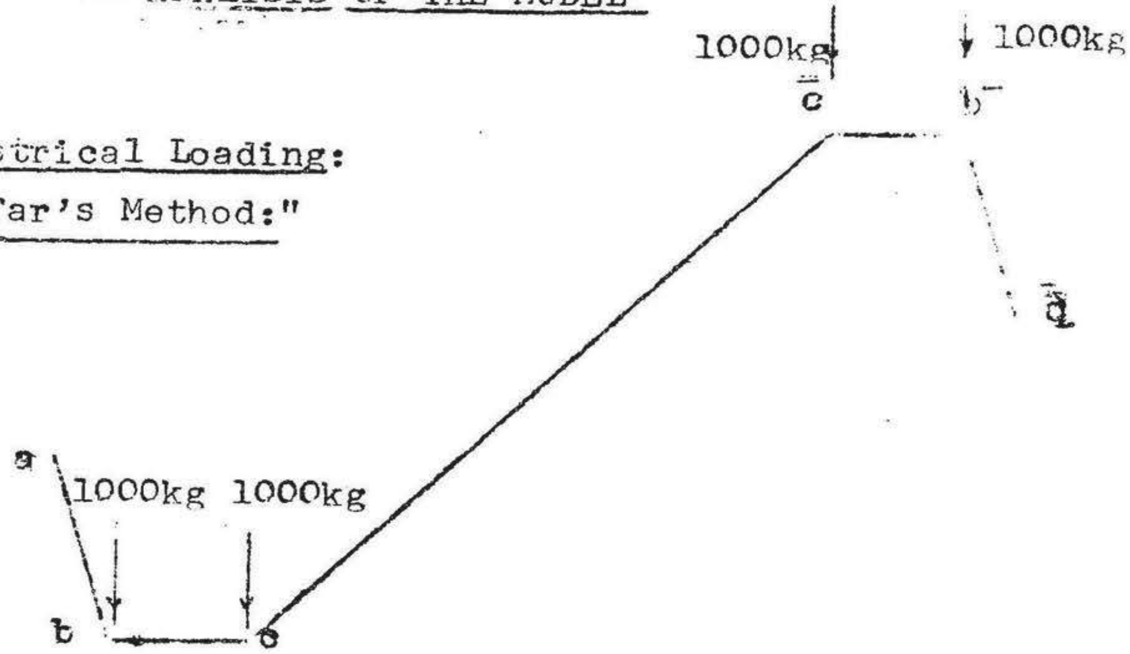
The Model & Strain Gages

Fig.(3-B)

2- ANALYSIS OF THE MODEL

a- Symmetrical Loading:

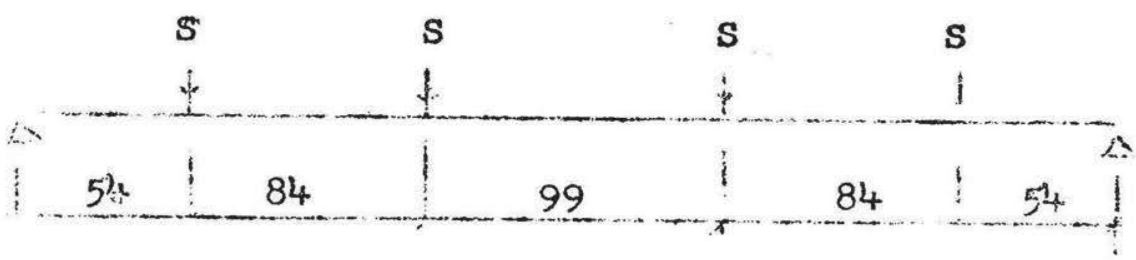
"Gaafar's Method:"



- Properties of the Different Plates

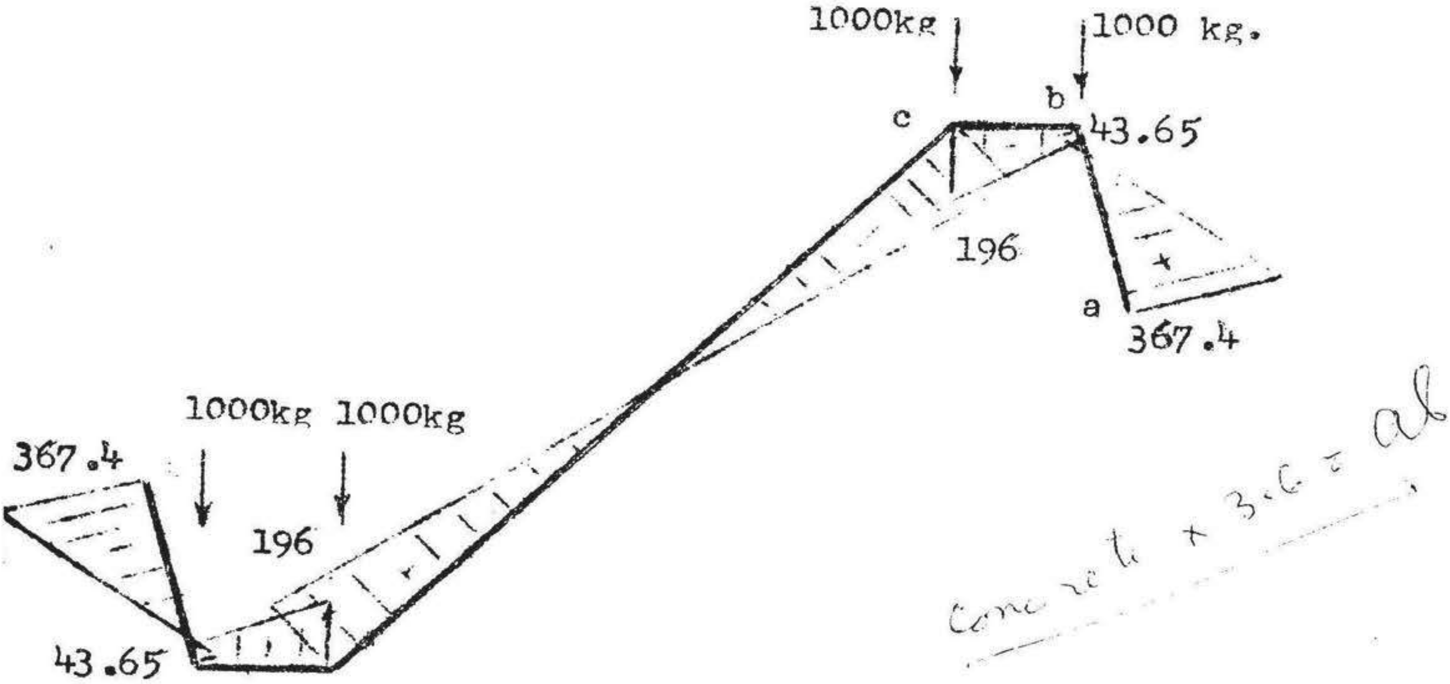
L = 375 cm. t = 3 cm.

Plate	Dim. cm $l \times t$	Area cm2	$Z = \frac{t \times l^2}{6}$ cm^3
AB, $\overline{AB}$	24 x 3	72	288
BC, $\overline{BC}$	15 x 3	45	112.5
C$\overline{C}$	112.5x3	337.5	6328



- Stresses

As the model dimensions as well as its loading is similar to the Aluminum model; the stresses can be concluded directly using the previous calculations.



Final Stresses

W = 16.0 tons

1000 Kg/loaded point

- Displacements

Elementary Correction

$$\delta_{ab} = \frac{374+51.3}{9.3 \times 24} \times \frac{375^2}{300/1000} - \frac{6.6+7.65}{24 \times \pi^2} \times \frac{375^2}{300/1000}$$

$= .9080 - 0.0287 \qquad = 0.8793$ cm

$$\delta_{bc} = \frac{193 - 51.3}{9.3 \times 15} \times \frac{375^2}{300/1000} + \frac{7.65 + 3.06}{15 \times \pi^2} \times \frac{375^2}{300/1000}$$

$= 0.4820 + 0.0343 \qquad = 0.5163$ cm

$$\delta_{cc} = \frac{193 \times 2}{9.3 \times 112.5} \times \frac{375^2}{300/1000} + \frac{3.06 \times 2}{\pi^2 \times 112.5} \times \frac{375^2}{300/1000}$$

$= 0.1760 + 0.0026 \qquad = 0.1786$ cm

$\Delta_{bv} = \delta_{ab}/\cos 15^\circ - \delta_{bc} \tan 15^\circ$

$= 0.912 - 0.137 \qquad = 0.7750$ cm

$\Delta_{cv} = \delta_{bc} \tan 45^\circ + \delta_{cc}/\tan 45^\circ$

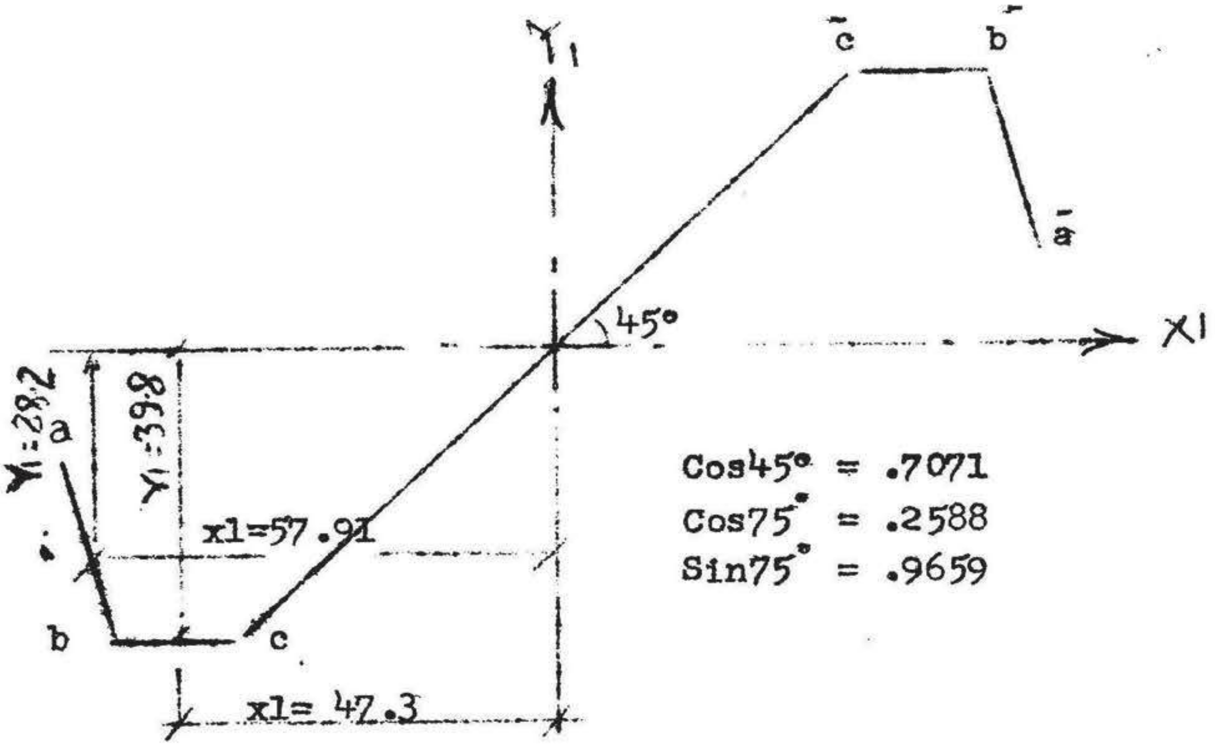
$= 0.5163 + 0.2525 \qquad = 0.7688$ cm

0.2525

Beam Analysis

t = 3 cm

Pl.	l cm.	$X = l_x$	$Y = l_y$	X_1	Y_1	A	$I_x = \frac{AY^2}{12}$	$I_y = \frac{AX^2}{12}$	$I_{xy} = \frac{AXY}{12}$	$A\bar{X}_1^2$	$A\bar{Y}_1^2$	AX_1Y_1
AB	24	6.22	23.2	57.91	28.2	72	3240	232	$\bar{8}66$	240980	57200	117500
BC	15	15	0	47.3	39.8	45	0	842	0	100660	71300	84700
$C\bar{C}$	112.5	79.6	79.6	0	0	337.5	177700	177700	177700	0	0	0
$\bar{C}\bar{B}$	15	15	0	47.3	39.8	45	0	842	0	100660	71300	84700
$\bar{B}\bar{A}$	24	6.22	23.2	57.91	28.2	72	3240	232	866	240980	57200	117500
Σ							184180	179850	175968	683280	257000	404400



$I_x = 184180 + 257000 = 441180 \; cm^4$

$I_y = 179850 + 683280 = 863130 \; cm^4$

$I_{xy} = 175968 + 404400 = 580368 \; cm^4$

$$\tan 2\theta = \frac{-2Ixy}{Ix - Iy} = \frac{-2 \times 580368}{-421950} = 2.7507$$

$\therefore\ 2\theta = 70^{\circ}$

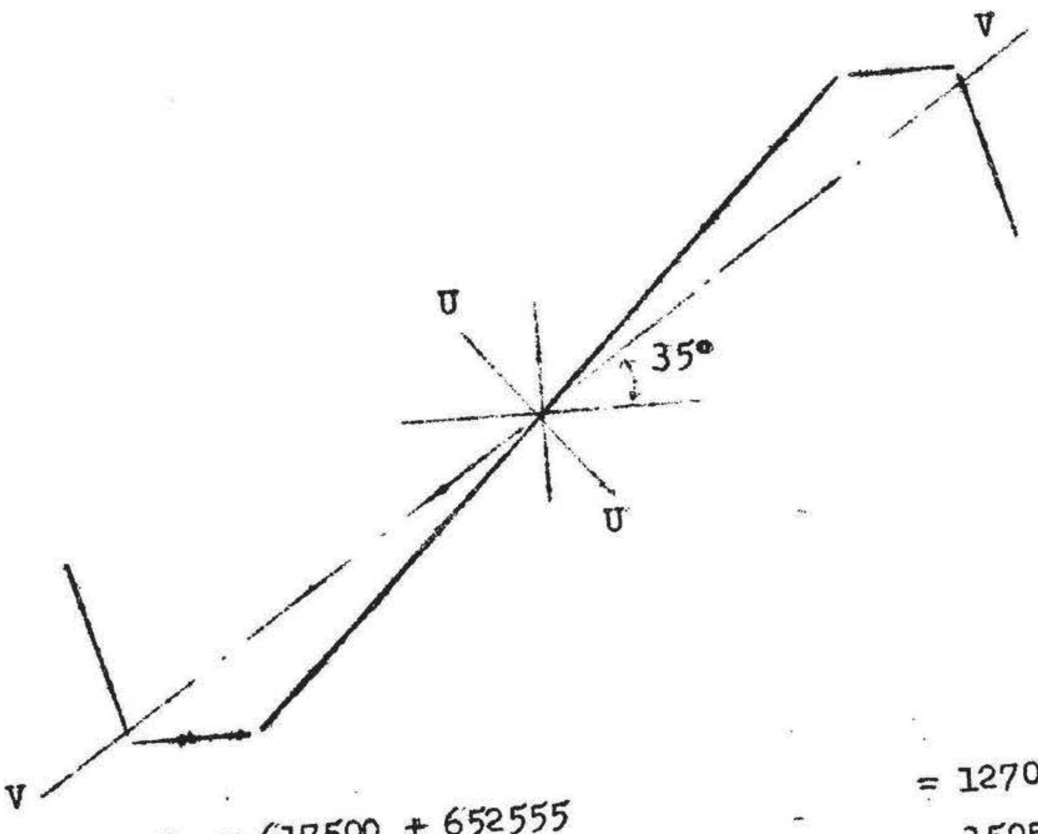
$\therefore\ \theta = 35^{\circ}$

$$\frac{Iy - Ix}{2} = 210970$$

$$\frac{Ix + Iy}{2} = = 652555$$

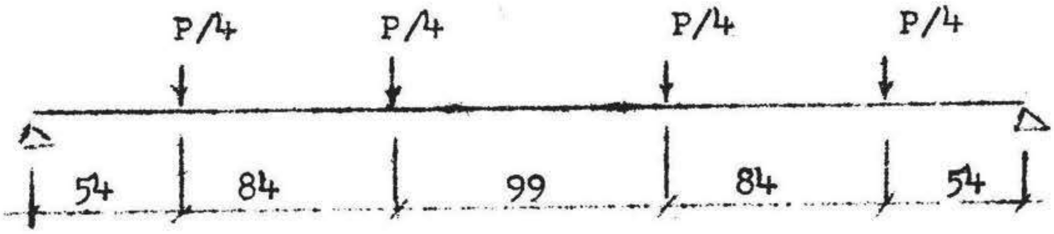
$$R = \left(\left(\frac{Ix - Iy}{2}\right)^2 + \overline{Ixy}^2\right)^{1/2} =$$

$$= (4.45 \times 10^{10} + 33.68 \times 10^{10})^{1/2} = 6.175 \times 10^5$$



$Iu = 617500 + 652555 = 1270055$

$Iv = 652555 - 617500 = 35055$



$$M = \frac{1}{2} P \times 138 - \frac{P}{4} \times 84 = 48P \text{ ton. cm}$$

$$M_u = M \sin 35^\circ = 48P \times .574 = 27.33P \text{ ton.cm.}$$

$$M_v = M \cos 35^\circ = 48P \times .819 = 39.44P \text{ ton.cm.}$$

$$\sigma = \frac{M_u}{I_v} .u + \frac{M_v}{I_u} .v$$

"P" is jack load

Point	X1	Y1	$v = \frac{a_1x_1+b_1y_1+c_1}{a_1^2+b_1^2}$	$u = \frac{a_1x_1+b_1y_1+c_1}{a_1^2+b_1^2}$
$\bar{A}$	61.02	16.6	21.75	59.40
$\bar{B}$	54.8	39.8	0.99	67.3
$\bar{C}$	39.8	39.8	9.73	55.4

For "P" = 16 tons

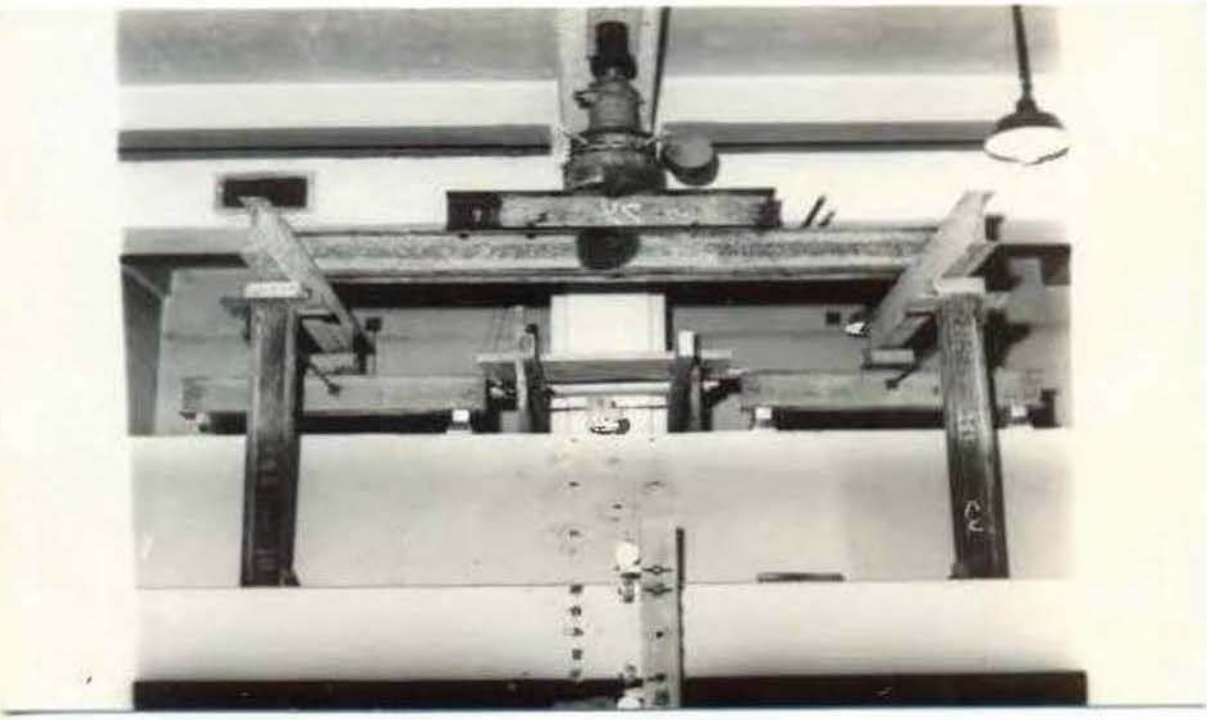
$$\sigma_{\bar{A}} = \frac{39440}{35055} \times 21.75 - \frac{27330}{1270055} \times 59.4 = 24.5 - 1.28 = 23.22P$$
= 371

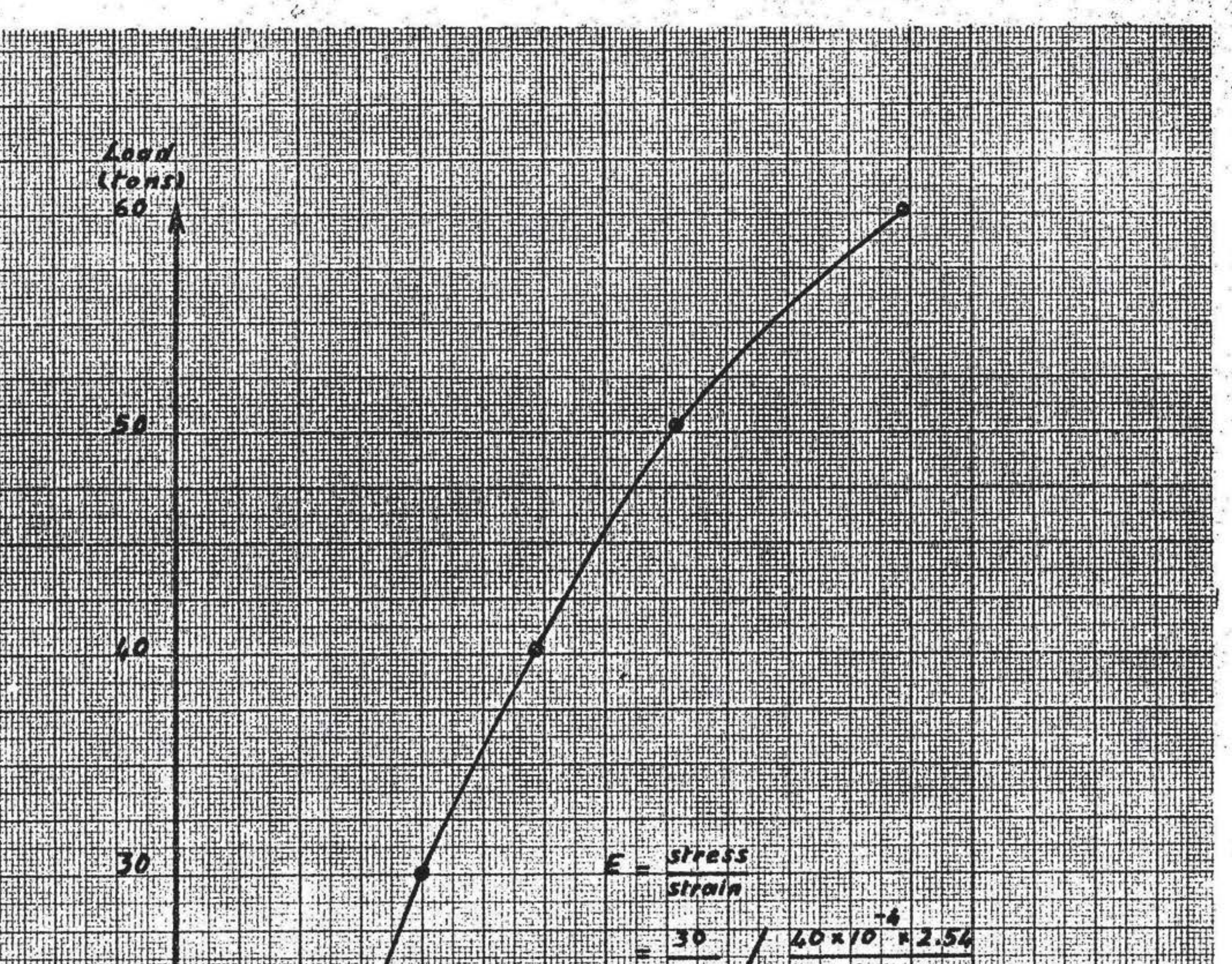
$$\sigma_{\bar{B}} = \frac{39440}{35055} \times 0.99 + \frac{27330}{1270055} \times 67.3 = 1.12 + 1.45 = +2.57P$$
= 41.2

$$\sigma_{\bar{C}} = \frac{39440}{35055} \times 9.73 + \frac{27330}{1270055} \times 55.4 = 10.95 + 1.19 = +12.14P$$
= 194.5

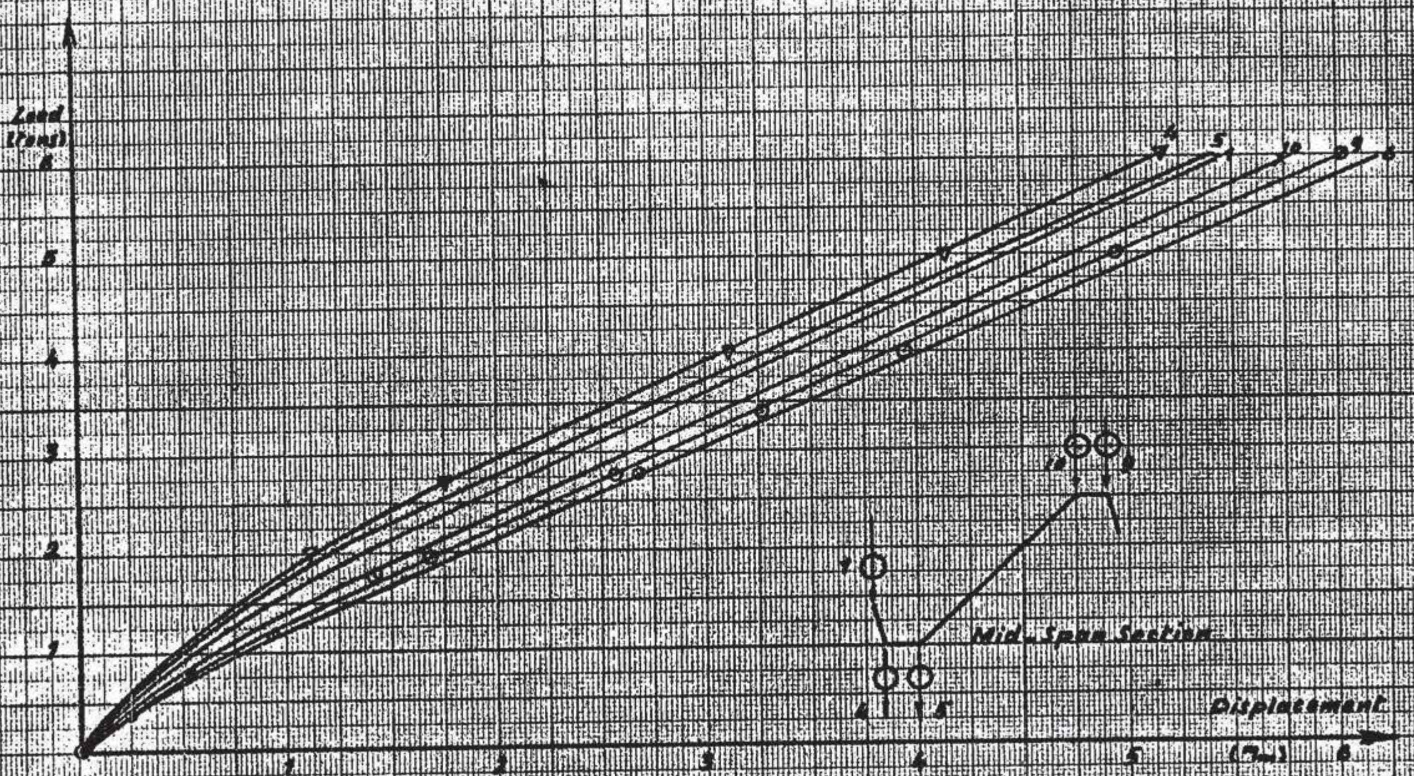
The same as that concluded using Gaafar's Method

- Experimental Results

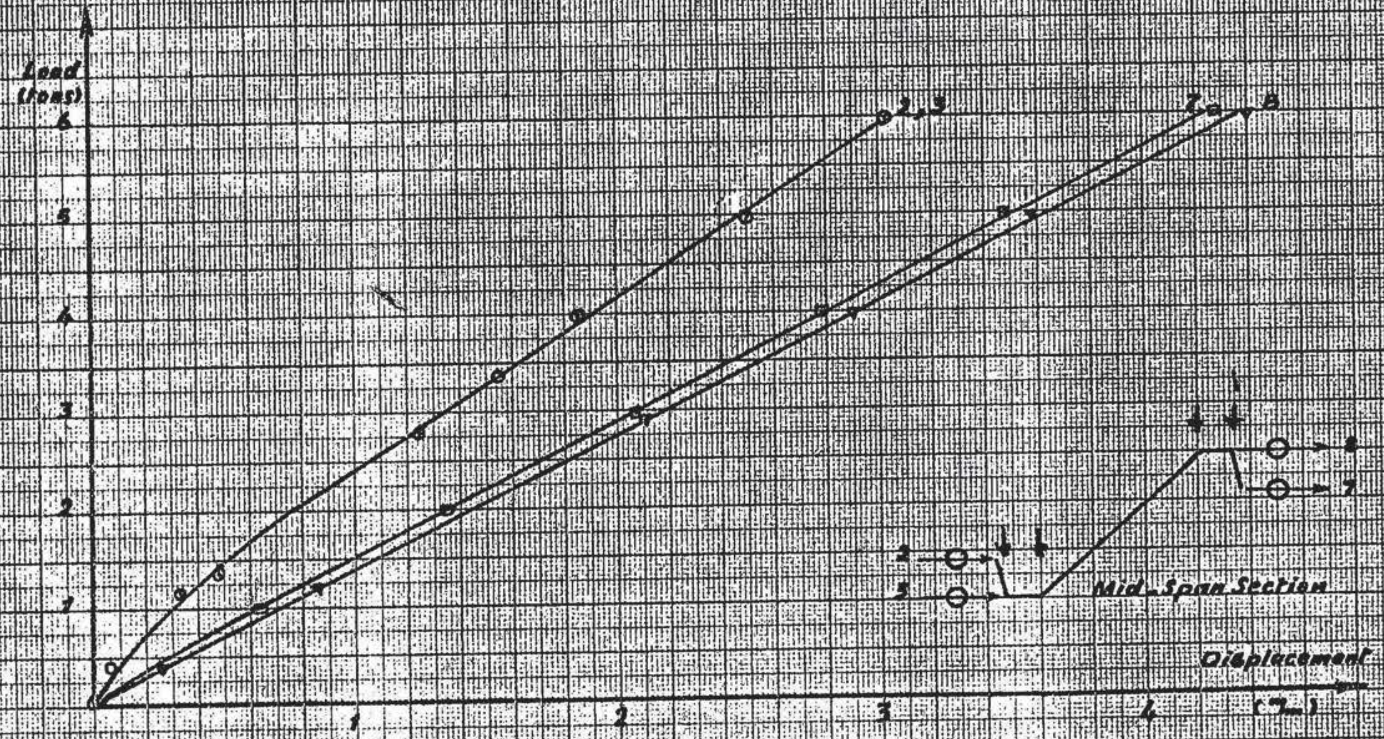




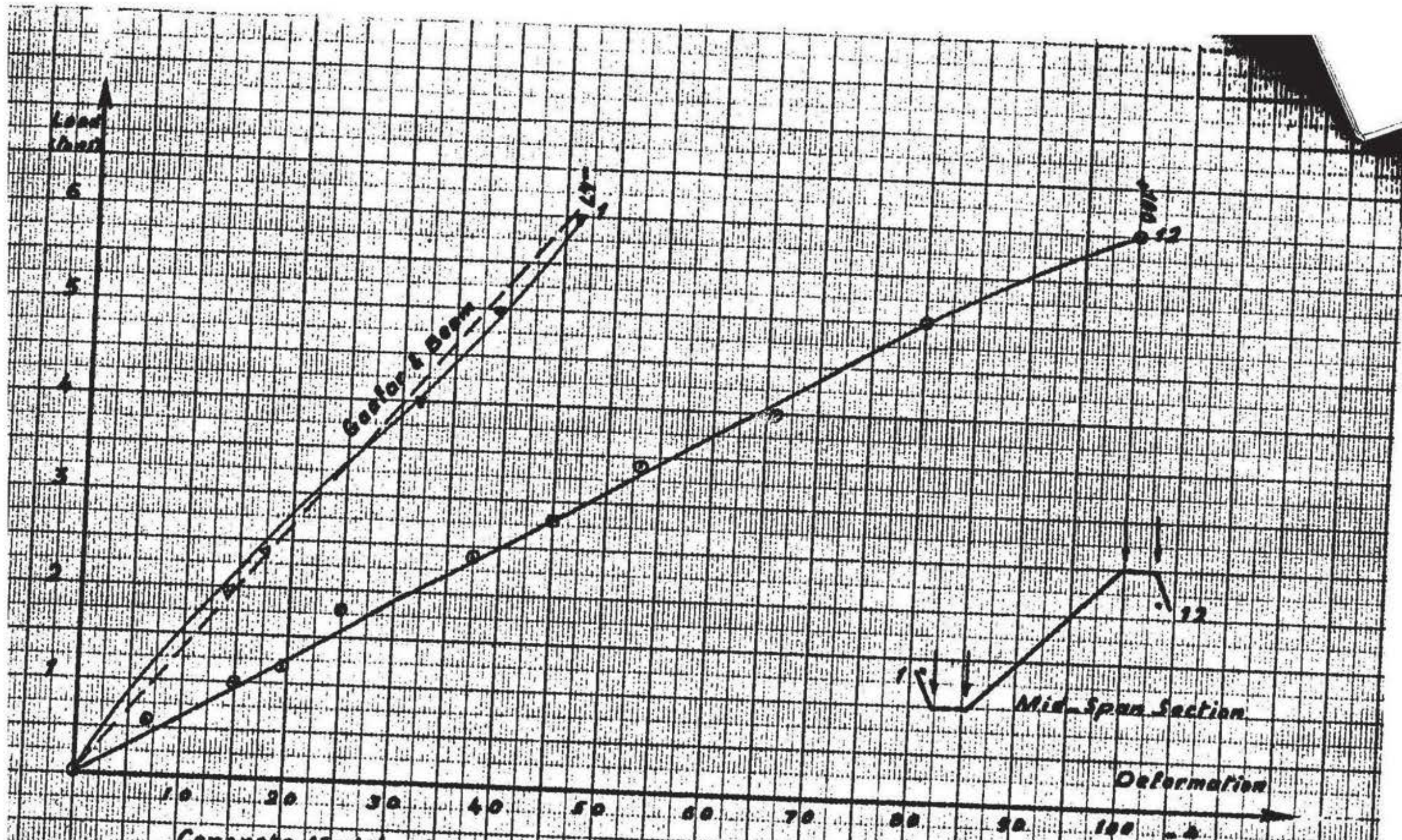
Load - Deformation Curve for P.C. Prism
15.8 x 15.8 x 47.4 Cm. G.L. = 25.4 Cm.



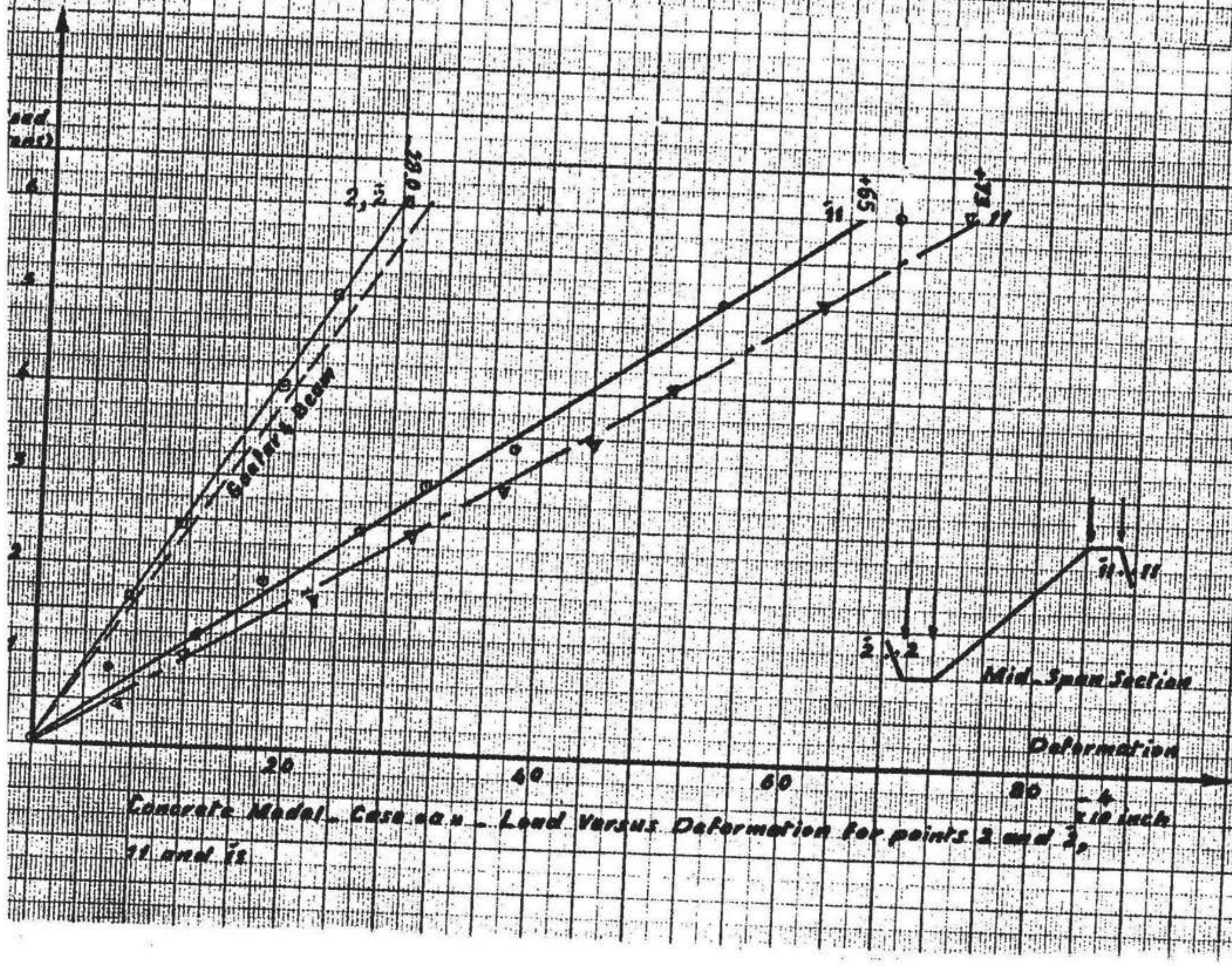
Concrete Model _ Case «a» _ Load Versus Displacement.



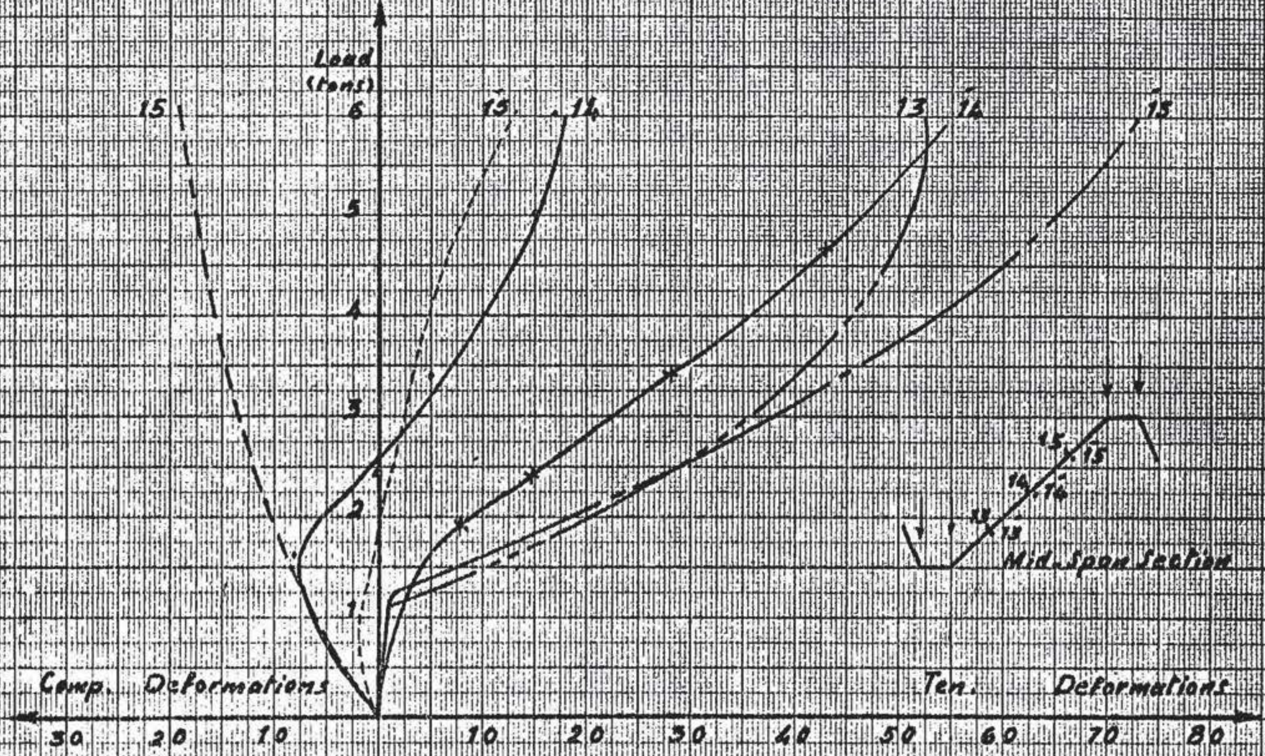
Concrete Model _ Case «a» _ Load Versus Displacement.



Concrete Model – Case «a» – Load Versus Deformation, for points 1, 12

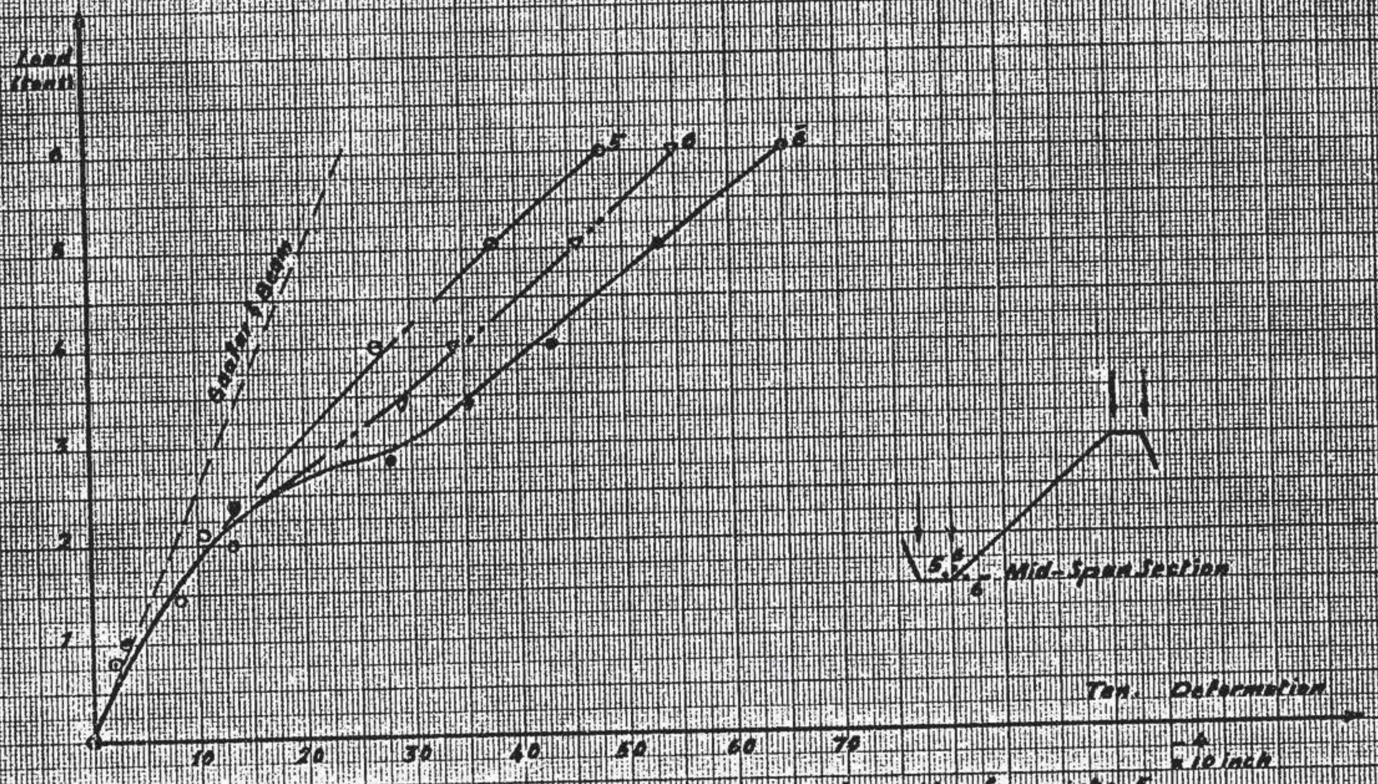


Concrete Model – Case «a» – Load Versus Deformation for points 2 and $\bar{2}$, 11 and $\bar{11}$

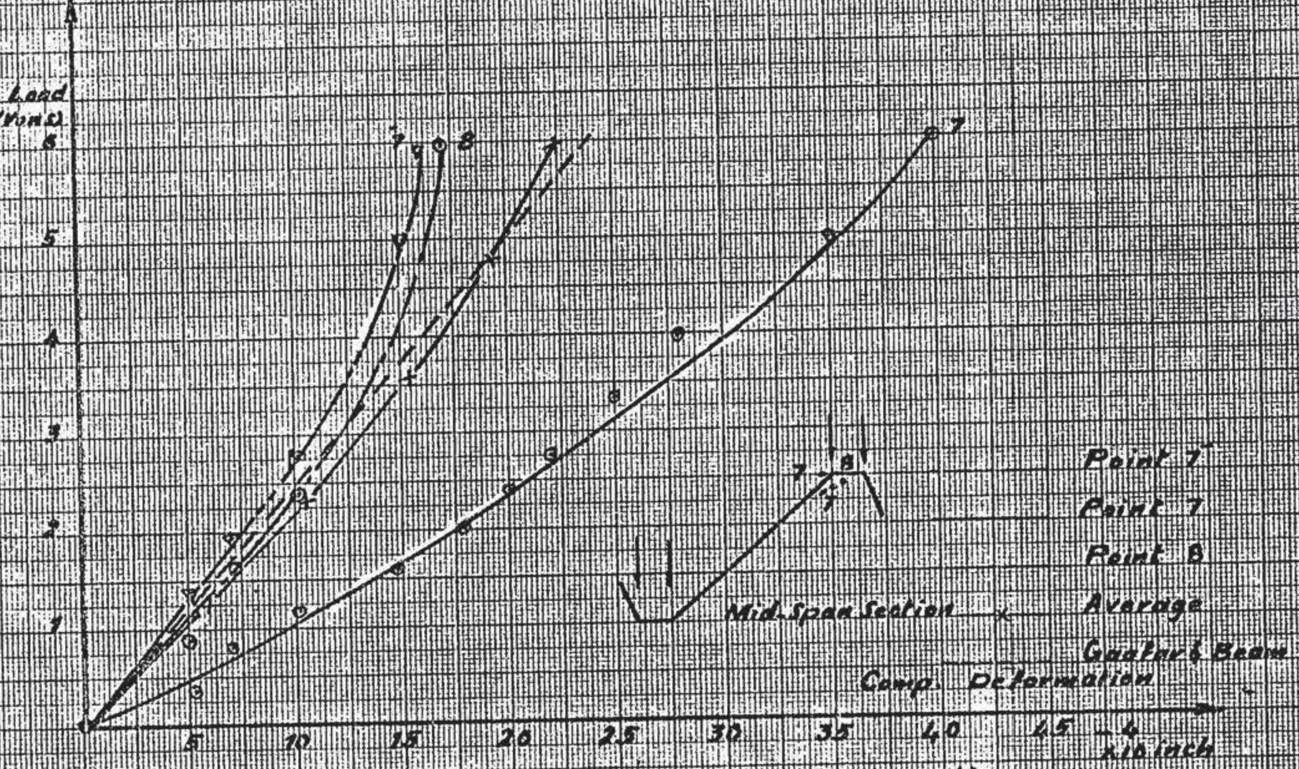


Concrete Model - Case «a» - Load Versus Deformation For points 13, 14, 15, 13̄, 14̄ and 15̄

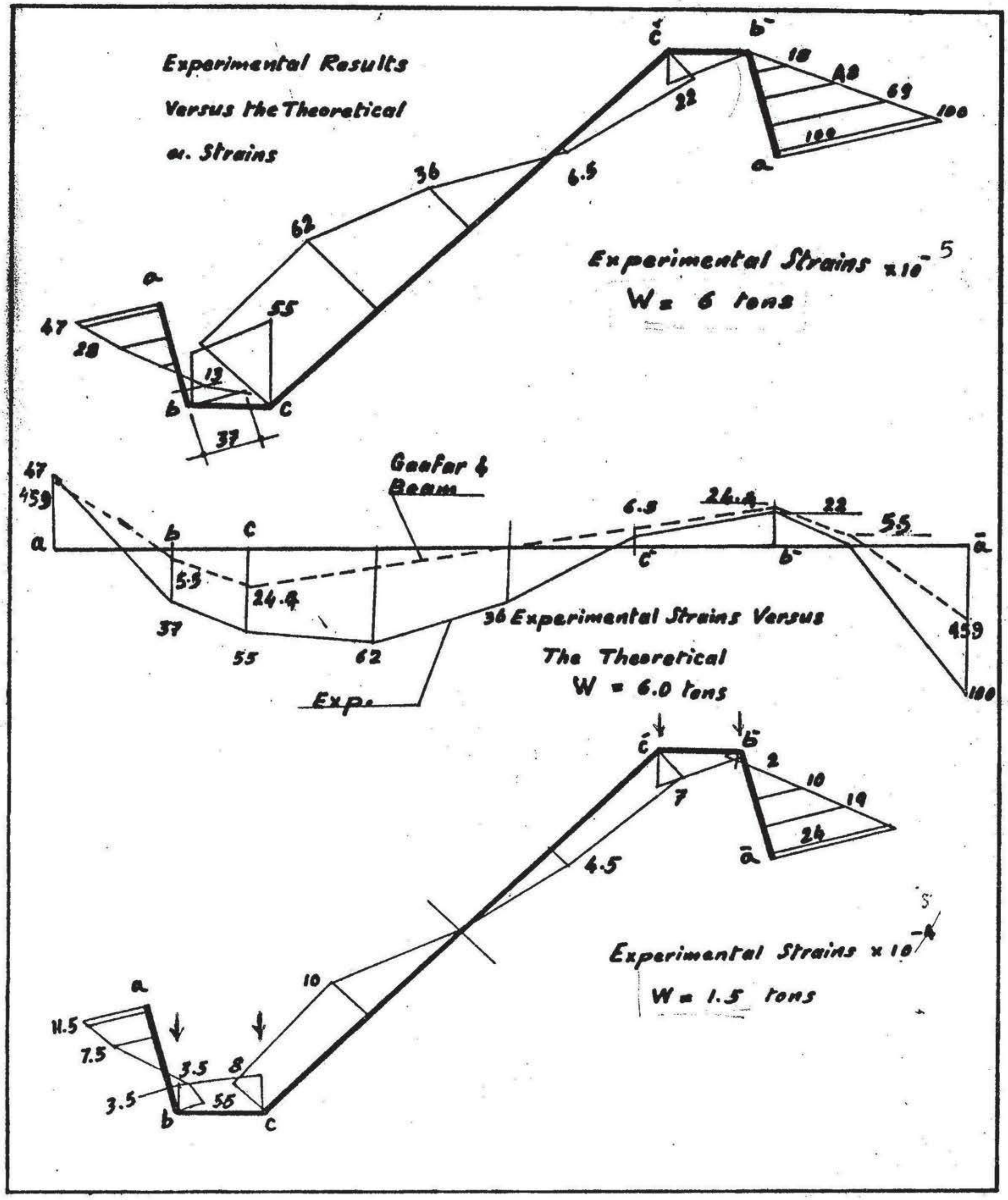
Concrete Model - Case «a» - Load Versus Deformation for points 3, 4 and 10



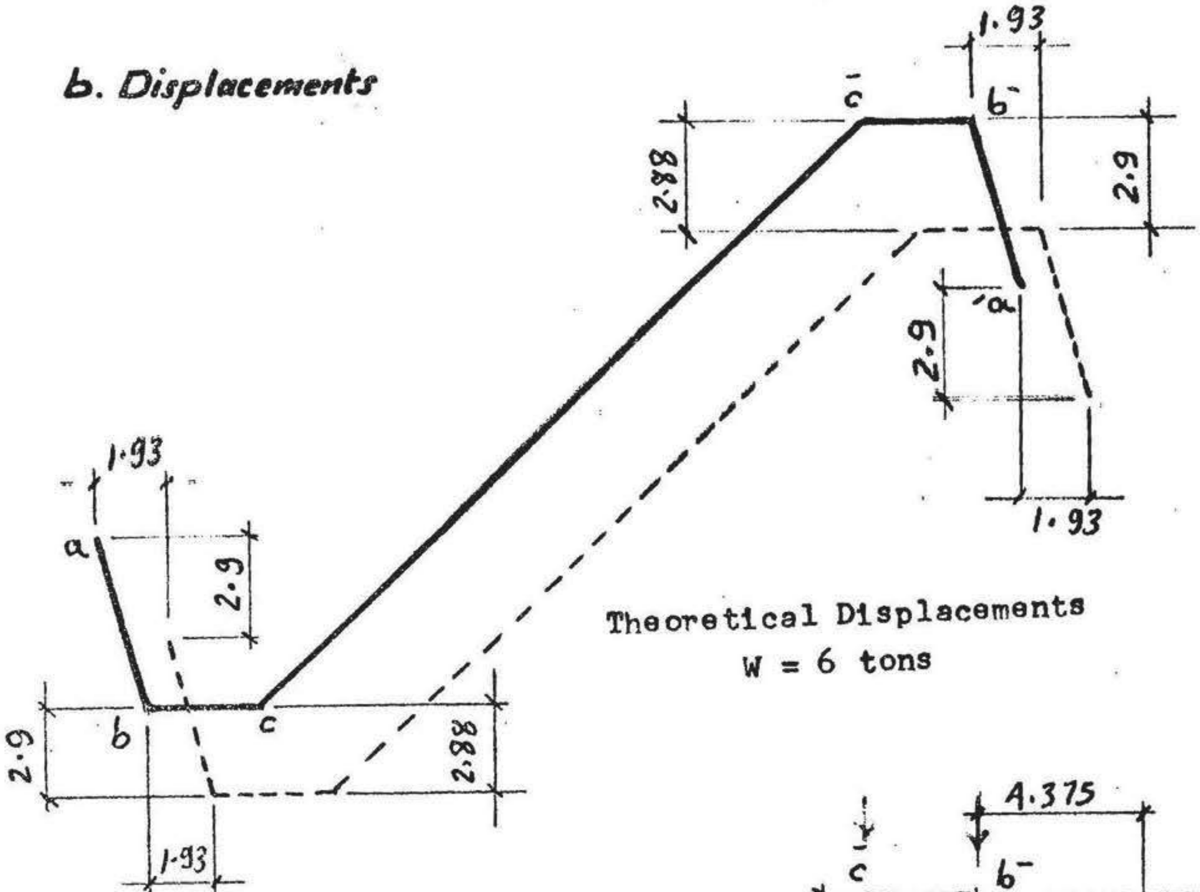
Concrete Model – Case «a» – Load Versus Deformation for points 5, 6 and 6'.



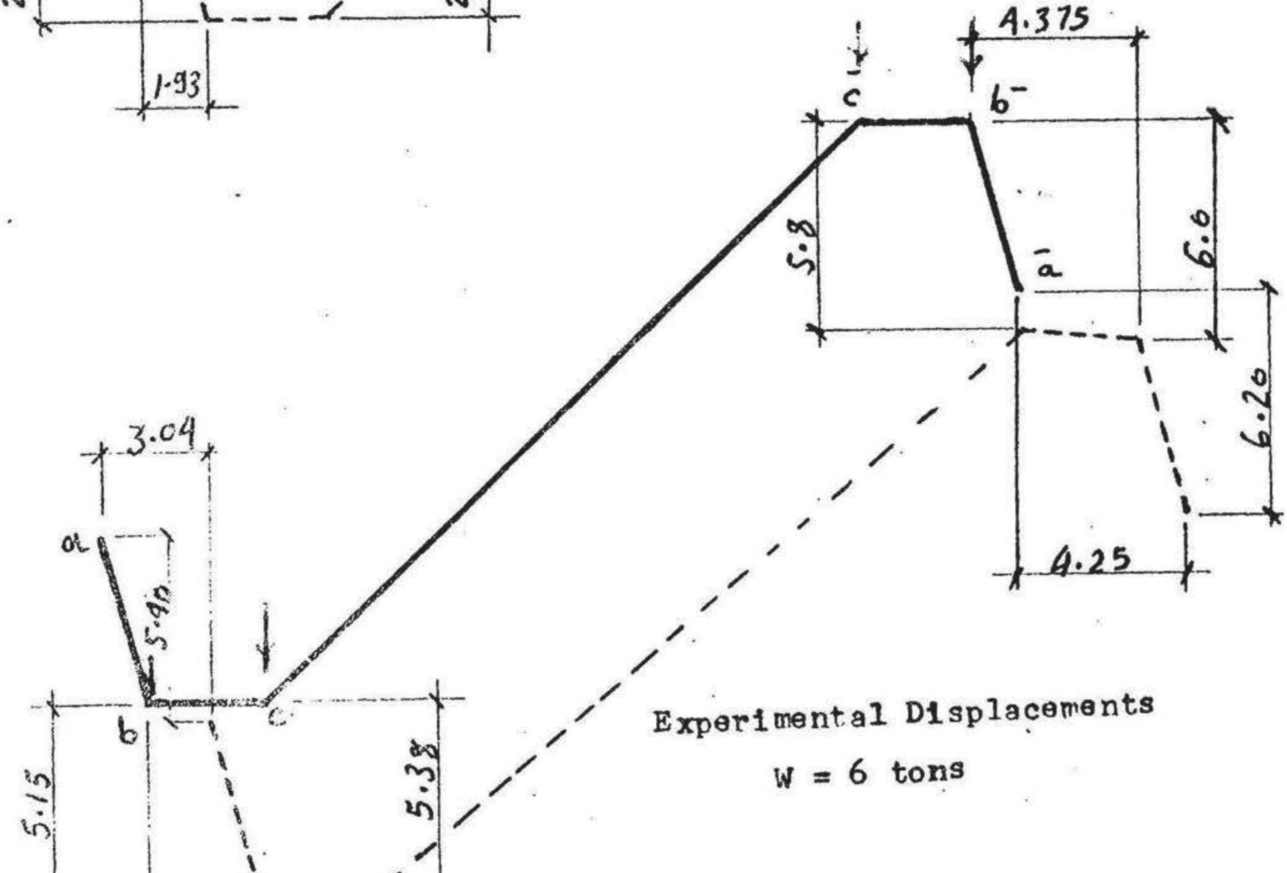
Concrete Model – Case «a» – Load Versus Deformation for points 7, 7 and 8.


Experimental Results
Versus the Theoretical
a. Strains
Experimental Strains x10⁻⁵
W = 6 tons
Gaafar & Beam
Exp.
Experimental Strains Versus
The Theoretical
W = 6.0 tons
Experimental Strains x10⁻⁵
W = 1.5 tons

b. Displacements



Theoretical Displacements
W = 6 tons

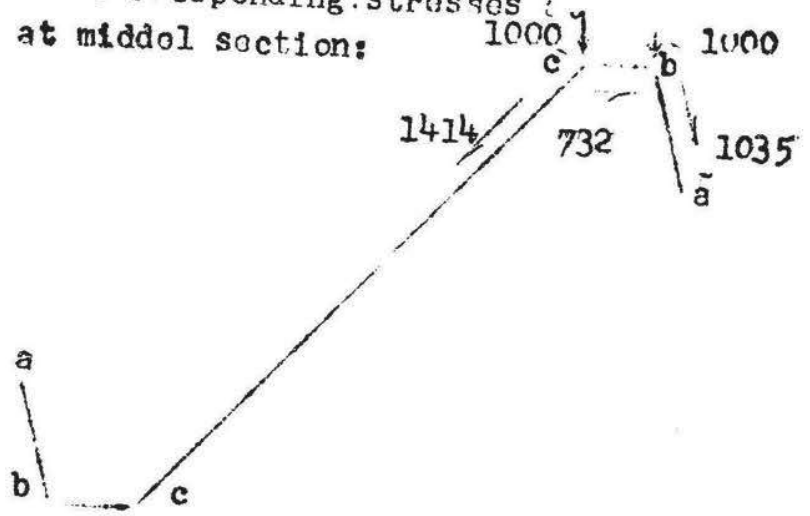


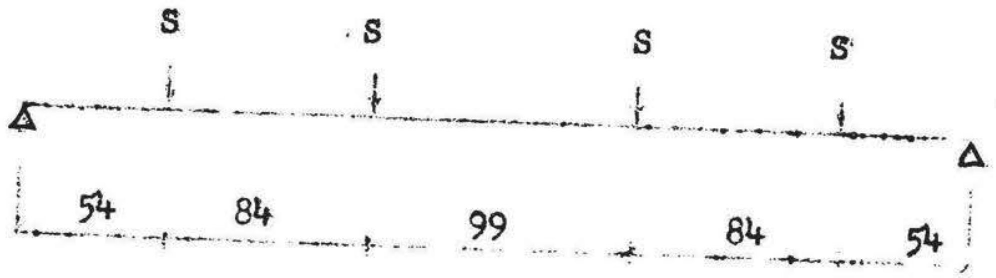
Experimental Displacements
W = 6 tons

b- Unsymmetrical Loading:

Loads on the upper edges(Gaafar's Method)

- Plate loads, bending moments and the corresponding stresses at middol section:



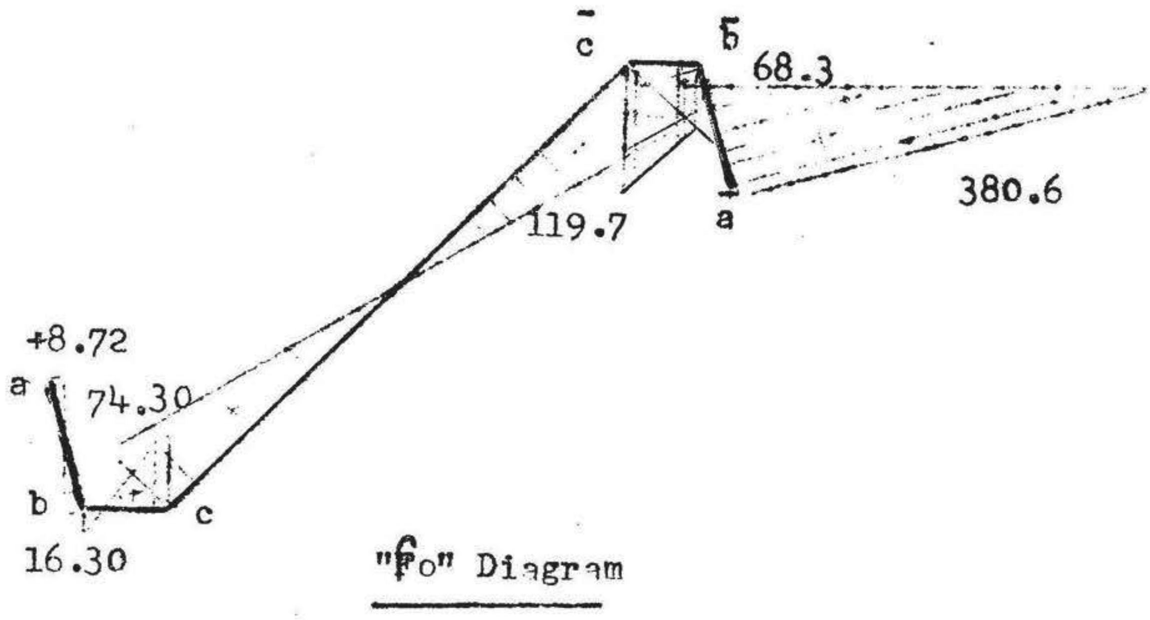


B.M = 2S x 138 - S x 84 = 192S

plate	loads	B.M = 192S	Z	$\sigma = \frac{M}{Z}$
$c\bar{c}$	1414	271488	6328	42.9
$\bar{C}\bar{B}$	732	140544	112.5	1250
$\bar{B}\bar{A}$	1035	198720	288	694

Stresses after distributions
neglecting the effect of joints displacement.

plate	a	b	c	$\bar{c}$	$\bar{b}$	$\bar{a}$
$\bar{A}\bar{B}$ loaded	+ 1.05	+ 2.8	− 22.2	+ 29.3	− 387	+ 540.9
$\bar{B}\bar{C}$ loaded	+ 5.8	− 11.4	+ 52.5	− 115	+ 311	− 157.7
$C\bar{C}$ loaded	+ 4.0	− 7.7	+ 34	− 34	+ 7.7	+ 4.0
Σ	+ 8.72	− 16.3	+ 74.3	− 119.7	− 68.3	+ 380.6

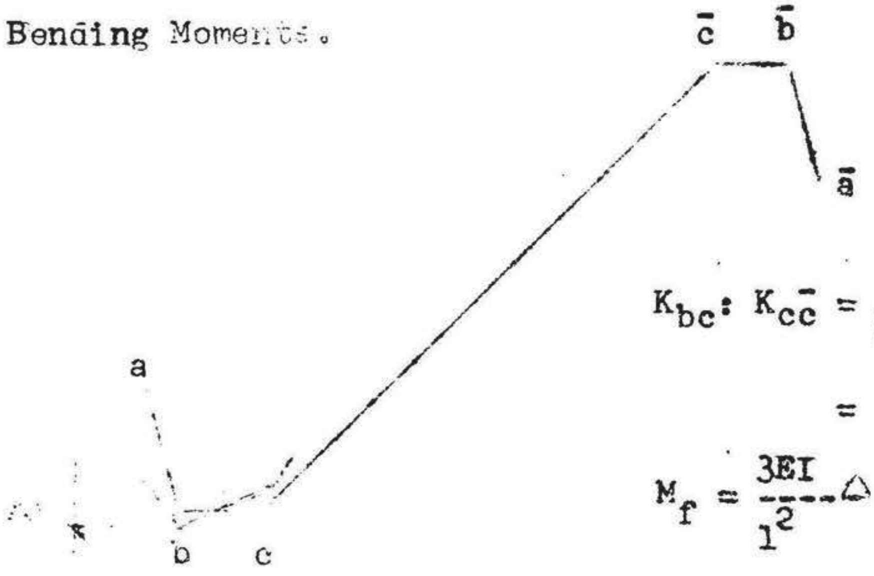


"Fo" Diagram

Joint Displacements:

- Effect of $\triangle 1$.

Bending Moments.



$$K_{bc} : K_{c\bar{c}} = \frac{3}{4} \times \frac{I}{15} : \frac{I}{112.5}$$

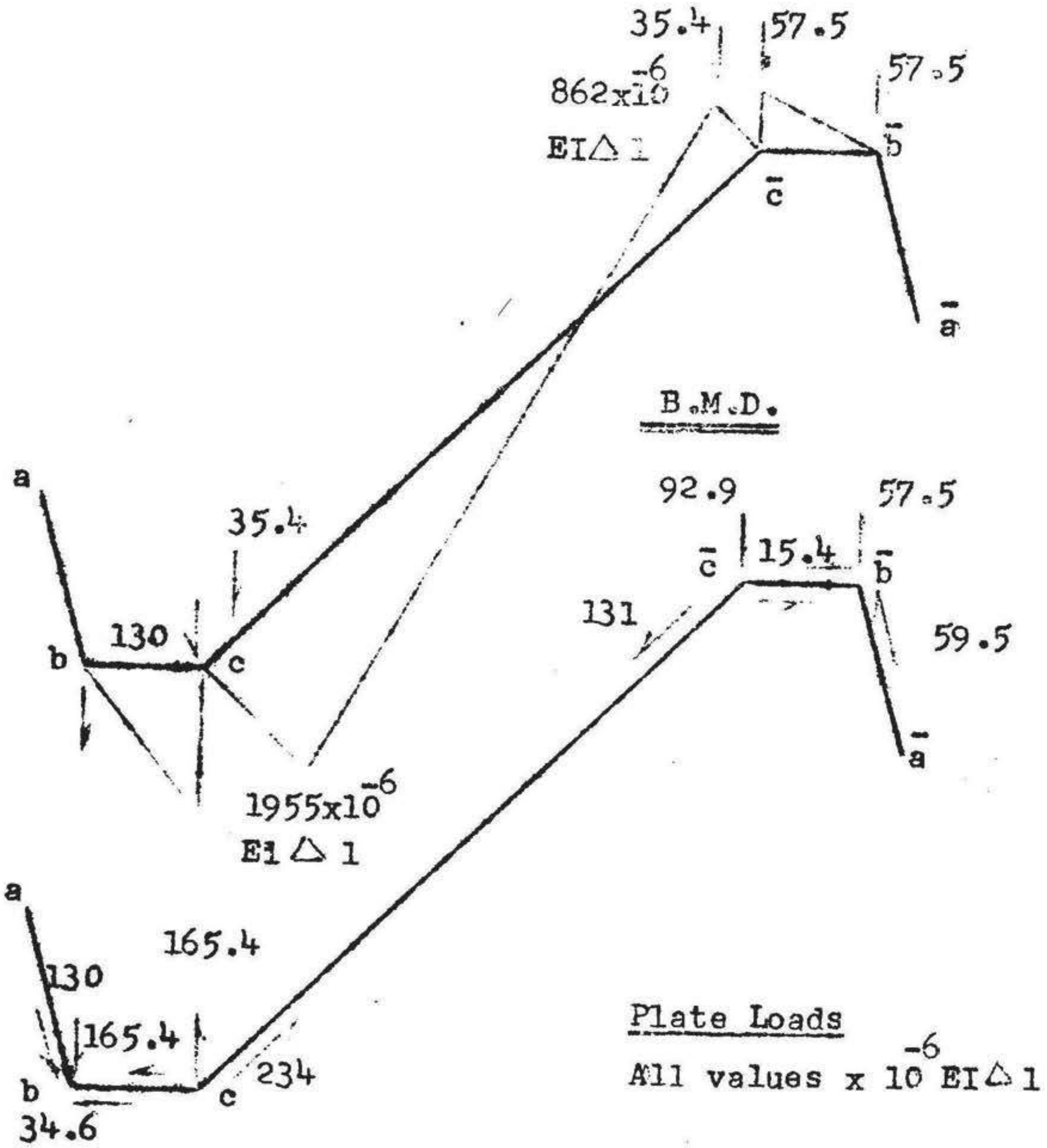
$$= .849 : .151$$

$$M_f = \frac{3EI}{l^2} \triangle 1 = \frac{3EI}{15 \times 15} \triangle 1$$

$$= 13.4 \times 10^{-3} EI \triangle 1$$

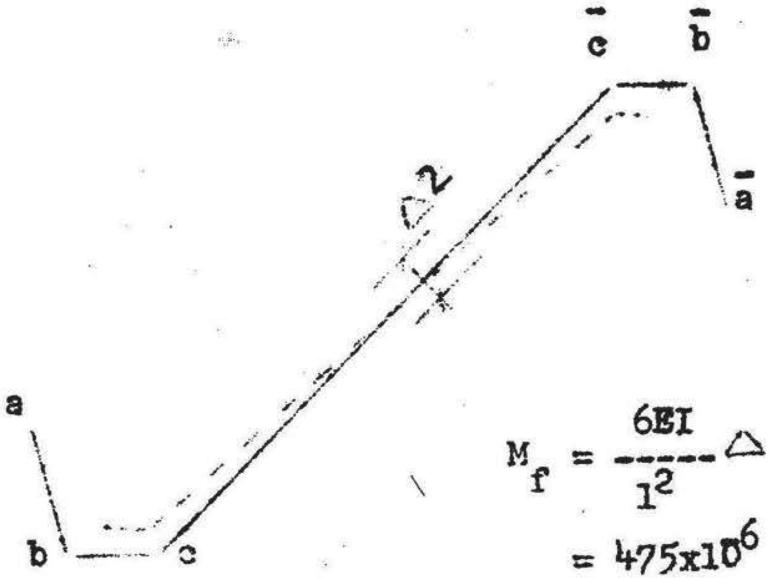
D.F.	.849	.151	.151	.849
F.E.M.	-13.4	0	0	0
Bal.	+11.38	+2.02		
C.O.			+1.01	
Bal.			-.153	-857
C.O.		-0.077		
Bal.	+.065	+.012		
C.O.			+.006	
Bal.			-.001	-.005
	-1.955	+1.955	+.862	.862

Plates Loads due to △1



Effect of $\Delta 2$

Bending Moments



$$M_f = \frac{6EI}{l^2}\Delta 2 = \frac{6EI}{112.5^2}\Delta 2$$

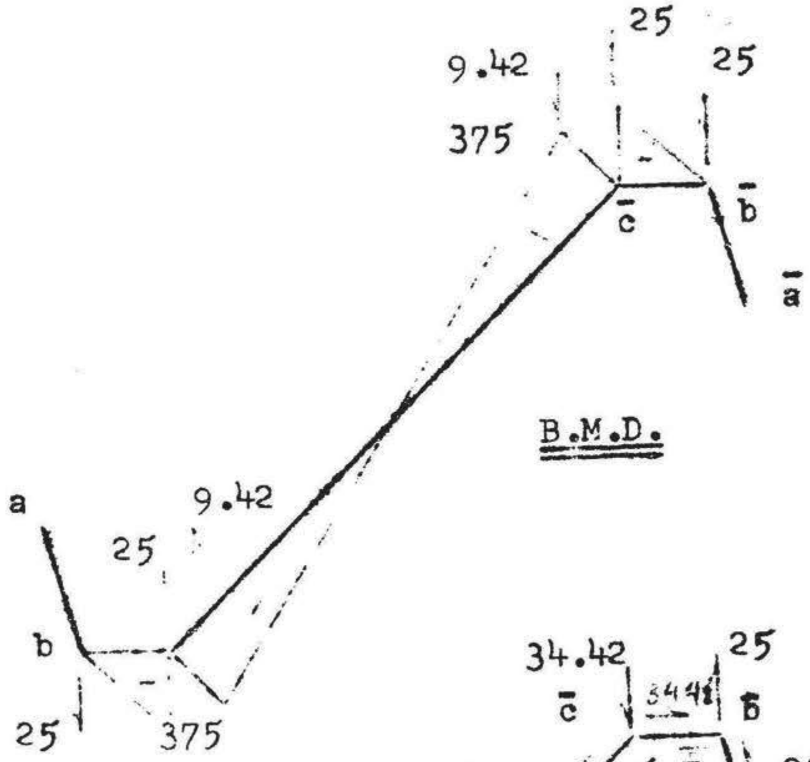
$$= 475\text{x}10^6\ EI\Delta 2$$

$$K_{bc}:K_{c\bar{c}} = \frac{3}{4}\text{x}\frac{I}{15} : \frac{3}{2}\ \frac{I}{112.5}$$

$$= .79 : .21$$

D.F.	.79	.21	.21	.79
F.E.M.		-475	-475	
Bal.	+375	+100	+100	+375
	+375	-375	+375	+375

Plate Loads due to $\triangle 2$



B.M.D.

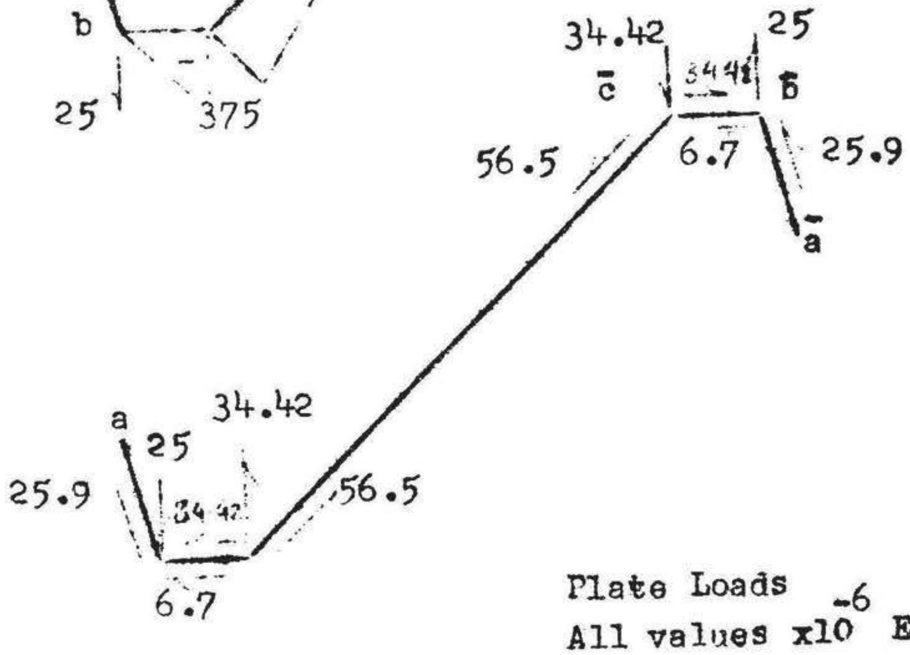
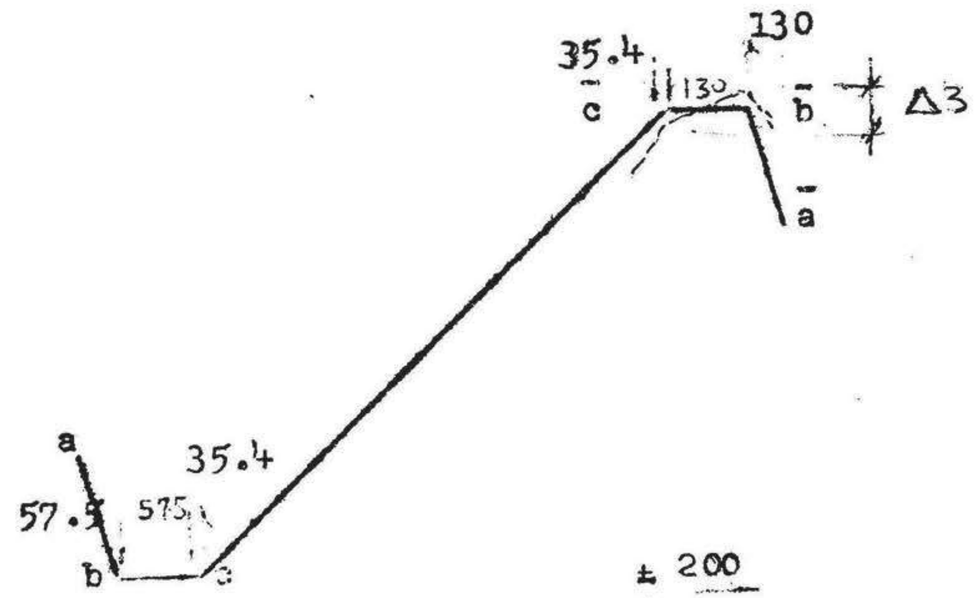


Plate Loads
All values $\times 10^{-6}$ EI $\triangle 2$

Plate Loads due to $\Delta 3$



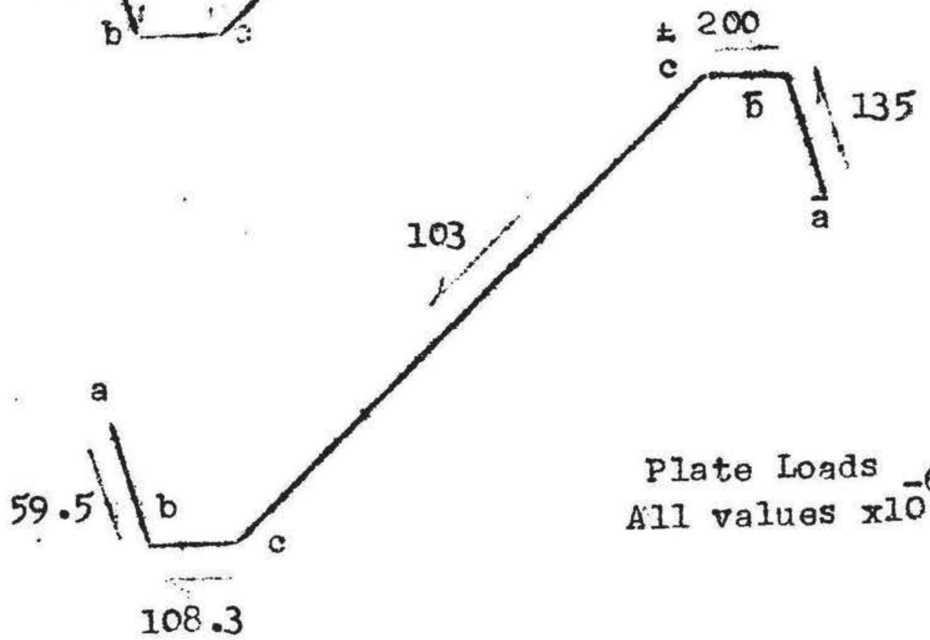
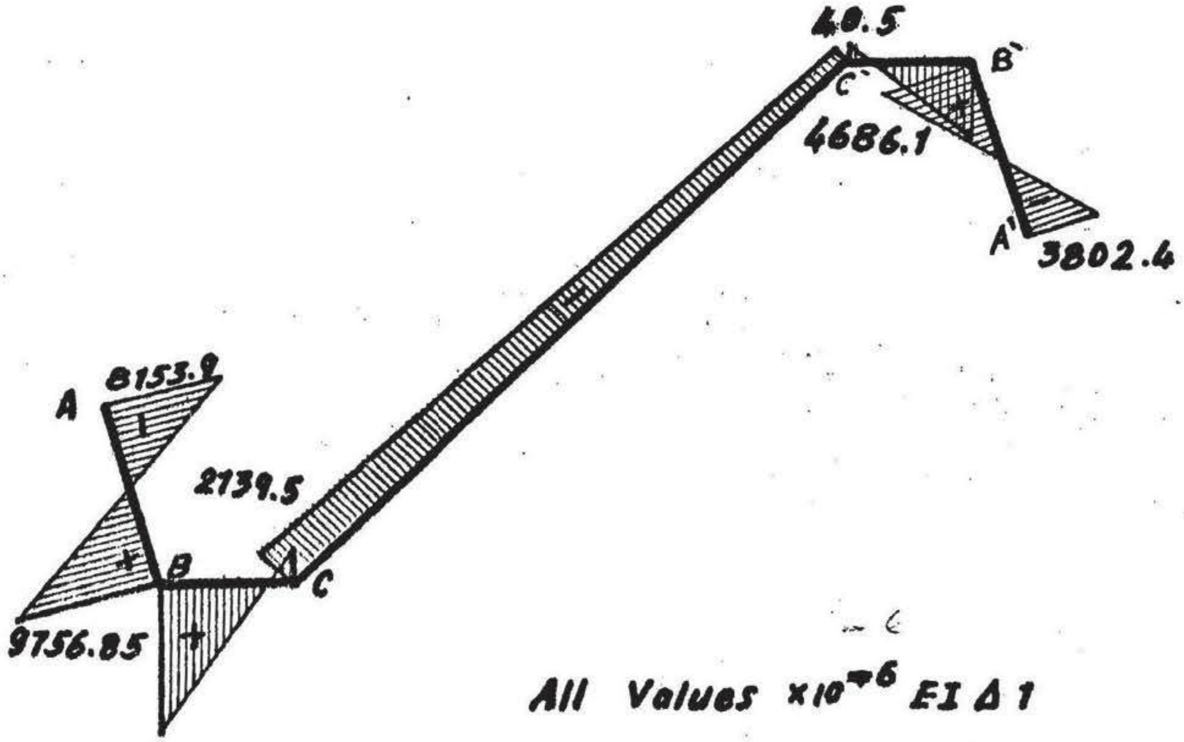


Plate Loads
All values $\times 10^{-6}$ EI$\Delta 3$

Stresses due to Δ1 :

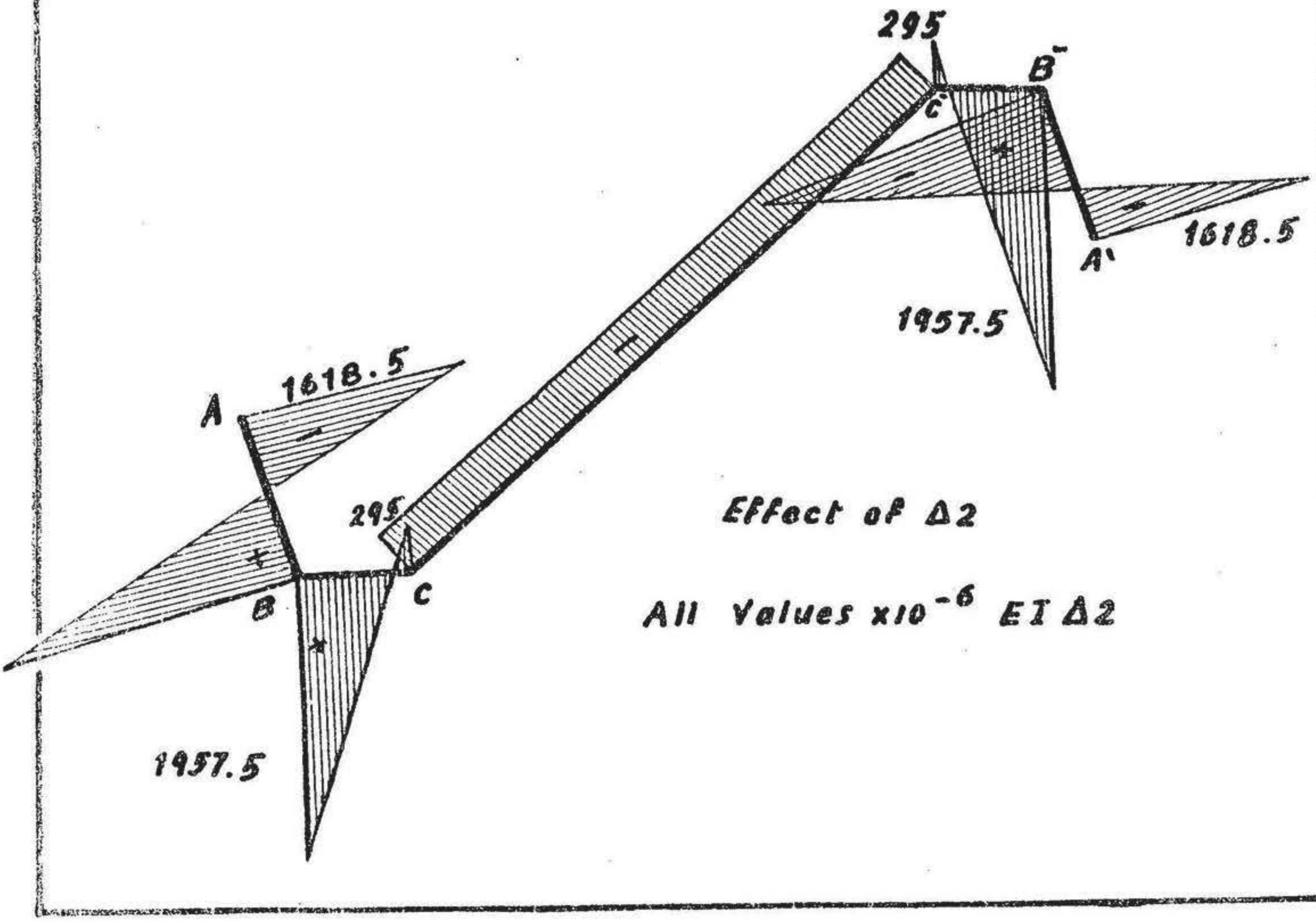
PL.	LD $\times 10^{-6}$	BM $\frac{pl^2}{12}$	$\sigma \times 10^{-6}$	$a \times 10^{-6}$	$b \times 10^{-6}$	$c \times 10^{-6}$	$c' \times 10^{-6}$	$b' \times 10^{-6}$	$a' \times 10^{-6}$
AB	135	1.900	6600	5100 (−)	3650 (+)	265 (−)	116 (+)	25.7 (−)	10.2 (+)
BC	200.	2.800	24900	3100 (−)	6200 (+)	2300 (−)	1020 (+)	227 (−)	116 (+)
CC̀	103	1.450	228	21.4 (−)	41.2 (+)	180 (−)	180 (+)	41.2 (−)	21.4 (+)
ĊḂ	1083	1.520	13500	63 (+)	123 (−)	555 (+)	1240 (−)	3370 (+)	1700 (−)
B̀À	59.5	.835	2900	4.5 (+)	11.35 (−)	50.5 (+)	116.5 (−)	1610 (+)	2250 (−)
Σ				8153.9 (−)	9756.85 (+)	2139.5 (−)	40.5 (−)	4686.1 (+)	3802.4 (−)



All Values $\times 10^{-6}$ EI Δ1

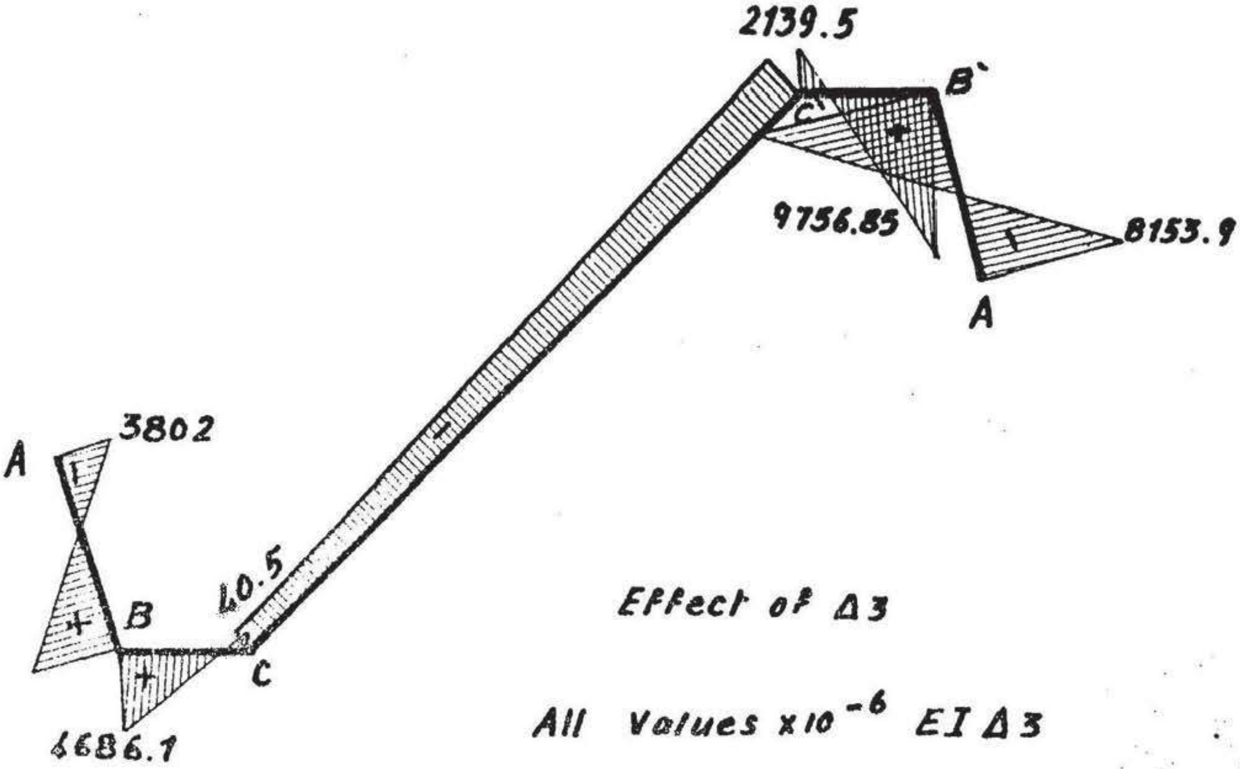
Stresses due to Δ2 :

PL.	LD $\times 10^{-6}$	B.M. 10^{-6}	σ $\times 10^{-6}$	a	b	c	$\bar{c}$	$\bar{b}$	$\bar{a}$
AB	25.91	.368	1280	−995	+710	−51.5	+22.5	−5.0	+2.0
BC	41.12 41d2	.585	5200	650−	1300+	480−	215+	47.5−	24.5+
C C̄	Zero	0	0	0	0	0	0	0	0
C̄ B̄	41.12	.585	5200	24.5+	47.5−	215+	480−	1300+	650−
B̄ Ā	25.91	.368	1280	2.0−	5.0−	22.5+	51.5−	710+	995−
Σ				1618.5−	1957.5+	295−	1157.5−	+1957.5	1618.5−



Stresses due to $\Delta 3$:

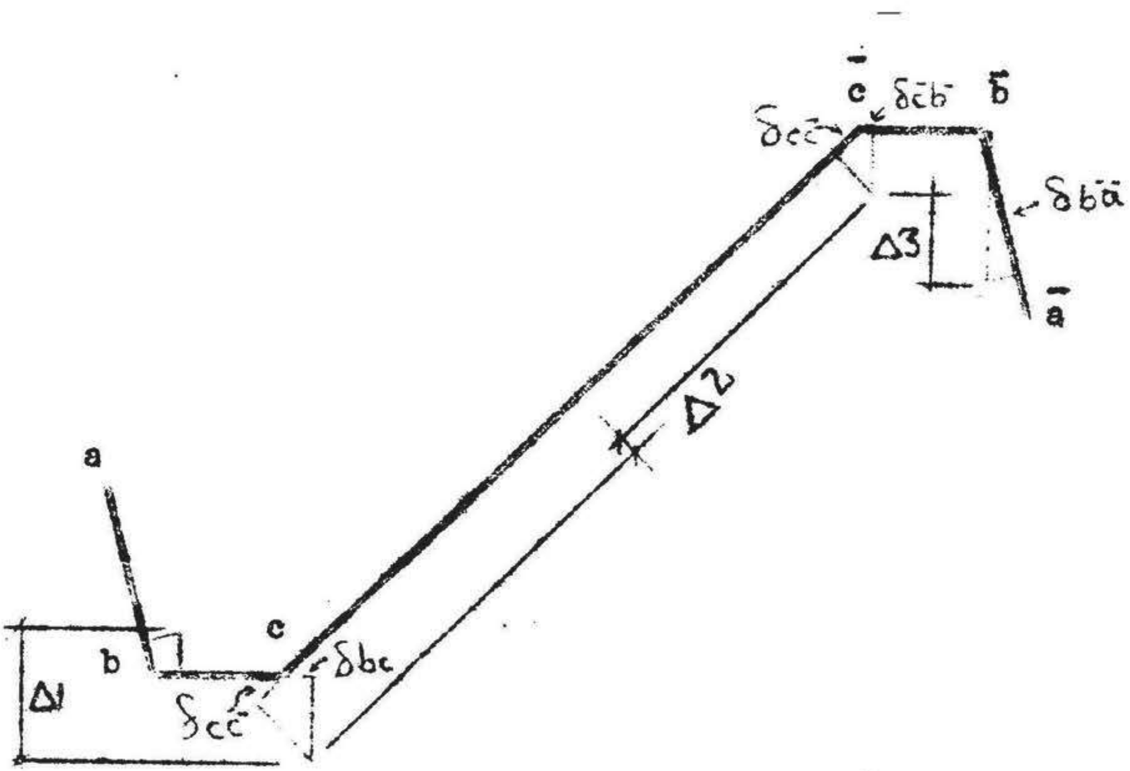
PL.	LD. $\times 10^{-6}$	B.M. $\times 10^{-3}$	$\sigma \times 10^{-3}$	$a \times 10^{-6}$	$b \times 10^{-6}$	$c \times 10^{-6}$	$\bar{c} \times 10^{-6}$	$\bar{b} \times 10^{-6}$	$\bar{a} \times 10^{-6}$
AB	59.5	83.5	2.9	− 2250	+ 1610	− 116.5	+ 50.5	− 11.35	+ 4.5
BC	108.3	1520	13.5	− 1700	+ 3370	− 1240	+ 555	− 123	+ 63
C $\bar{C}$	103	1450	0.228	+ 21.4	− 41.2	+ 180	− 180	+ 41.2	− 21.4
$\bar{C}\bar{B}$	200	2800	24.9	+ 116	− 227	+ 1020	− 2300	+ 6200	− 3100
$\bar{B}\bar{A}$	135	1900	6.6	+ 10.2	− 25.7	+ 116	− 265	+ 3650	− 5100
Σ				− 3802.4	+ 4686.1	− 40.5	− 2139.5	+ 9756.85	− 8153.9



Effect of $\Delta 3$

All Values $\times 10^{-6}$ EI $\Delta 3$

Geometrical Relations



$$\Delta_1 = \delta ab/\cos 75^\circ + \delta bc \tan 15^\circ + \delta c\bar{c}/\cos 45^\circ + \delta bc \tan 45^\circ$$

$$\therefore \Delta_1 = 1.035\ \delta ab + 1.268\ \delta bc + 1.414\ \delta c\bar{c}$$

$$\Delta_2 = (\delta bs/\cos 45^\circ + \delta c\bar{c} \tan 45^\circ) - (\delta \bar{c}\bar{b}/\cos 45^\circ + \delta c\bar{c} \tan 45^\circ)$$

$$\therefore \Delta_2 = 1.414\ \delta bc - 1.414\ \delta \bar{b}\bar{c}$$

$$\Delta_3 = (\delta \bar{b}\bar{a}/\cos 15^\circ - \delta \bar{b}\bar{c} \tan 15^\circ) - (\delta \bar{b}\bar{c} \tan 45^\circ + \delta c\bar{c}/\cos 45^\circ)$$

$$\therefore \Delta_3 = 1.035\ \delta \bar{b}\bar{a} - 1.268\ \delta \bar{b}\bar{c} - 1.414\ \delta c\bar{c} \qquad \text{(A)}$$

<u>The values of δ in terms of the relative displacements Δ:</u>

$$\delta ab = \frac{(8.72+16.3)}{24x9.3} x \frac{l^2}{E} - 10^{-6} \left(\frac{8153.9+9756.85}{\pi^2 x 24} x l^2 \right.$$

$$\left. + \frac{1618.6+1957.5}{\pi^2 x\ 24} + \frac{4686.1+3802}{\pi^2 x\ 24} \right)$$

$$\therefore \delta ab = \frac{25.02}{24\ x9.3} \frac{l^2}{E} - 10^{-6} \left(\frac{17710.75}{\pi^2 x24} L^2 \Delta 1 + \right.$$

$$\left. + \frac{3576}{\pi^2 x\ 24} L^2 \Delta 2 + \frac{8488.1}{\pi^2 x\ 24} L^2 \Delta 3 \right) xI$$

$$\therefore \delta ab = 0.113 \frac{L^2}{E} - 10^{-6} l^2 (75 \Delta 1 + 15.10 \Delta 2 + 35.80 \Delta 3)\ I$$

$$\delta bc = \frac{(74.3+16.3)}{15\ x\ 9.3} \frac{l^2}{E} - 10^{-6} \left(\frac{2139.5+9756.85}{\pi^2 x\ 15} L^2 \Delta 1 \right.$$

$$\left. + \frac{1957.5\ +295.0}{\pi^2\ x\ 15} l^2 \Delta 2 + \frac{40.5+4686.1}{\pi^2 x\ 15} l^2 \Delta 3 \right)$$

$$\therefore \delta bc = \frac{90.6}{15\ x9.3} \frac{l^2}{E} - 10^{-6} \left(\frac{11896.35}{\pi^2 x15} L^2 \Delta 1 \right.$$

$$\left. + \frac{2252.5}{\pi^2 x\ 15} L^2 \Delta 2 + \frac{4726.6}{\pi^2 x\ 15} L^2 \Delta 3 \right)$$

$$\therefore \delta bc = .648 \frac{L^2}{E} - 10^{-6} L^2 (80.5 \Delta 1 + 15.2 \Delta 2 + 31.80 \Delta 3)\ I$$

$$\delta c\bar{c} = \frac{74.3+119.7}{112.5x9.3} \frac{L^2}{E} + 10^{-6} L^2 \left(\frac{-2139.5-40.5}{112.5\ x\ \pi^2} \Delta 1 \right.$$

$$\left. + \frac{2139.5-40.5}{112.5\ x\ \pi^2} \Delta 3 \right)$$

$$\therefore \delta_{\bar{c}\bar{c}} = \frac{194}{112.5 \times 9.3} \frac{L^2}{E} + 10^{-6} L^2 \left(- \frac{2099}{112.5} \Delta 1 \; \frac{2099}{112.5} \Delta 3\right)$$

$$\therefore \delta_{\bar{c}\bar{c}} = 0.184 \frac{L^2}{E} + 10^{-6} L^2 (-1.88 \Delta 1 + 1.88 \Delta 3) \; I$$

$$\delta_{\bar{c}\bar{b}} = \frac{51.4}{15 \times 9.3} \frac{L^2}{E} + 10^{-6} \left(\frac{4726.6}{\pi^2 \times 15} \Delta 1 L^2 \right.$$

$$\left. + \frac{225.5}{\pi^2 \times 15} \Delta 2 L^2 \pm \frac{11896.35}{\pi^2 \times 15} L^2 \Delta 3 \right)$$

$$\therefore \delta_{\bar{c}\bar{b}} = 0.368 \frac{L2}{E} + 10^{-6} L^2 (+31.8 \Delta 1 + 15.20 \Delta 2 + 80.5 \Delta 3) I$$

$$\delta_{\bar{b}\bar{a}} = \frac{448.9}{24 \times 9.3} \frac{L^2}{E} + 10^{-6} \left(- \frac{8488.1}{\pi^2 \times 24} L^2 \Delta 1 + \frac{3576.0}{\pi^2 \times 24} L^2 \Delta 2 \right.$$

$$\left. - \frac{17710.75}{\pi^2 \times 24} L^2 \Delta 3 \right)$$

$$\therefore \delta_{\bar{b}\bar{a}} = 2.02 \frac{L^2}{E} - 10^{-6} L^2 (35.80 \Delta 1 + 15.10 \Delta 2 + 75 \Delta 3) I$$

(B)

<u>Substituting the values of δ in the Geometrical Relations:</u>

From A & B

$$\Delta 1 = 0.117 \frac{L^2}{E} - 10^{-6} L^2 (77.9 \Delta 1 + 15.65 \Delta 2 + 37 \Delta 3) \; I$$

$$+ .815 \frac{L2}{E} - 10^{-6} L^2 (101.5 \Delta 1 + 19.30 \Delta 2 + 40 \Delta 3) \times I$$

$$+ 0.26 \frac{L^2}{E} + 10^{-6} L^2 (-2.66 \Delta 1 + 0.0 \Delta + 2.66 \Delta 3) \; \times I$$

$$\text{i.e.} \; \Delta 1 = 1.192 \frac{L^2}{E} - 10^{-6} L^2 (182.06 \Delta 1 + 34.95 \Delta 2 + 74.34 \Delta 3) \times I$$

$$\therefore 1.192 \frac{L^2}{E} - 58.5\Delta 1 - 11.0\Delta 2 - 23.5\Delta 3 = 0 \qquad (1)$$

$$\Delta 2 = .917 \frac{L^2}{E} - 10^{-6} L^2 (113.2\Delta 1 + 21.5\Delta 2 + 45\Delta 3) xI$$

$$-.520 \frac{L^2}{E} - 10^{-6} L^2 (45\Delta 1 + 21.5\Delta 2 + 113.2\Delta 3) xI$$

$$\text{i.e.} \Delta 2 = .397 \frac{L^2}{E} - 10^{-6} L^2 (158.2\Delta 1 + 43.0\Delta 2 + 158.2\Delta 3) xI$$

$$\therefore 0.397 \frac{L^2}{E} - 50.\Delta 1 - 14.60\Delta 2 - 50.\Delta 3 = 0 \qquad (2)$$

$$\Delta 3 = 2.095 \frac{L^2}{E} - 10^{-6} L^2 (37\Delta 1 + 15.65\Delta 2 + 77.9\Delta 3) xI$$

$$-0.467 \frac{L^2}{E} - 10^{-6} \frac{L^2}{E} (+40\Delta 1 + 19.30\Delta 2 + 101.5\Delta 3) xI$$

$$-0.26 \frac{L^2}{E} + 10^{-6} L^2 (2.66\Delta 1 + 0.0 - 2.66\Delta 3) xI$$

$$\text{i.e.} \Delta 3 = 1.368 \frac{L^2}{E} - 10^{-6} L^2 (74.34\Delta 1 + 34.95\Delta 2 + 182.06\Delta 3) xI$$

$$\therefore 1.368 \frac{L^2}{E} \; 23.50\Delta 1 - 11.0\Delta 2 - 58.5\Delta 3 = 0 \qquad (3)$$

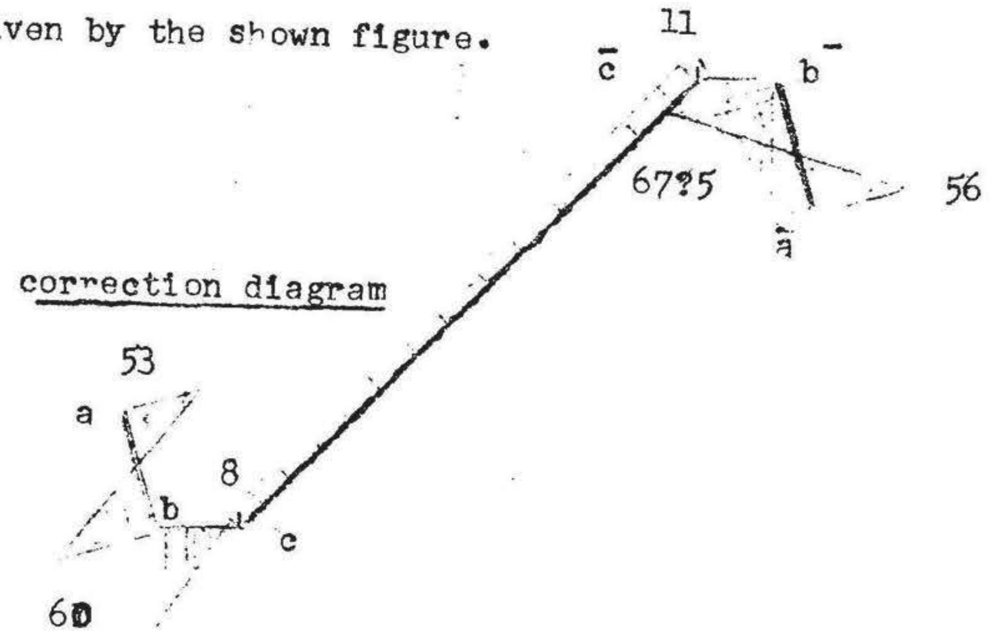
From thèse equations we get :

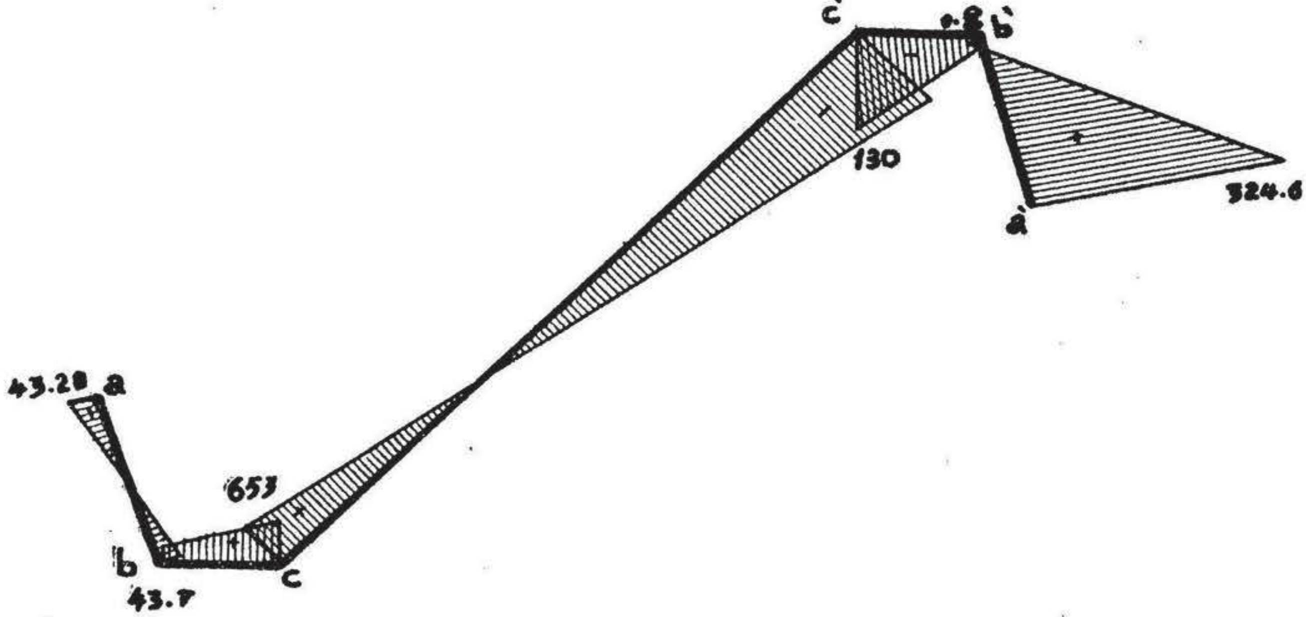
$$\Delta 1 = .1375 \frac{L^2}{E}$$

$$\Delta 2 = - .93 \frac{L^2}{E}$$

$$\Delta 3 = .1425 \frac{L^2}{E}$$

The correction diagram associated to this case of loading is given by the shown figure.





Final Stresses

loads applied at points c`-b`

Displacements

$$\delta ab = 0.113 \times \frac{\overline{375}^2}{300000} - \frac{60 + 49}{24 \pi^2} \times \frac{\overline{375}^2}{300000}$$

$$\therefore \delta ab = 0.054 - 0.225 \qquad = 0.171 \text{ cm}$$

$$\delta bc = 0.648 \times \frac{\overline{375}^2}{300000} - \frac{60 + 8}{15 \pi^2} \times \frac{\overline{375}^2}{300000}$$

$$= 0.308 - 0.218 \qquad = .090 \text{ cm}$$

$$\delta c\bar{c} = 0.184 \times \frac{\overline{375}^2}{300000} + \frac{11 - 8}{112.5 \pi^2} \times \frac{\overline{375}^2}{300000}$$

$$= 0.875 + .0013 \qquad = .0888 \text{ cm}$$

$$\delta \bar{b}\bar{c} = 0.368 \times \frac{\overline{375}^2}{300000} + \frac{11 + 67.5}{15 \pi^2} \quad \frac{\overline{375}^2}{300000}$$

$$= .175 + .250 \qquad = 0.425 \text{ cm}$$

$$\delta \bar{b}\bar{a} = 2.02 \times \frac{\overline{375}^2}{300000} - \frac{123.5}{24 \pi^2} \times \frac{\overline{375}^2}{300000}$$

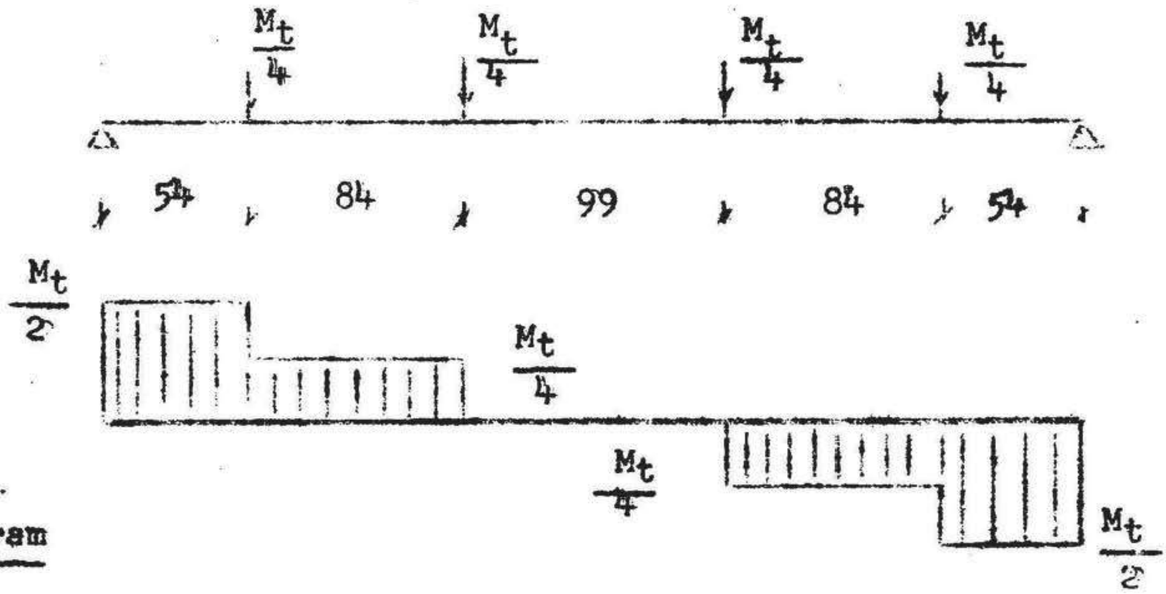
$$= .96 - .247 \qquad = 0.713 \text{ cm}$$

$$\Delta b_v = \delta ab / \cos 15 - \delta bc \tan 15 = .175 - 0.234 \qquad = 0.1516 \text{ cm}$$

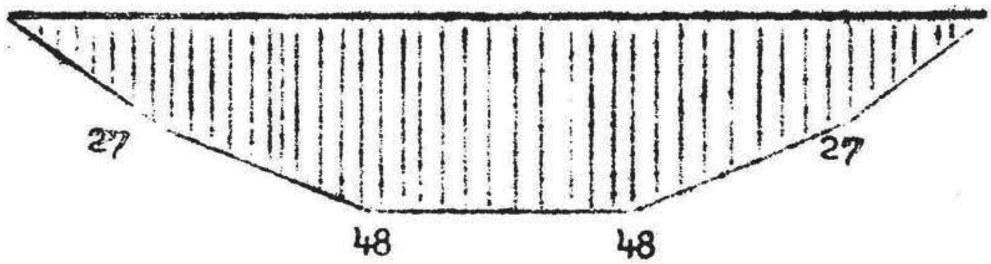
$$\Delta c_v = \delta c\bar{c} / \cos 45 + \delta bc \qquad = 0.125 + 0.091 \qquad = 0.216 \text{ cm}$$

$$\Delta \bar{c}_v = .125 + 0.425 = 0.560 \quad , \quad \Delta \bar{b}_v = .738 - .110 = \qquad .628 \text{ cm}$$

Torsion Analysis and Comments



M_t-Diagram



Ø-Diagram

All values x .468 x 10^{-8}

$$\phi = \frac{3\,M_t\,l}{G\,\Sigma(t^3 . l)} = \frac{3\,M_t\,l}{G t^3 \Sigma (l)} = \frac{3\,M_t \times l}{G \times 27 \times (24\times2+15\times2+112.5)}$$

$$= \frac{3\,M_t\,l}{5143.5G}$$

but we have $G = \frac{E}{2(1+\mu)} = \frac{E}{2\times1.165} = 0.428E$

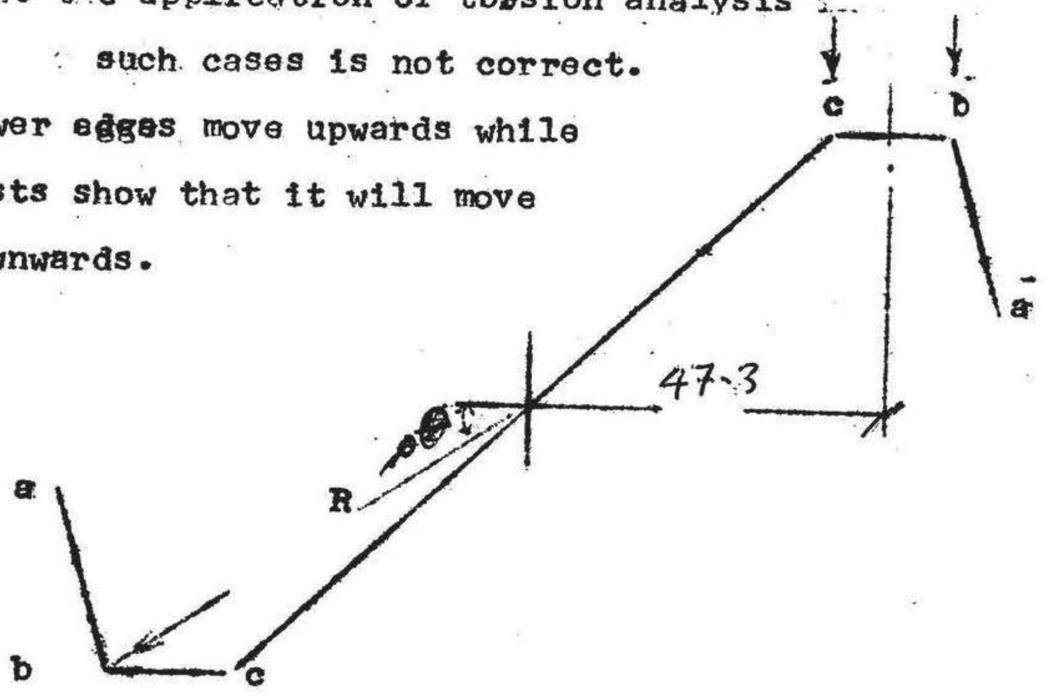
$$\therefore \phi = \frac{3\,M_t\,l}{5143.5\times0.428\times300000} = 0.463 \times 10^{-8}\,M_t\,l$$

$$M_t = 1000\,P \times 47.3 = 47300\,P\ \text{kg.cm}$$

$$\therefore \phi_{max.} = 47300P \times 48 \times 0.463\times10^{-8} = 1.05\times10^{-2}\,P$$

Comments :

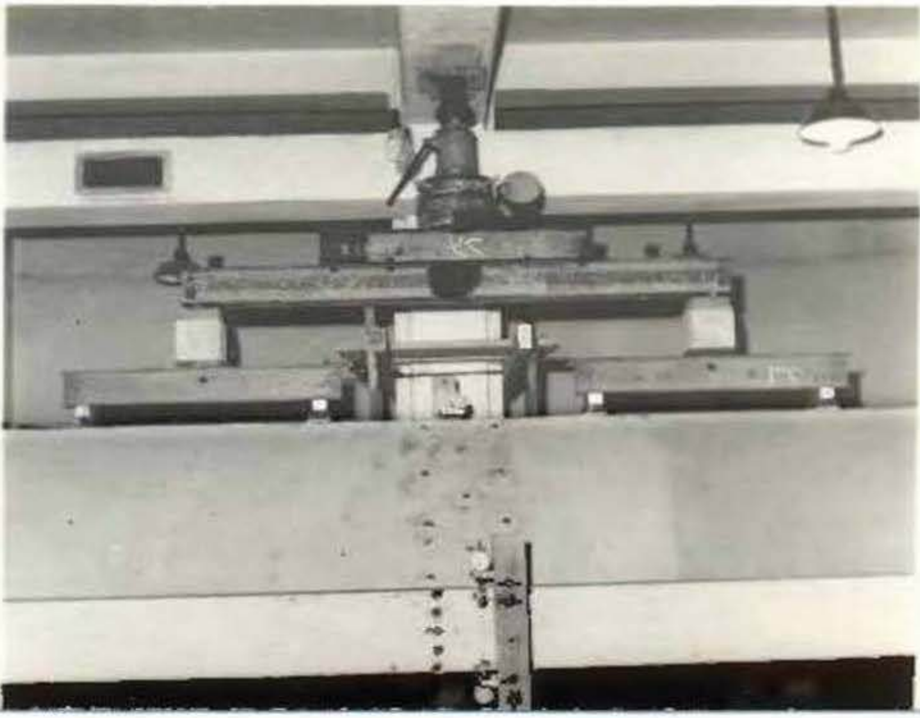
It is concluded from the next table that the application of torsion analysis in such cases is not correct. Lower edges move upwards while tests show that it will move downwards.

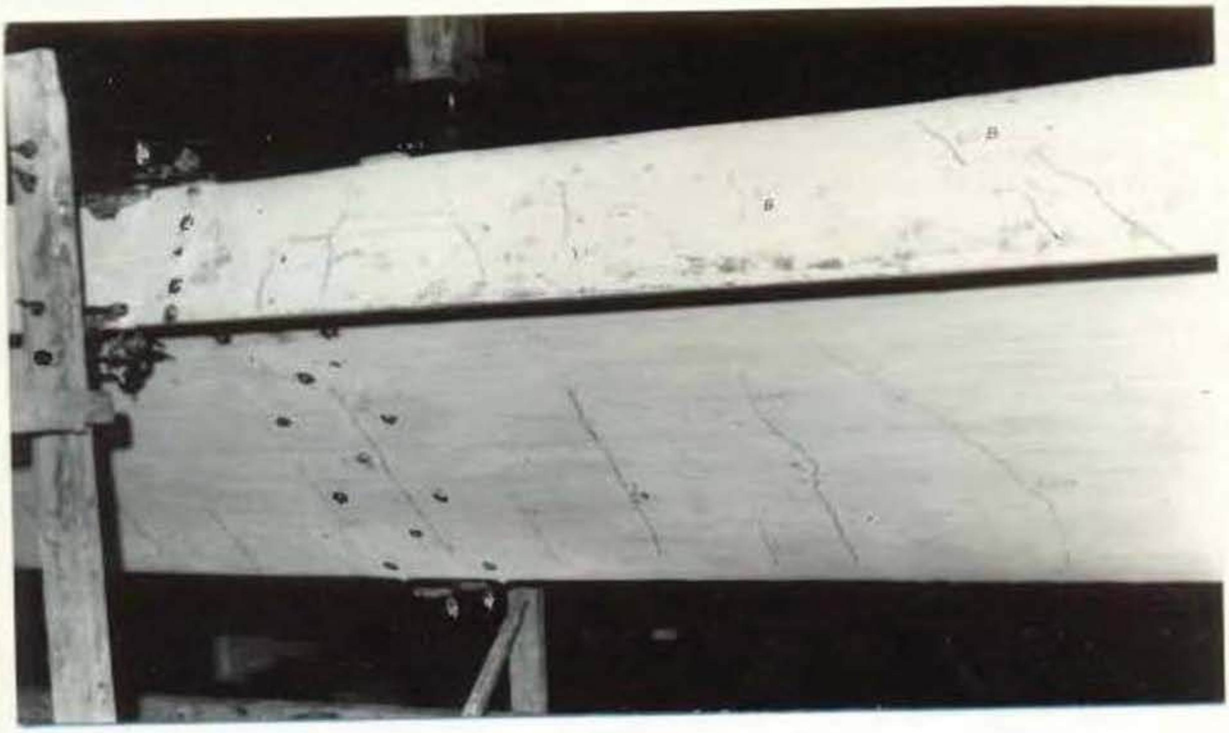


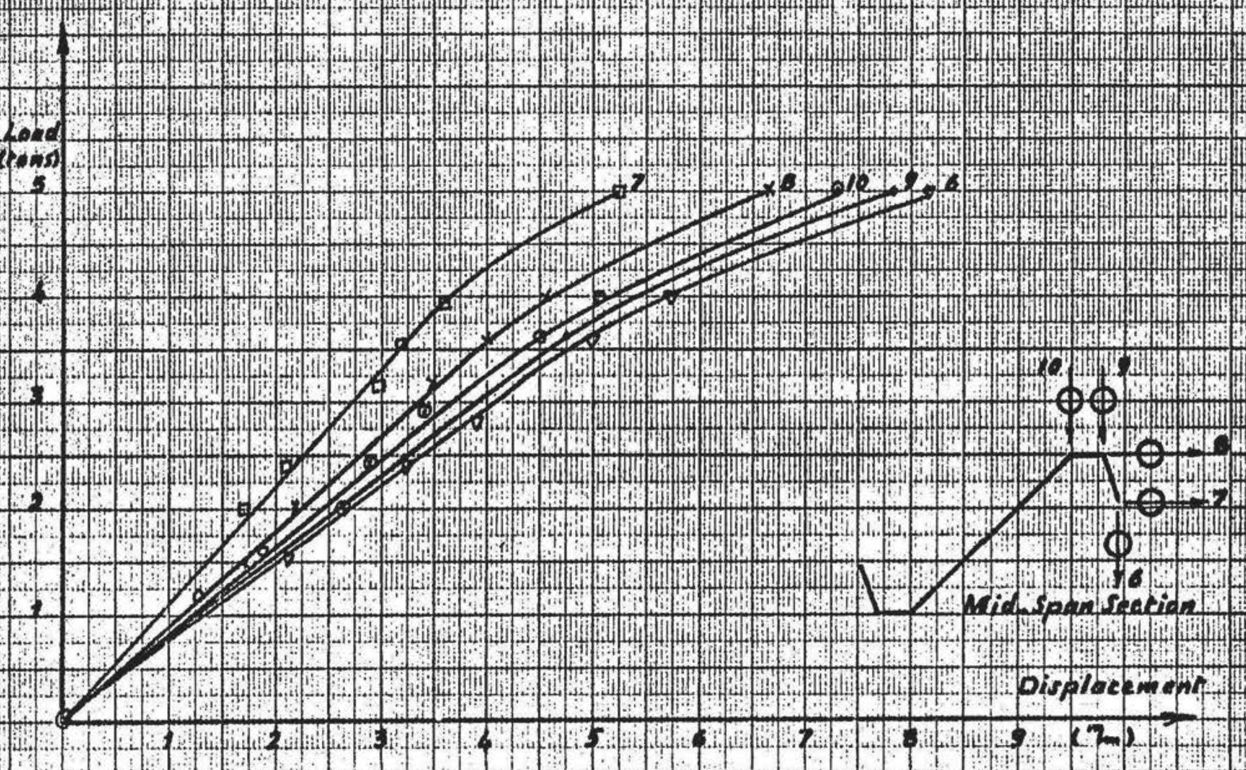
$\phi = 1.05 \times 10^{-2}$ P

point	x	y	$R=\sqrt{x^2+y^2}$	$\tan\theta = y/x$	$\rho = R\phi$	$x_1 = \rho \cdot Y/R$	$Y_1 = \rho X/R$
A	61.02	16.6	63.40	16.6 / 61.02	0.666	.174P	0.64P ↑
B	54.8	39.8	67.73	39.8 / 54.8	0.710	0.418P	0.575P ↑
C	39.8	39.8	56.25	1	.59	0.417P	0.417P ↑

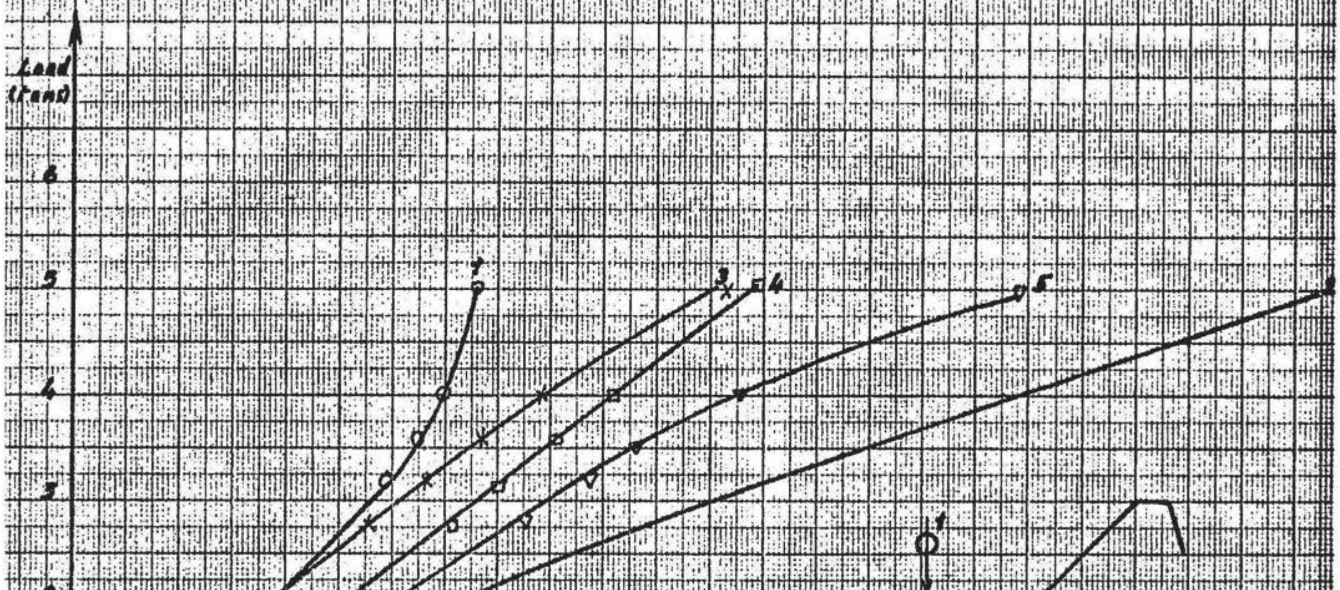
- Experimental Results



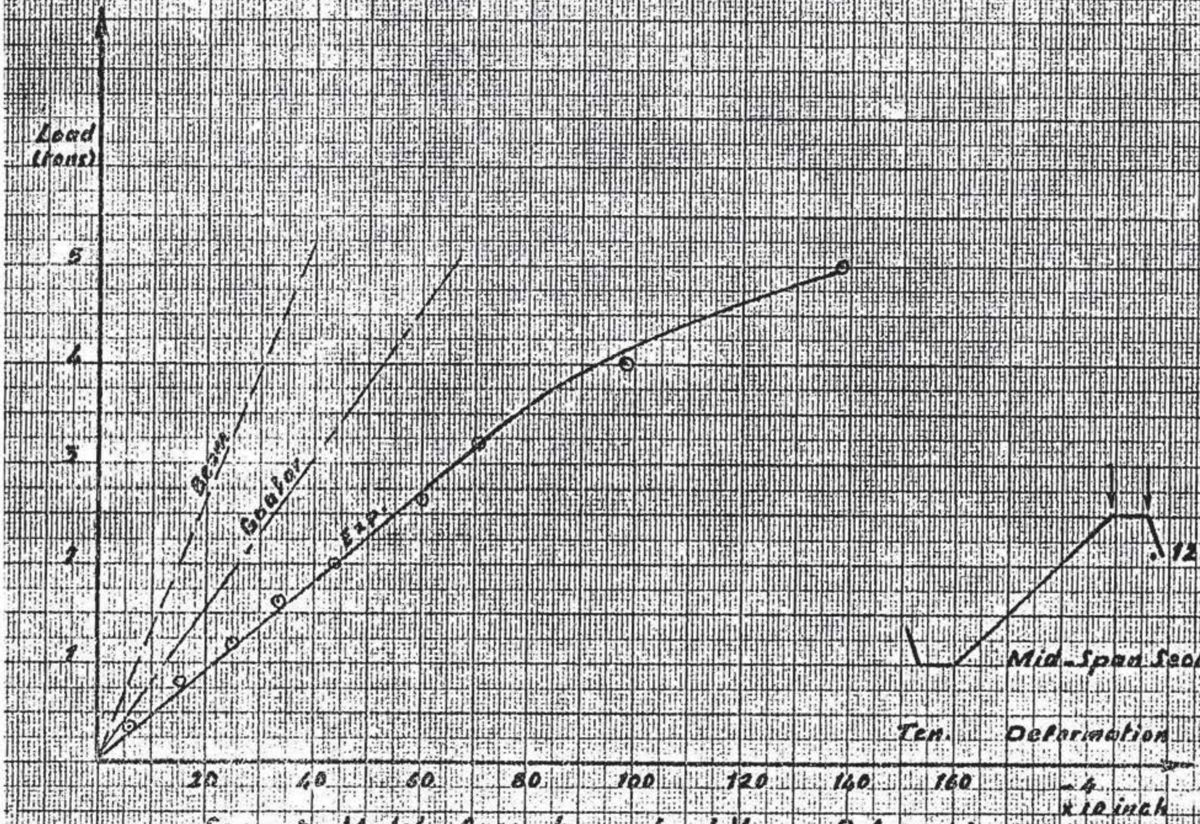




Concrete Model _ Case «b» _ Load Versus Displacement.

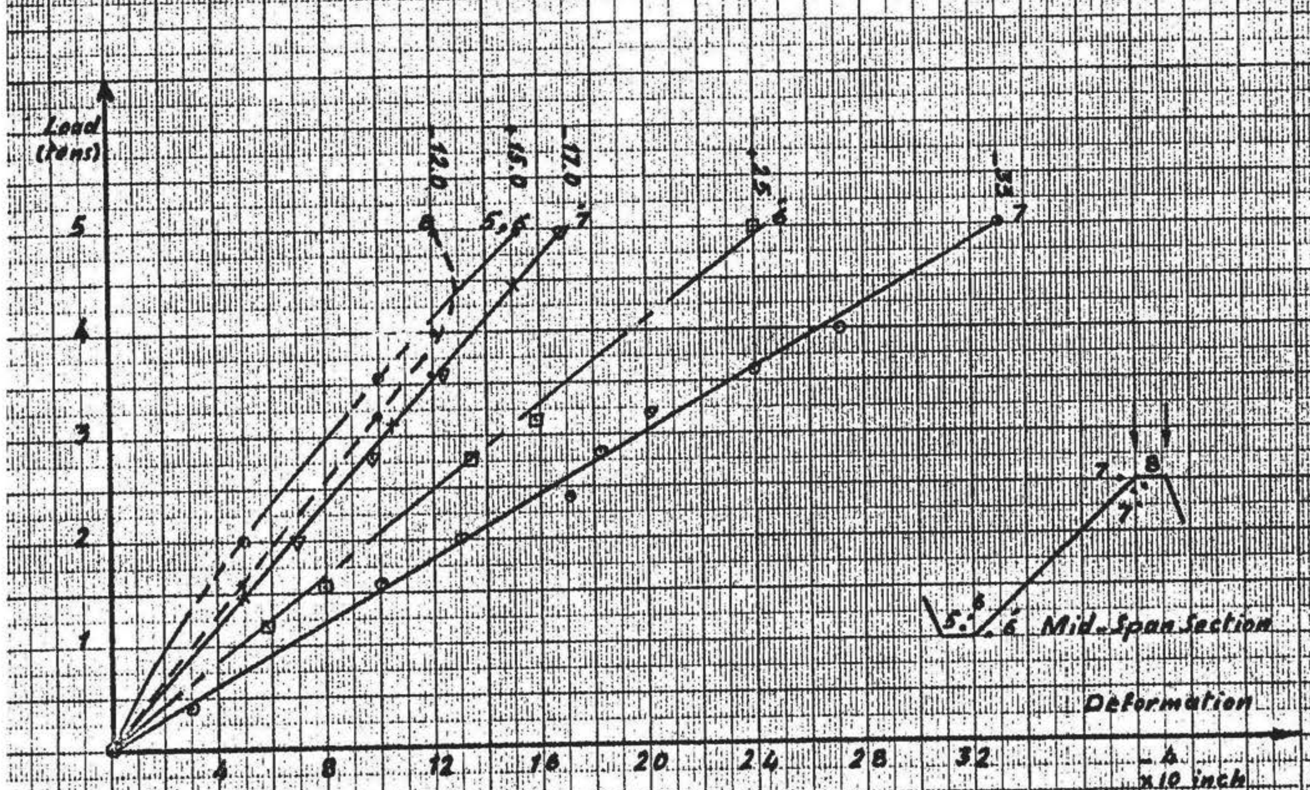


Concrete Model _ Case «b» _ Load Versus Displacement.



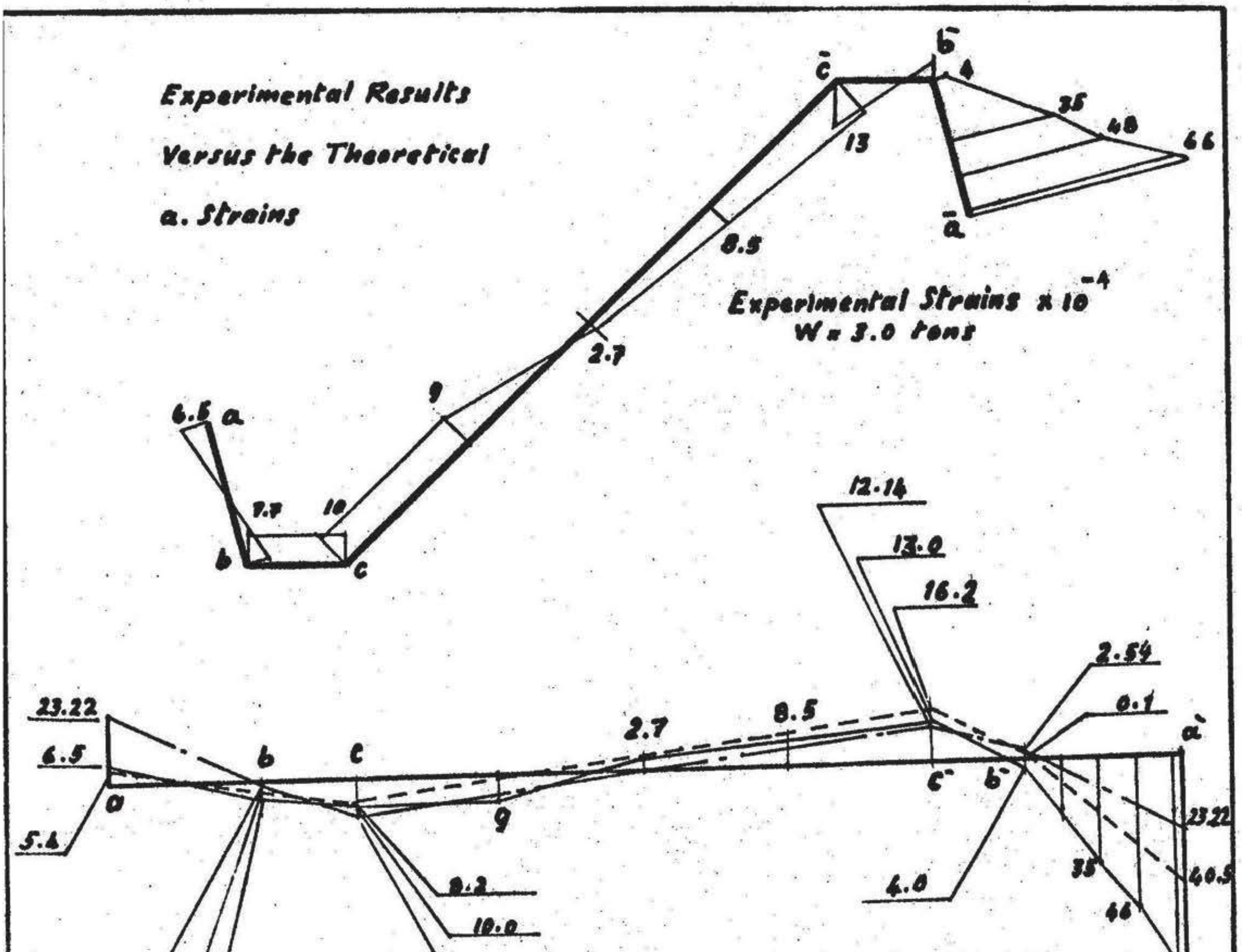
Concrete Model - Case «b» - Load Versus Deformation for point 12.

Concrete Model - Case «b» - Load Versus Deformation for point 1.

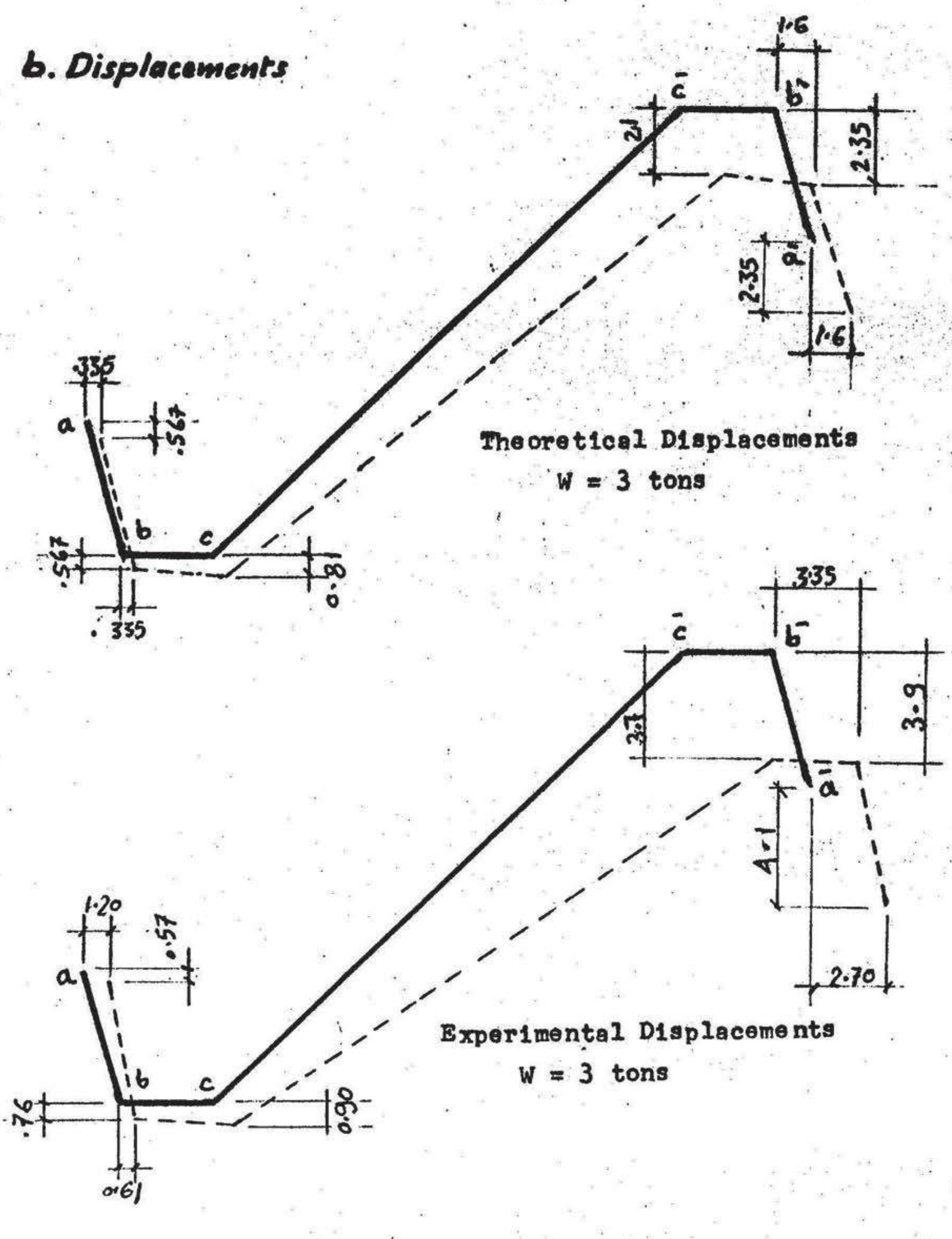


Concrete Model – Case "b" Load Versus Deformation, for points 5, 6, 6, 7, 7 and 8

Concrete Model – Case "b" Load Versus Deformation for points 4, 11, 11, 13, 13, 14, 14, 15, 15, 16 & 16

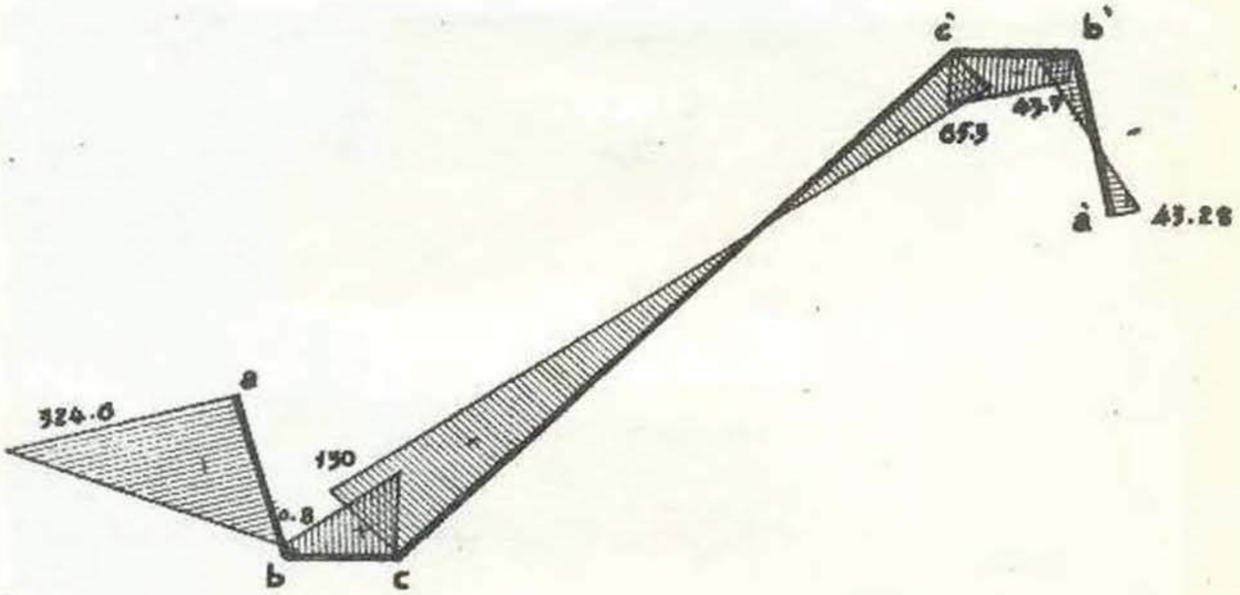

Experimental Results
Versus the Theoretical
a. Strains
Experimental Strains x 10^-4
W = 3.0 tons
6.5
a
7.7
10
b
c
9
2.7
8.5
13
c̄
b̄
4
35
48
66
ā
12.14
13.0
16.2
2.54
0.1
23.22
6.5
5.4
a
b
c
2.7
8.5
c̄
b̄
ā
9
8.2
10.0
12.14
2.54
5.4
7.7
4.0
23.22
40.5
35
46
66
Experimental Strains Versus
The Theoretical
W = 3.0 tons
Exp.
Gaafar
Beam
c̄
b̄
.1
16.2
ā
40.5
Theoritical Strains x 10^-4
W = 3.0 tons
5.4
a
5.4
8.2
b
c

b. Displacements



c- Unsymmetrical loading (case c)

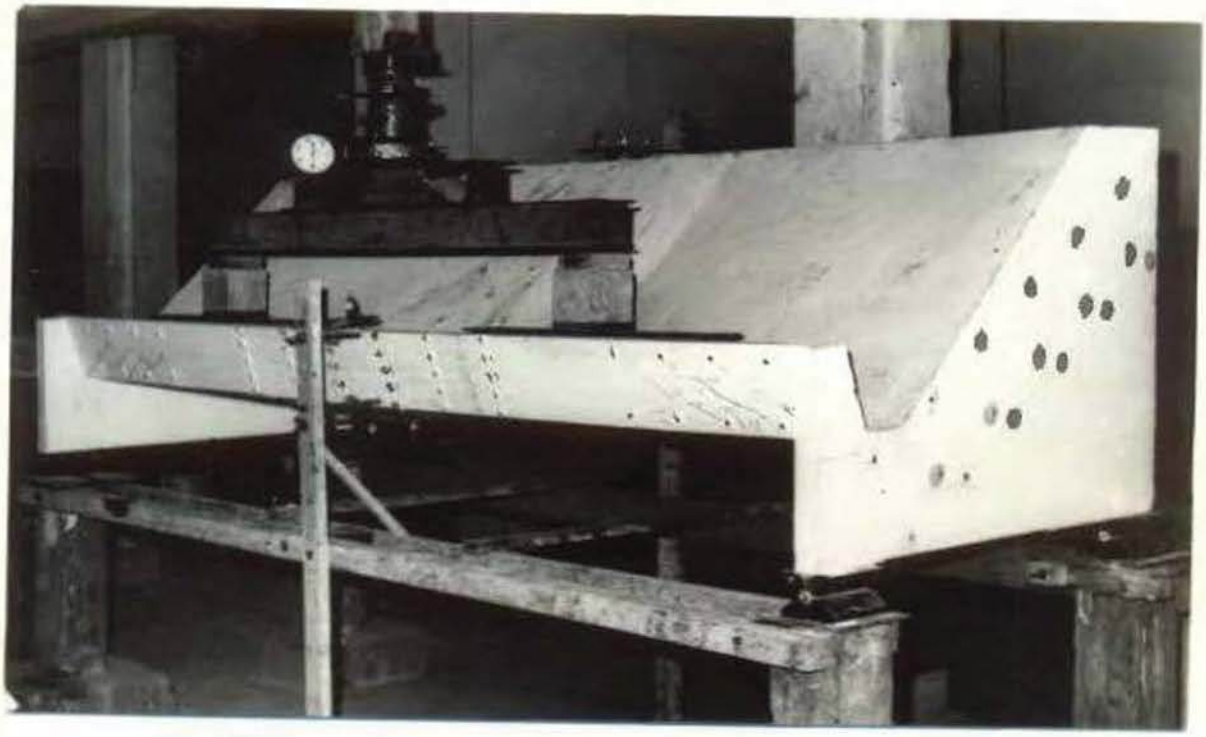
In this case stresses and displacements can be concluded directly from the previous case "b" .

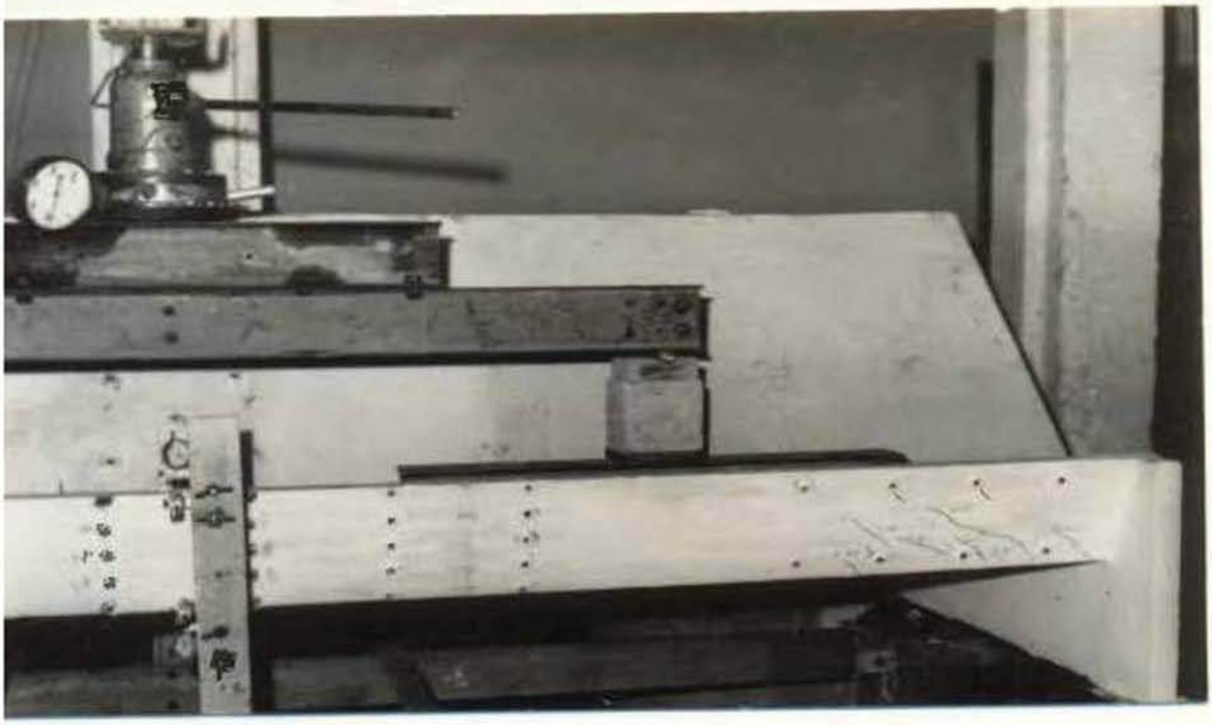


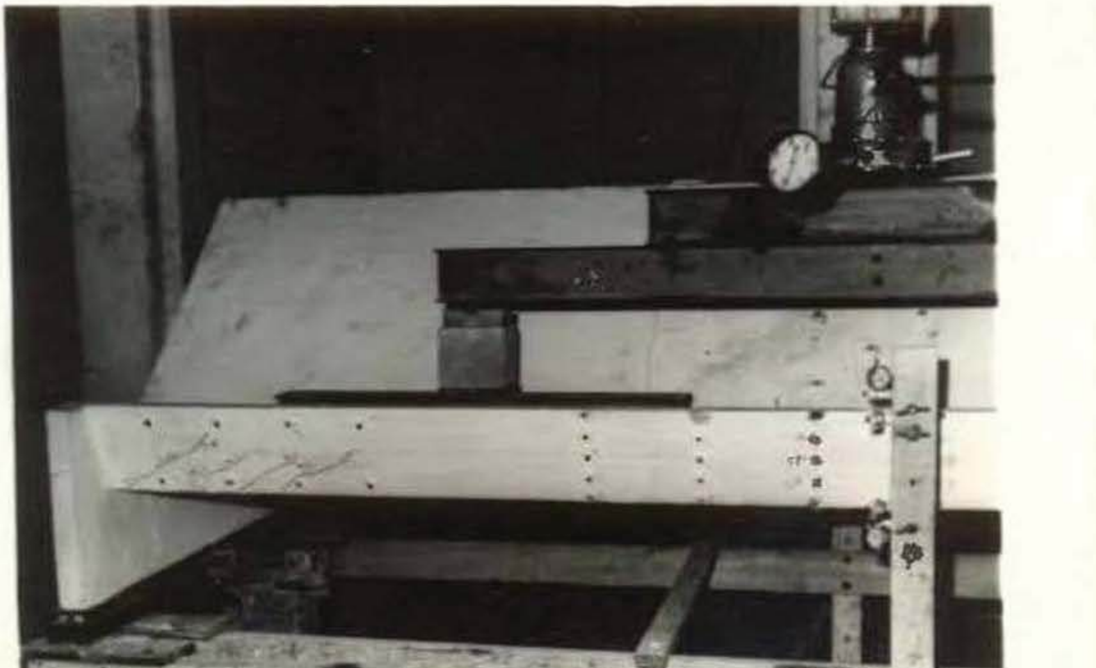
Final Stresses

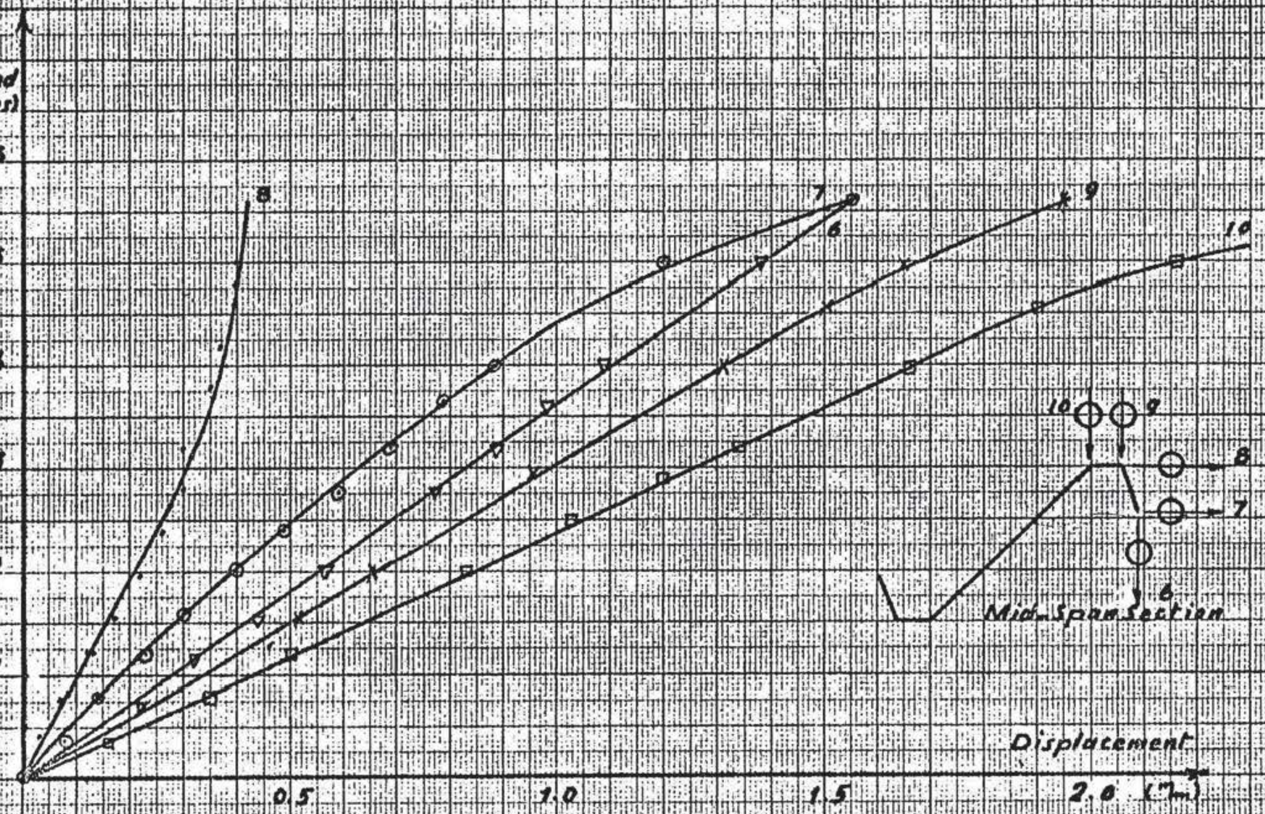
loads applied at points b. c

- Experimental Results

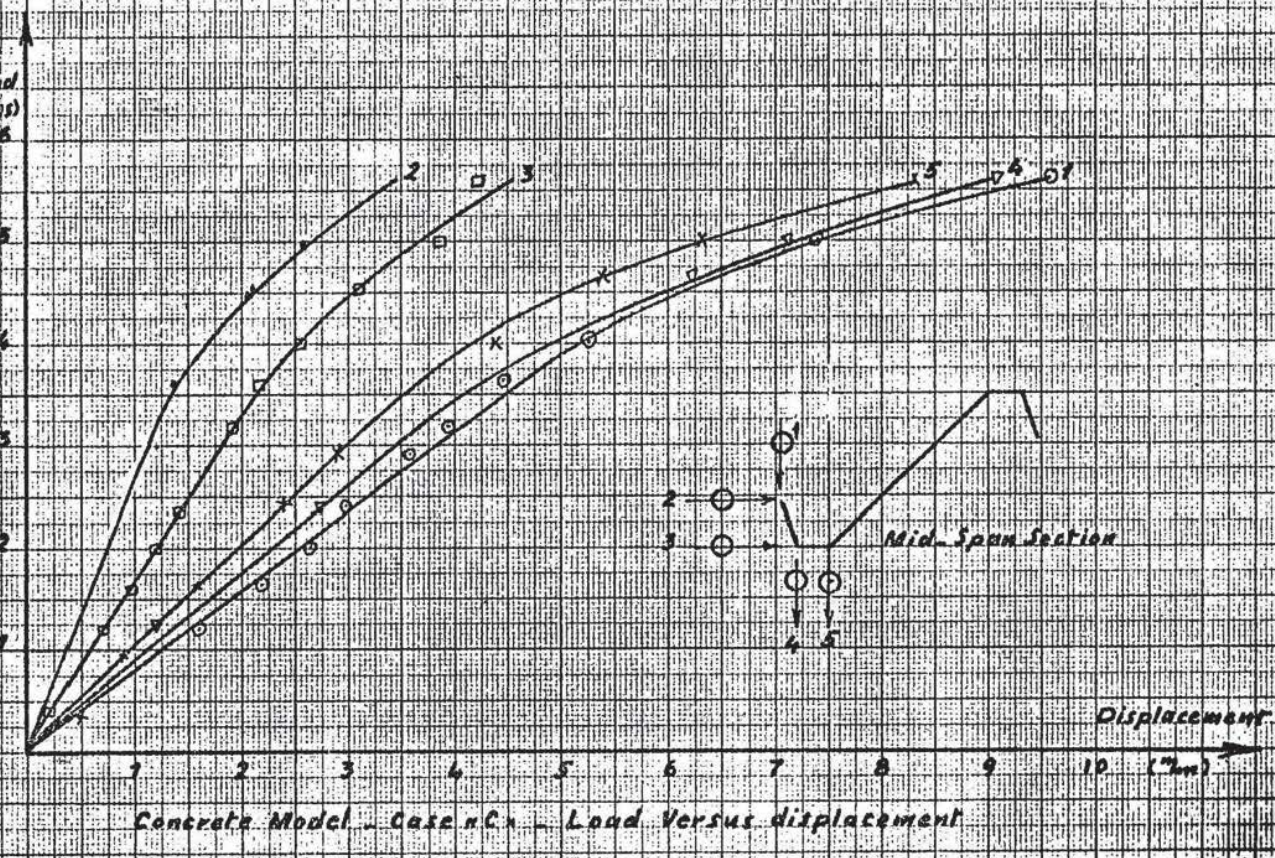




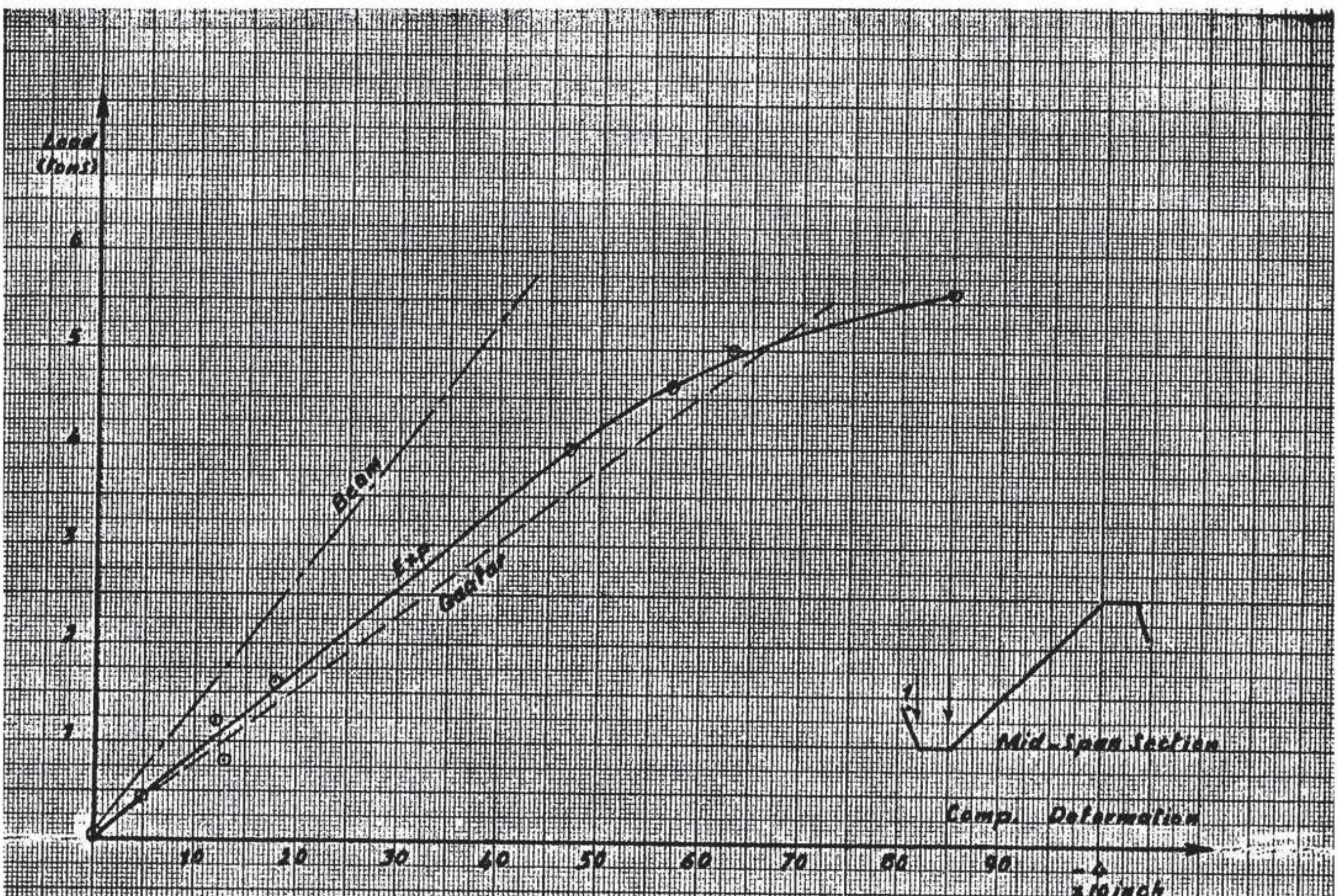




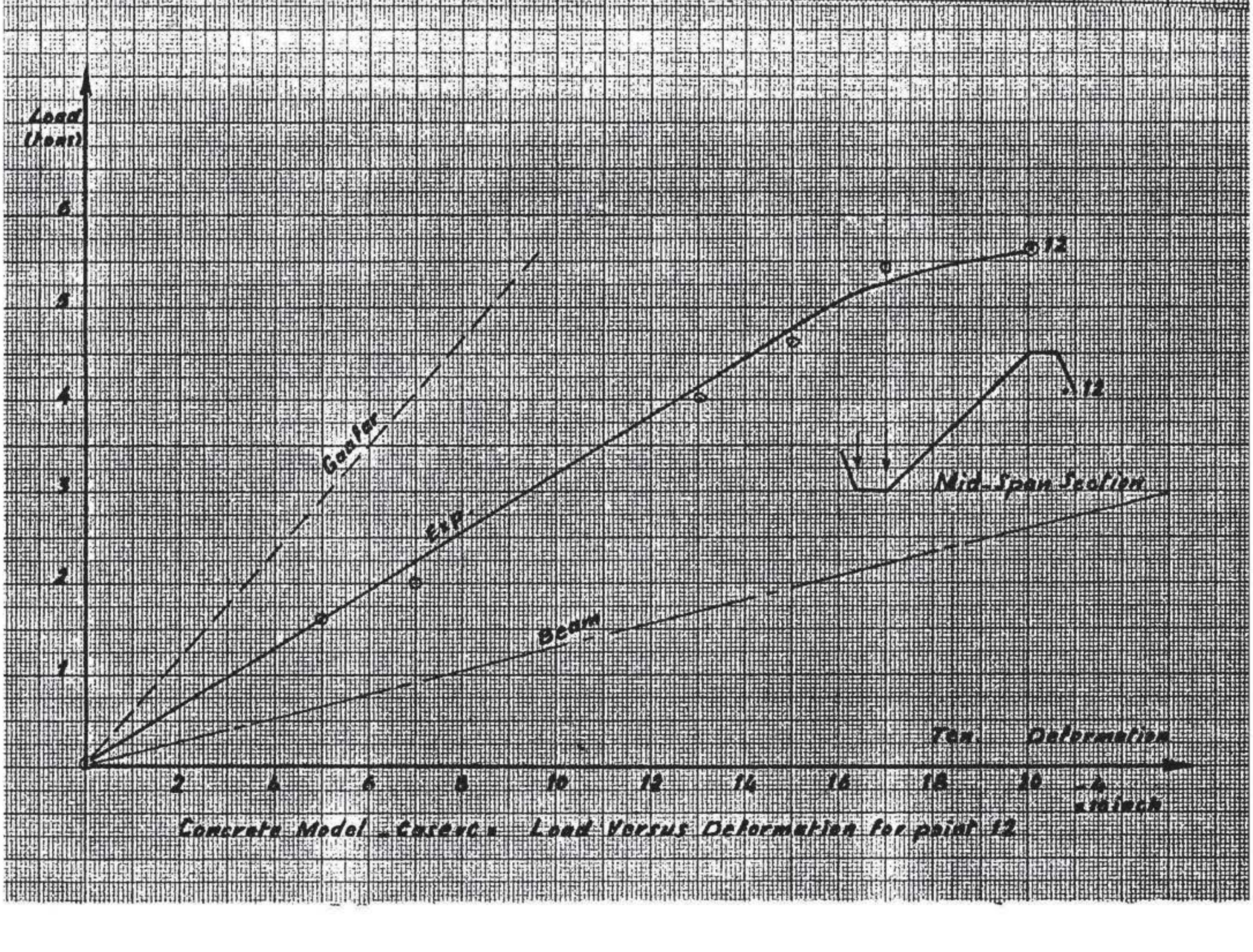
Concrete Model _ Case « C » Load Versus Displacement.



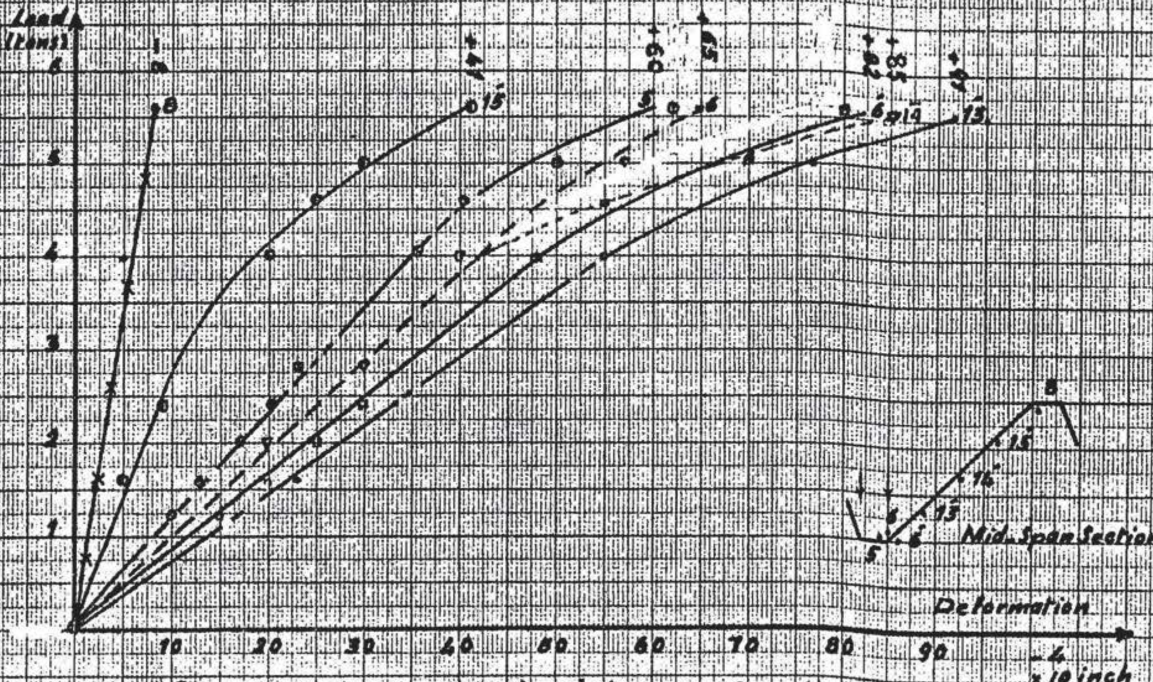
Concrete Model _ Case « C » _ Load Versus displacement



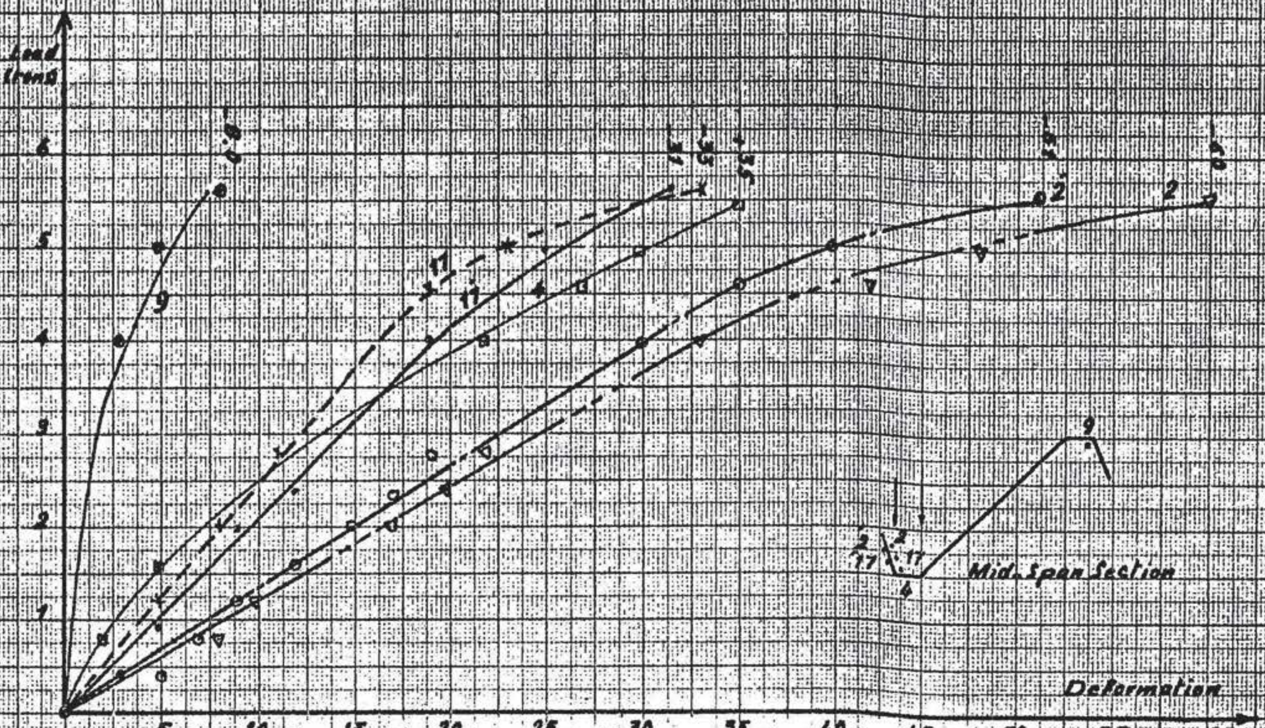
Concrete Model - Case «C» - Load Versus Deformation for point 1.



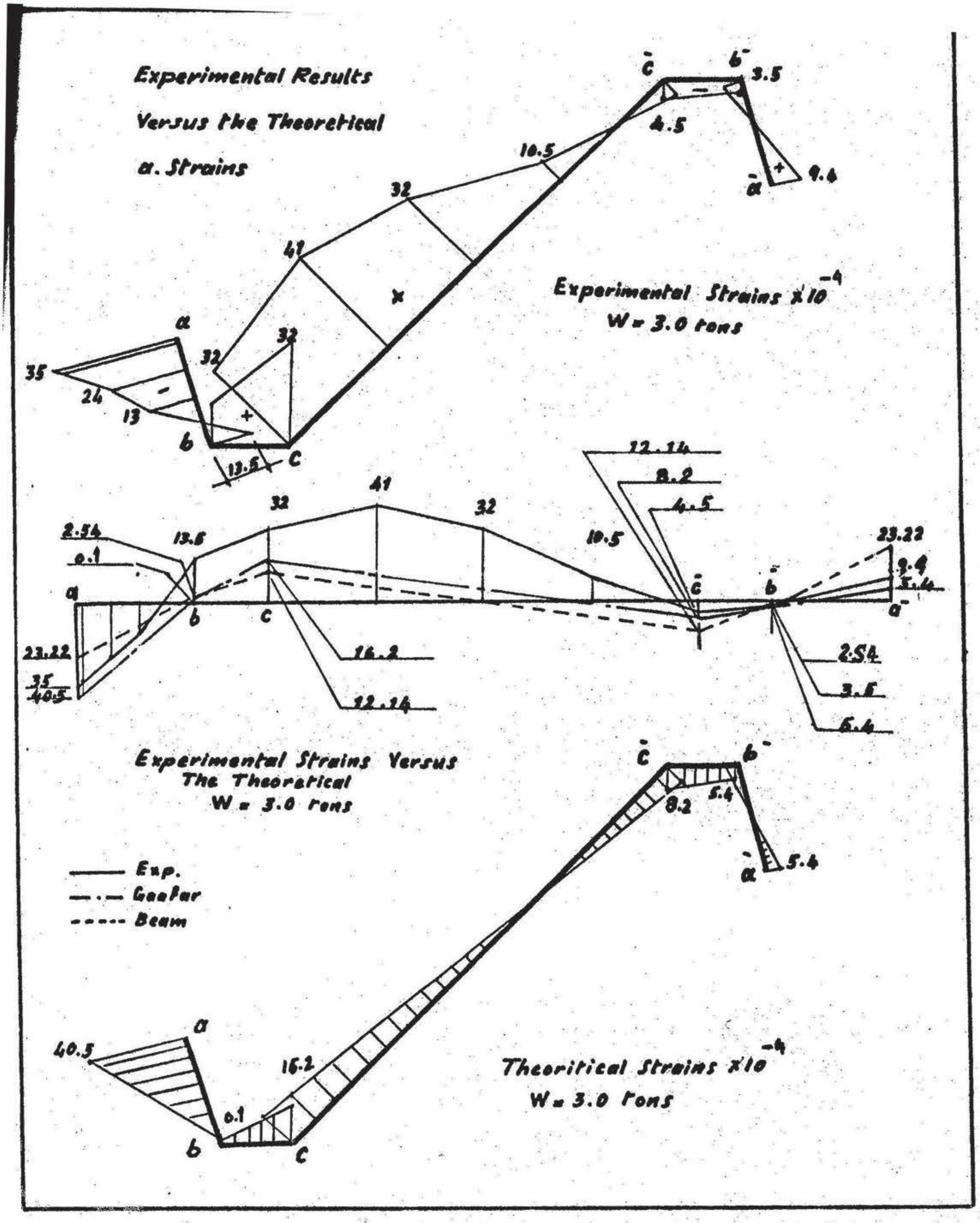
Concrete Model - Case«C» - Load Versus Deformation for point 12.



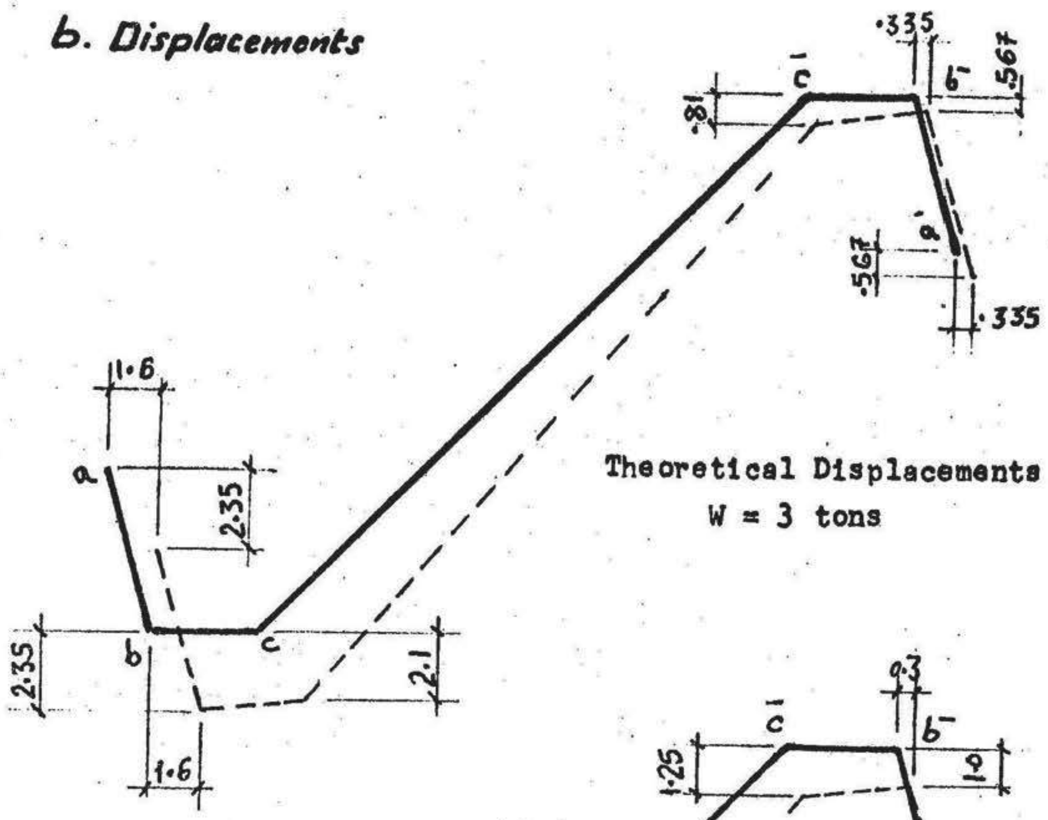
Concrete Model - Case «C» Load Versus Deformation for points 5, 6, 6', 8, 13, 14 and 15



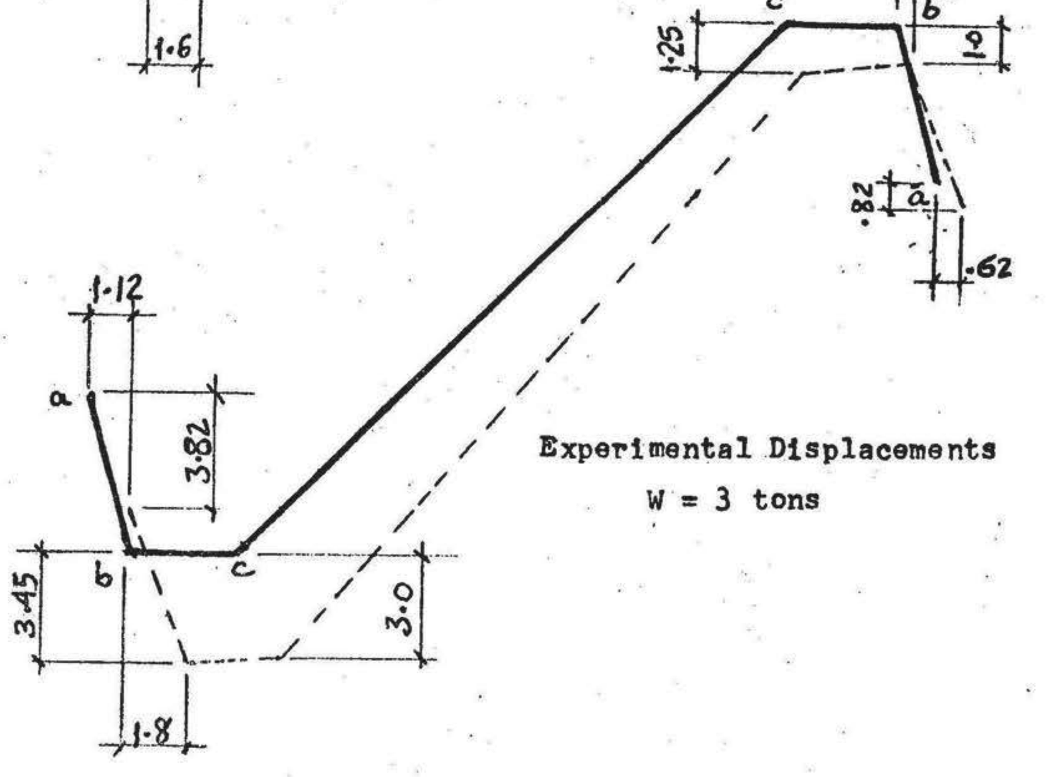
Concrete Model - Case «C» Load Versus Deformation for points 2, 2', 4, 9, 17 and 17'


Experimental Results
Versus the Theoretical
a. Strains
Experimental Strains x10^-4
W = 3.0 tons
Experimental Strains Versus
The Theoretical
W = 3.0 tons
Exp.
Gaafar
Beam
Theoritical Strains x10^-4
W = 3.0 tons

b. Displacements



Theoretical Displacements
W = 3 tons



Experimental Displacements
W = 3 tons

d- General Solution
based on (Gaafar's Methods)

A more general solution is introduced. This solution depends on correcting the stresses produced by each case of plate loading. The advantages of this method appear when solving a given problem for different cases of loadings

The solution is summerized in 3-steps:

1. Elementary analysis:

Where the stresses are calculated for any arbit-rary values of each plate loading. These values were chosen to give the solution of the unsymmetrical case as a check.

2. Correction analysis :

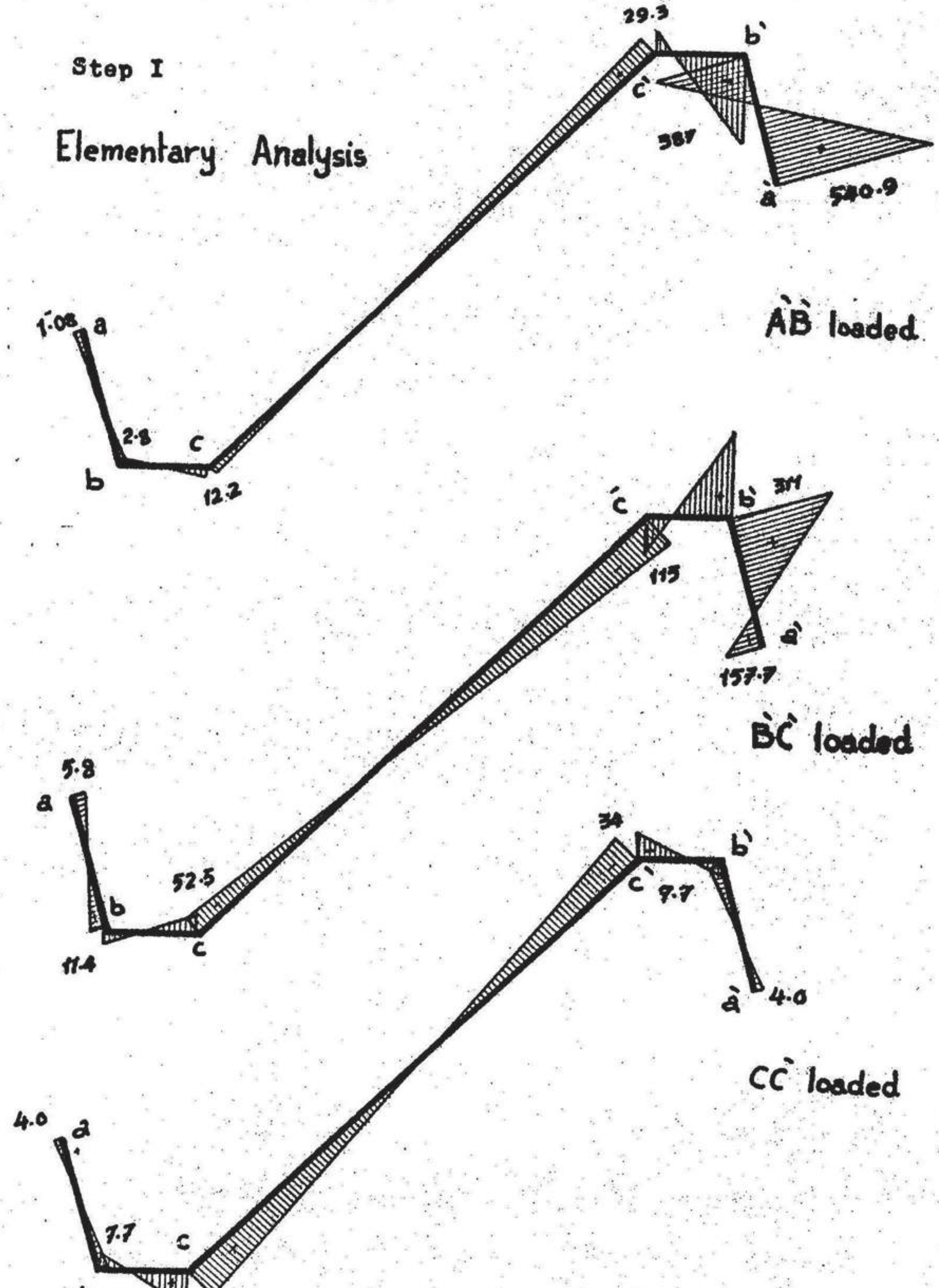
In this case we make use of the pre-calculated values of stresses corresponding to $\Delta 1$, $\Delta 2$, and $\Delta 3$ values .

3. Final stresses :

It is obtained by summing the stresses of steps 1 and 2. as in the previous solutions .

Step I

Elementary Analysis



Step 2 : CORRECTION ANALYSIS

Plate $\bar{a}\bar{b}$ loaded :

$$\delta_{ab} = 0.017\frac{L^2}{I} + L^2 \times 10^{-6} (-75\,\Delta 1 + 15.1\Delta 2 + 35.8\Delta 3) \times I$$

$$\delta_{bc} = .108\frac{L^2}{I} + L^2 \times 10^{-6} (-80.5\Delta 1 + 15.2\Delta 2 + 31.8\Delta 3) \times I$$

$$\delta_{c\bar{c}} = .0395\frac{L^2}{E} + L^2 \times 10^{-6} (-1.88\Delta 1 - 1.88\Delta 3) \times I$$

$$\delta_{\bar{b}\bar{c}} = 2.98\frac{L^2}{E} + L^2 \times 10^{-6} (31.8\,\Delta 1 - 15.2\Delta 2 - 80.5\Delta 3) \times I$$

$$\delta_{\bar{b}\bar{a}} = 4.17\frac{L^2}{E} + L^2 \times 10^{-6} (35.8\,\Delta 1 - 15.1\,\Delta 2 - 75\,\Delta 3) \times I$$

GEOMETRICAL RELATIONS :

$$\Delta_1 = 1.035\,\delta_{ab} + 1.268\,\delta_{bc} + 1.414\,\delta_{c\bar{c}}$$

$$\Delta 2 = 1.414\,\delta_{\bar{b}\bar{c}} - 1.414\,\delta_{bc}$$

$$\Delta_3 = 1.035\,\delta_{\bar{a}\bar{b}} + 1.268\,\delta_{\bar{b}\bar{c}} + 1.414\,\delta_{c\bar{c}}$$

Substituting the values of δ in the geometrical relations :

$$\Delta_1 = .210 - 57.5\Delta_1 + 11.0\,\Delta_2 + 23.5\,\Delta 3 \qquad 1$$

$$\Delta_2 = 3.63 + 50\,\Delta_1 - 13.6\,\Delta_2 - 50\,\Delta 3 \qquad 2$$

$$\Delta_3 = 8.1358 + 23.5\Delta_1 - 11.0\,\Delta_2 - 57.5\,\Delta 3 \qquad 3$$

Solving these equations we get :

$$\Delta_1 = -0.0568\frac{L^2}{E}$$

$$\Delta_2 = -.955\,\frac{L^2}{E}$$

$$\Delta_3 = +.296\,\frac{L^2}{E}$$

Plate $\bar{b}\bar{c}$ loaded:

$$\delta ab = 0.077\frac{L^2}{E} + L^2 \times 10^{-6}(-75\,\Delta 1 + 15.1\Delta 2 + 35.8\,\Delta 3) \times I$$

$$\delta bc = .458\,\frac{L^2}{E} + L^2 \times 10^{-6}(-80.5\Delta 1 + 15.2\,\Delta 2 + 35.8\,\Delta 3) \times I$$

$$\delta c\bar{c} = 0.159\frac{L^2}{E} + L^2 \times 10^{-6}(-1.88\Delta 1 - 1.88\Delta 3) \times I$$

$$\delta \bar{c}\bar{b} = 3.05\,\frac{L^2}{E} + L^2 \times 10^{-6}(35.8\,\Delta 1 - 15.2\,\Delta 2 - 80.5\Delta 3) \times I$$

$$\delta \bar{b}\bar{a} = 2.1\,\frac{L^2}{E} + L^2 \times 10^{-6}(35.8\,\Delta 1 - 15.1\,\Delta 2 - 75\,\Delta 3) \times I$$

GEOMETRICAL RELATIONS:

$$\Delta 1 = 1.035\ \delta ab + 1.26\ \delta bc + 1.414\ \delta c\bar{c}$$

$$\Delta 2 = 1.414\ \delta \bar{b}\bar{c} - 1.414\ \delta bc$$

$$\Delta 3 = 1.035\ \delta \bar{a}\bar{b} + 1.26\ \delta \bar{b}\bar{c} + 1.414\ \delta c\bar{c}$$

Substituting the values of δ in the geometrical relations:

$$\Delta 1 = .8910\,\frac{L^2}{E} - 57.5\Delta 1 + 11.0\Delta 2 + 23.5\,\Delta 3 \qquad 1$$

$$\Delta 2 = 3.245\,\frac{L^2}{E} + 50\,\Delta 1 - 13.6\Delta 2 - 50\,\Delta 3 \qquad 2$$

$$\Delta 3 = 6.24\,\frac{L^2}{E} + 23.5\Delta 1 - 11.0\Delta 2 - 57.5\,\Delta 3 \qquad 3$$

Solving these equations we get:

$$\Delta 1 = 0.0682\,\frac{L^2}{E}$$

$$\Delta 2 = 0.00925\,\frac{L^2}{E}$$

$$\Delta 3 = 0.1355\,\frac{L^2}{E}$$

Plate $c\bar{c}$ loaded :

$$\delta ab = 0.052\frac{L^2}{E} + L^2 \times 10^{-6}(-75\,\Delta 1 + 35.8\,\Delta 3) \times I$$

$$\delta bc = .298\,\frac{L^2}{E} + L^2 \times 10^{-6}(-80.5\,\Delta 1 + 35.8\,\Delta 3) \times I$$

$$\delta c\bar{c} = 0.0645\frac{L^2}{E} + L^2 \times 10^{-6}(-1.88\,\Delta 1 + 1.88\,\Delta 3) \times I$$

$$\delta \bar{c}\bar{b} = 0.298\frac{L^2}{E} + L^2 \times 10^{-6}(-35.8\,\Delta 1 + 80.5\,\Delta 3) \times I$$

$$\delta \bar{b}\bar{a} = 0.052\frac{L^2}{E} + L^2 \times 10^{-6}(-35.8\,\Delta 1 + 75\,\Delta 3) \times I$$

GEOMETRICAL RELATIONS:

$$\Delta 1 = 1.035\,\delta ab + 1.268\,\delta bc + 1.414\,\delta c\bar{c}$$

$$\Delta 2 = 0$$

$$\Delta 3 = 1.035\,\delta \bar{a}\bar{b} + 1.268\,\delta \bar{b}\bar{c} + 1.414\,\delta c\bar{c}$$

Substituting the values of δ in the geometrical relations:

$$\Delta = 0.523\frac{L^2}{E} - 57.5\Delta + 23.5\,\Delta$$

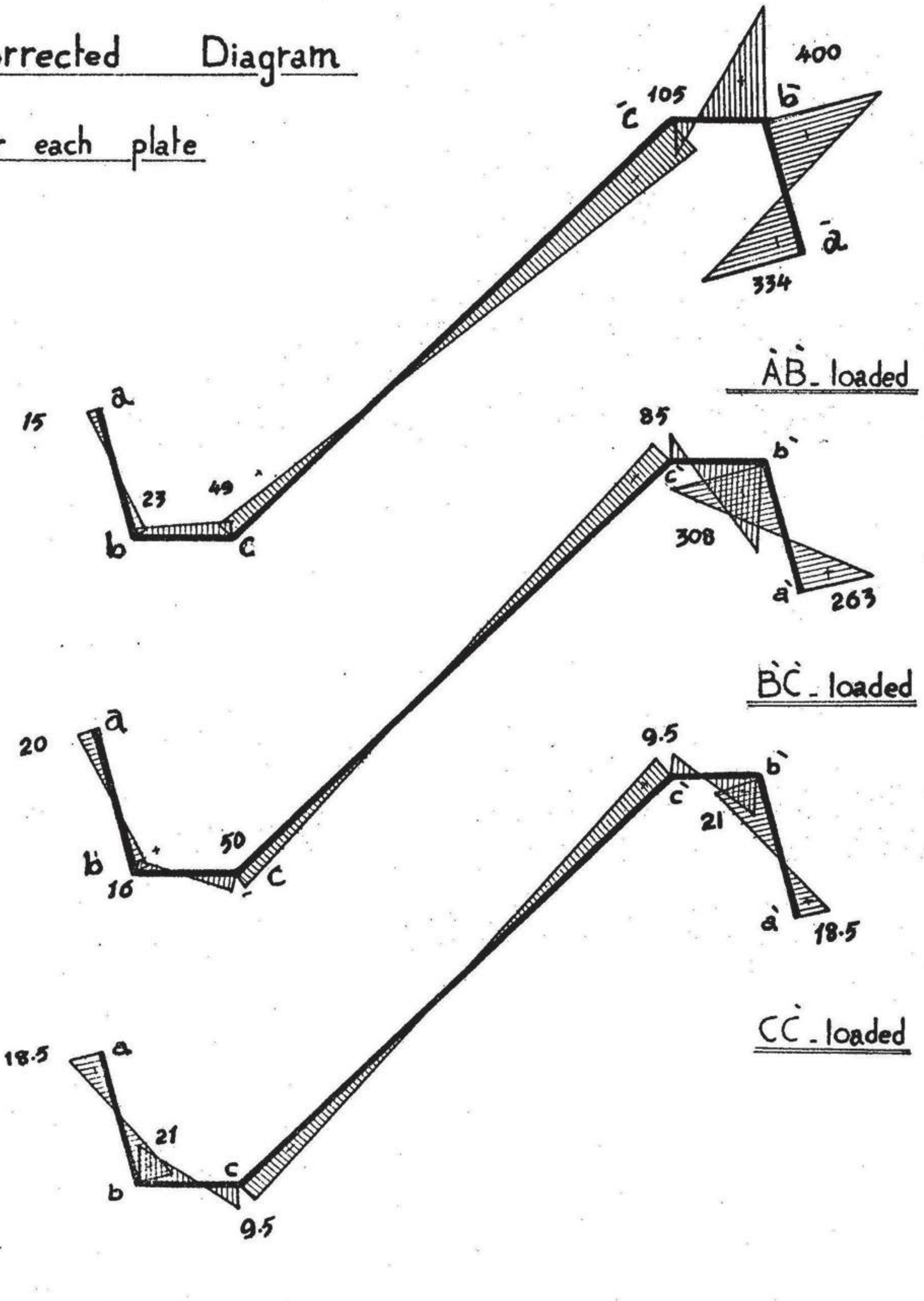
$$\text{ie}\,\Delta = 0.523\,\frac{L^2}{E} - 34\,\Delta$$

$$\therefore \Delta = \frac{0.523}{35}\,\frac{L^2}{E}$$

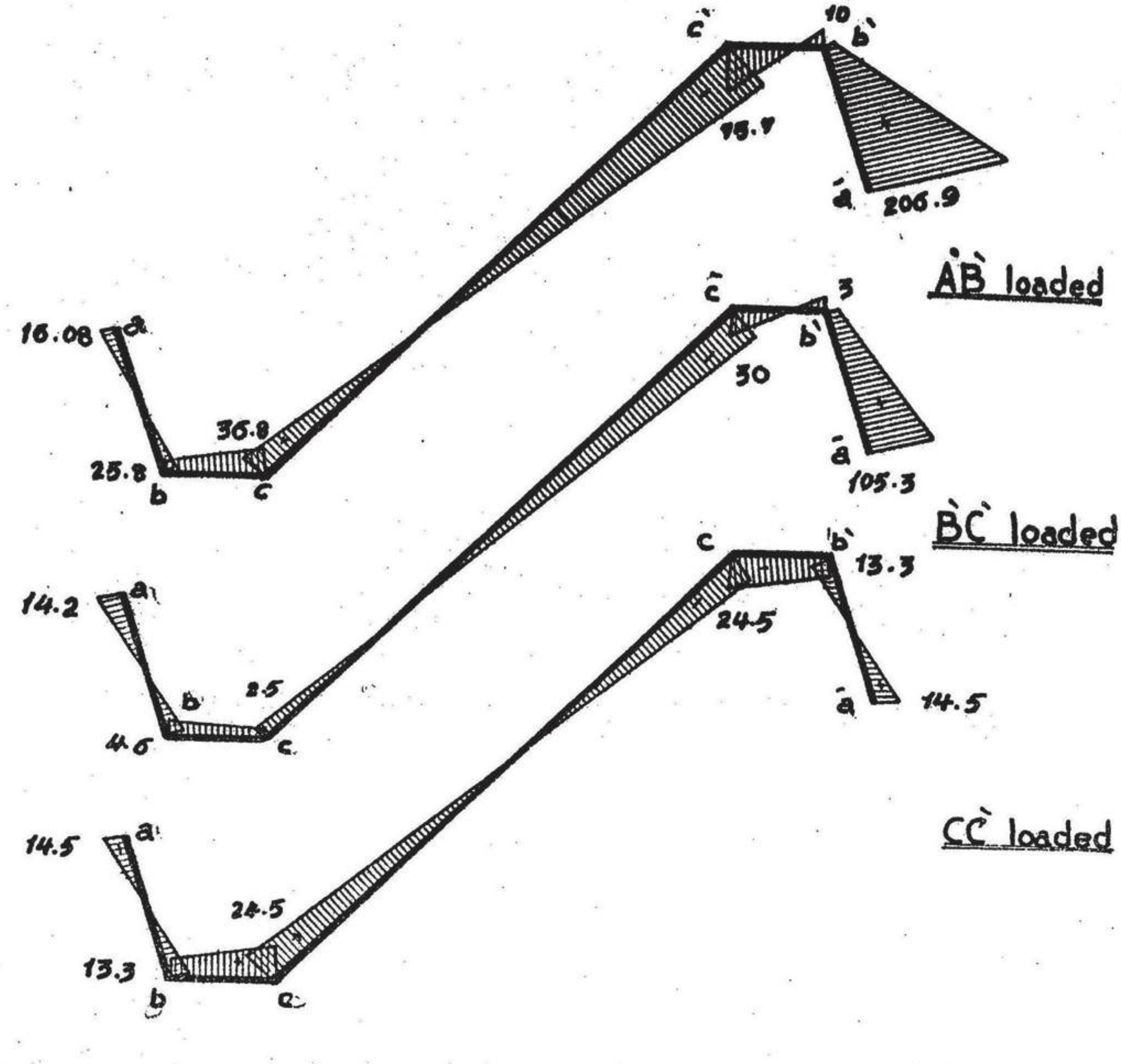
$$= 1.495 \times 10^{-2}\,\frac{L^2}{E}$$

Corrected Diagram

for each plate



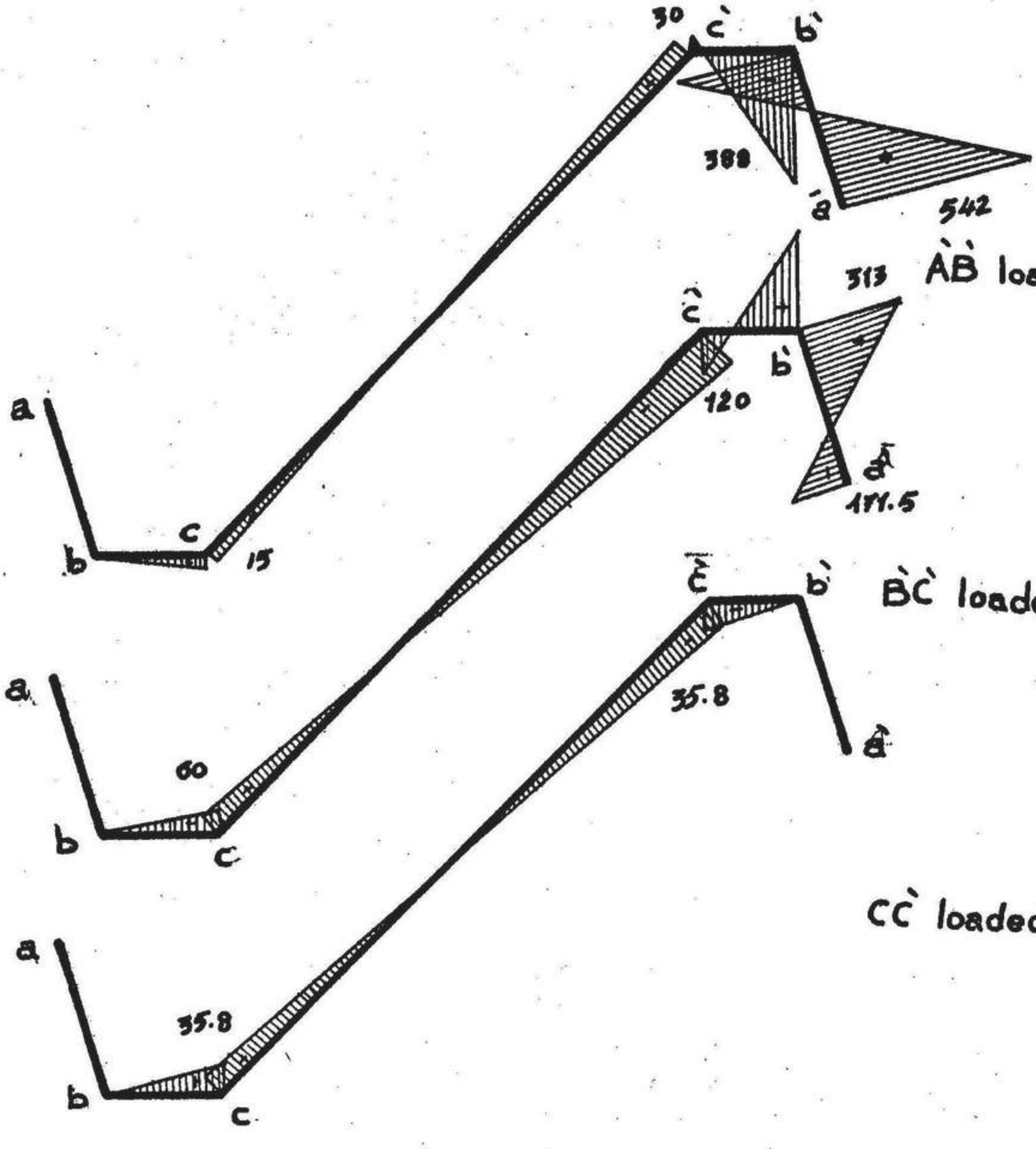
Final Stresses



e - Proposed Simplifications

Engineers always tend to simplify complicated problems . Terms of small effects are neglected. Refering to the general solution we show that stresses induced in plates ab or bc when plate ab is loaded are very small. So these two plates are omitted. In general , when loading any plate , not far than two plates are considered. The results obtained using this simplification are acceptable. The deviations from the usual solution do not exceed five percent as shown in the next sheets .

Elementary Analysis

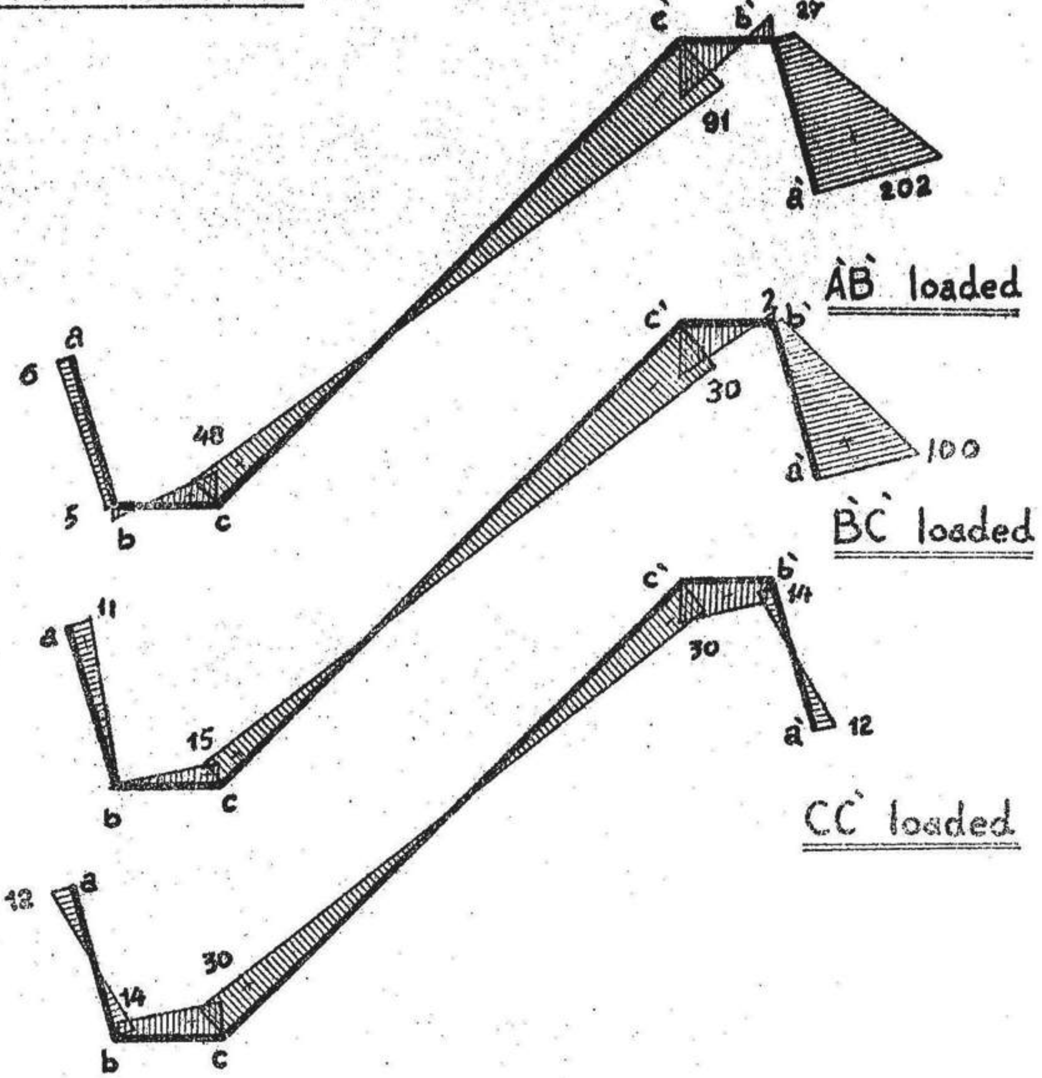


ÀB̀ loaded

B̀C̀ loaded

CC̀ loaded

Final Stresses



C H A P T E R
IV
THE FINITE ELEMENT METHOD

BRIEF STUDY OF THE FINITE ELEMTN METHOD

The finite element method is also used for the theortical investigation of stresses and deflections allover the domain. The structure is separated by imaginary lines into a number of finite elements Fig.(4.1).The elements are assumed to be interconnected at a discrete number of nodal points situated on their boundaries. The displacements of these nodal points are interpreted as the actual displacements of the corresponding points on the structure.

If the displacements at any point within an element "e" can be expressed in terms of the nodal known displacements, in the form of

$$\{f\ (x,y)\} = N.\ \delta$$

where $\{f\}, \{\delta\}$: represent possible movement of a typical point within the element and the corresponding displacement of a node respecitvely .

$[N]$: Position matrix

Then with the aid of this function, both strains and stresses at any point can be determined using the following relationships :

$$\{\varepsilon\} = [B]\cdot\{\delta\}$$
$$\{\sigma\} = [D]\cdot\{\delta\}$$

Where $[B]$ is the strain matrix

$[D]$ is the elasticity matrix

And also the stiffness properties of the assembled structure made up from the idealized elements can be put in the form

$$\{F\} = [K]\cdot[\delta]$$

Where $[K]=\left[\iint [B]^T [D] [B]\, d\,(vol)\right]$ and is called the stiffness matrix of the element

$\{F\}$ = Nodal force vector which are equivalent statically to the boundary stresses and distributed loads on the element.

In the problem of folded plate a difficulty arises when all the elements joining at a particular node are in one plane, because in globle coordinates six equations that are singular are obtained. This is due to the fact that only five of the equations can then be independent, due to the omission of the rotation perpendicular to the plane. For such nodes, the assembly should be made in the local coordinate system.

This means that the equilibrium equations written for the nodal points are referred to two systems of axes.

First for all nodal points situated at the fold lines, the equilibrium equations relating forces to displacements at every point are referred to one globle system X,Y and Z axes and the available degrees of freedom per node are equal to six. The second for all nodal points situated at one common plate, the equilibrium equations are referred to local system X,Y & Z axes corresponding to this plate . The available degrees of freedom per node for this case are equal to five . The transformation process adopted to relate nodal points with five degrees of freedom with other nodal points with six degrees of freedom are as follows :

1- In the type number one where the nodal point "r" lies on fold line and "s" lies on plane plate the equilibrium equation is

$$\{F_r\} = [L]^T [K_{rs}] \{\delta_s\}$$

2- In type number two where nodal point "r" lies on plane plate and "s" lies on fold line the equilibrium equation is :

$$\{F_r\} = [K_{rs}] [L] \{\delta_s\}$$

3- In the type number three where nodal points "r" and "s" lie on common plate the equilibrium equation is.

$$\left\{ F_r \right\} = \left[K_{rs} \right] \left\{ \delta s \right\}$$

4- In the type number four where nodal points "r" and "s" lie on the fold line the equilibrium equation is

$$\left\{ F_r \right\} = \left[L \right]^T \left[K_{rs} \right] \left[L \right] \left\{ \delta s \right\}$$ 1

Where L is given by

$$\left[L \right] = \left[\begin{array}{ccc|ccc} \lambda_{x\bar{x}} & \lambda_{x\bar{y}} & \lambda_{x\bar{z}} & & & \\ \lambda_{y\bar{x}} & \lambda_{y\bar{y}} & \lambda_{y\bar{z}} & & & \\ \lambda_{z\bar{x}} & \lambda_{z\bar{y}} & \lambda_{z\bar{z}} & & & \\ \hline & & & \lambda_{x\bar{x}} & \lambda_{x\bar{y}} & \lambda_{x\bar{z}} \\ & & & \lambda_{y\bar{x}} & \lambda_{y\bar{y}} & \lambda_{y\bar{z}} \\ & & & \lambda_{z\bar{x}} & \lambda_{z\bar{y}} & \lambda_{z\bar{z}} \end{array} \right]$$ 2

In which λ is the direction cosine between axes.
For the right hand system of axes shown in Fig.(4-2).
direction cosines of X axes are : 3

$$\lambda_{x\bar{x}} = 1$$
$$\lambda_{x\bar{y}} = 0$$
$$\lambda_{x\bar{z}} = 0$$

The direction cosines of the y axis in terms of the co-ordinates of the various model points are :

$$\lambda_{y\bar{x}} = 0$$

$$\lambda_{y\bar{y}} = \frac{\bar{y}_j - \bar{y}_i}{\sqrt{(\bar{z}_j - \bar{z}_i)^2 + (\bar{y}_j - \bar{y}_i)^2}}$$

$$\lambda_{y\bar{z}} = \frac{\bar{z}_j - \bar{z}_i}{\sqrt{(\bar{z}_j - \bar{z}_i)^2 + (\bar{y}_j - \bar{y}_i)^2}}$$

Similarly the direction cosines of the Z axis are

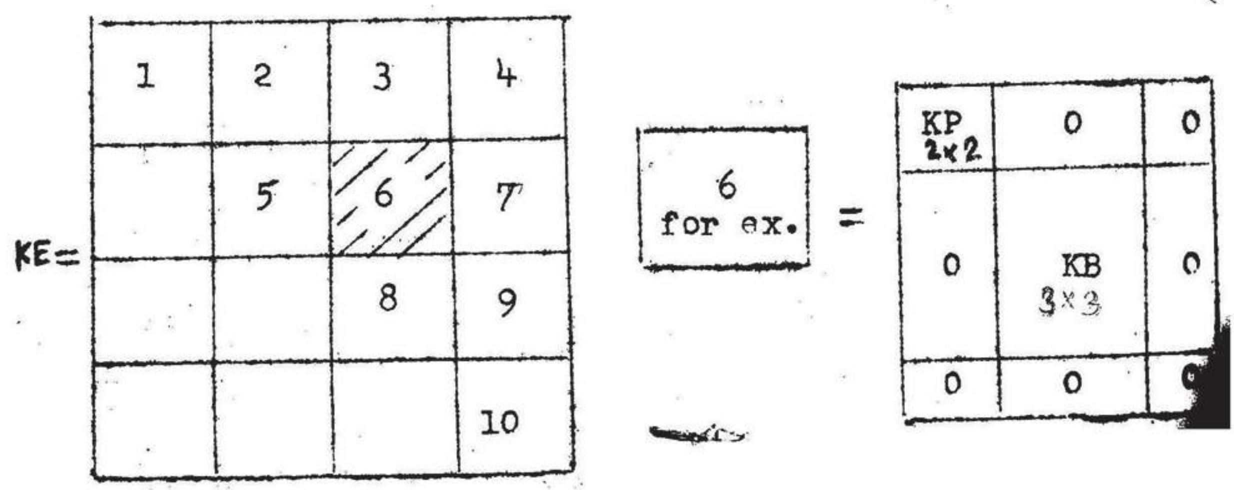
$$\lambda_{z\bar{x}} = 0$$

$$\lambda_{z\bar{y}} = - \frac{\bar{z}_j - \bar{z}_i}{\sqrt{(\bar{z}_j - \bar{z}_i)^2 + (\bar{y}_j - \bar{y}_i)^2}}$$

$$\lambda_{z\bar{z}} = \frac{\bar{y}_j - \bar{y}_i}{\sqrt{(\bar{z}_j - \bar{z}_i)^2 + (\bar{y}_j - \bar{y}_i)^2}}$$

The main steps required for the analysis are :

1- The space stiffness matrix of element(KE) is composed of number of sub-matrices. Each of these matrices is formed from the inplane and out of plane stiffness matrices (KP & KB).



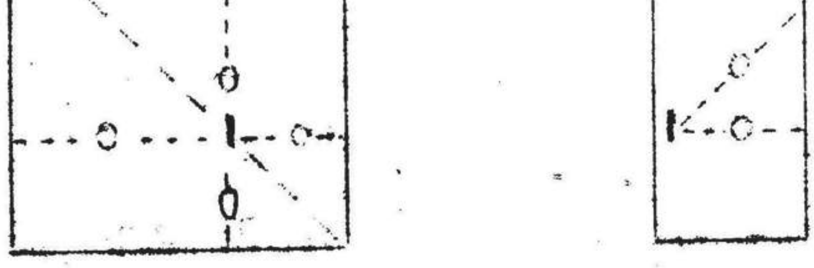
2- Using the foregoing equilibrium equations, each sub-martix is transformed to the global axes according to its type.

3- The space rectangular element in the global axes(K_1) formed in step two can be assembled to form the globle stiffness matix (KK). This matrix is banded to have a reasonable core size in the computer.

4 - Imposing the boundary conditions.

5 - The results.

Clearly, without substitution of the minimum number of prescribed displacements to prevent rigid body movement of the structure, it is impossible to solve this system. A unique solution will be presented by multiplying raw and column corresponding to each prescribed support displacement by zero number. The diagonal coefficient of the matrix(K) for the prescribed displacement is developed by unity number. This is equivalent to reducing the number of equilibrium equations.



K KK

The boundary condition used for the folded plate shown in Fig.(4-1) are:

a- For nodal points on the end diaphragm:

Linear displacements : $u \neq 0$, $v = 0$ & $w = 0$

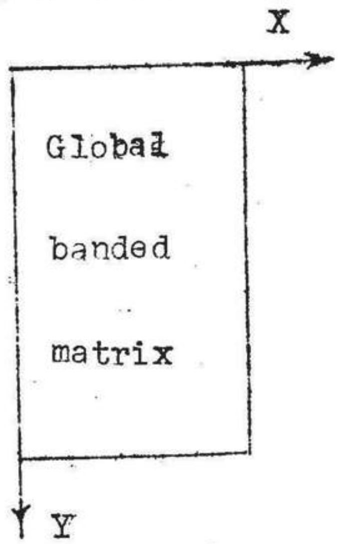
Rotations : $\theta_x = 0$; $\theta_y \neq 0$ & $\theta_z \neq 0$

b- For nodal points on axix of symmetry parallel to y-axis:

Linear displacements : $u = 0$, $v \neq 0$ & $w \neq 0$.

Rotations : $\theta_x \neq 0$, $\theta_y = 0$ & $\theta_z = 0$

The program possesas a special technic to form the global banded stiffness matrix.

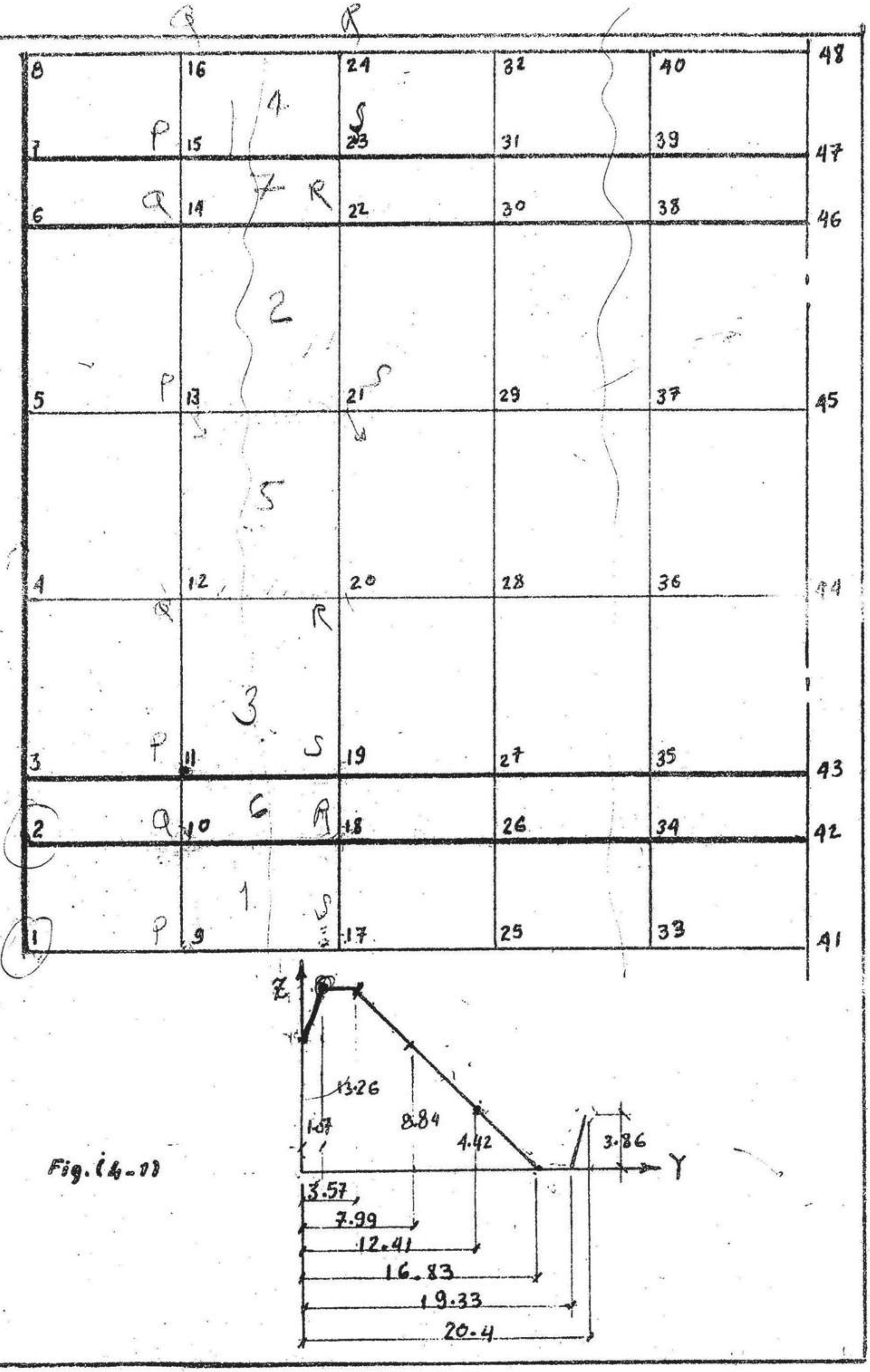


The vertical of any node say "J" is determined by ~~the~~ Summing summation of the degrees of freedom from joint one to the joint number (J-1). The Joint "J" will occupy a distance in the y direction equal to its degree of freedom.

The abscissa of the node "21" for example in the Fig(4-1) element 12-13-21-20 will begin by summing the degrees of freedom from joint 12 till joint 20 i.e. (5+5+6+6+5+5+6+6+5)=49 and this node will occupy a distance in the x direction equal to its degree of freedom. For any element the maximum abscissa is called the band width.

The input data contains nodal number, nodal point local and global co-ordinates, number of element types, number of elements of the same type, nodal loads, Material properties and boundary conditions.

The output data gives nodal points in-plane and out of plane displacements and rotations. The displacements, of nodes on fold lines are referred to the global axes while for the other nodes, the displacements are referred to the plate local axes, displacements of nodes on fold lines are then transferred to the local axis of each of the adjoining plates. Stresses σ_x, σ_y & σ_{xy} at different nodal points are calculated. Also the moments M_x, M_y & M_{xy} are calculated at all nodal points.



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6
1
Z
Y
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8.84
4.42
3.86
3.57
7.99
12.41
16.83
19.33
20.4

Fig. (4-11)

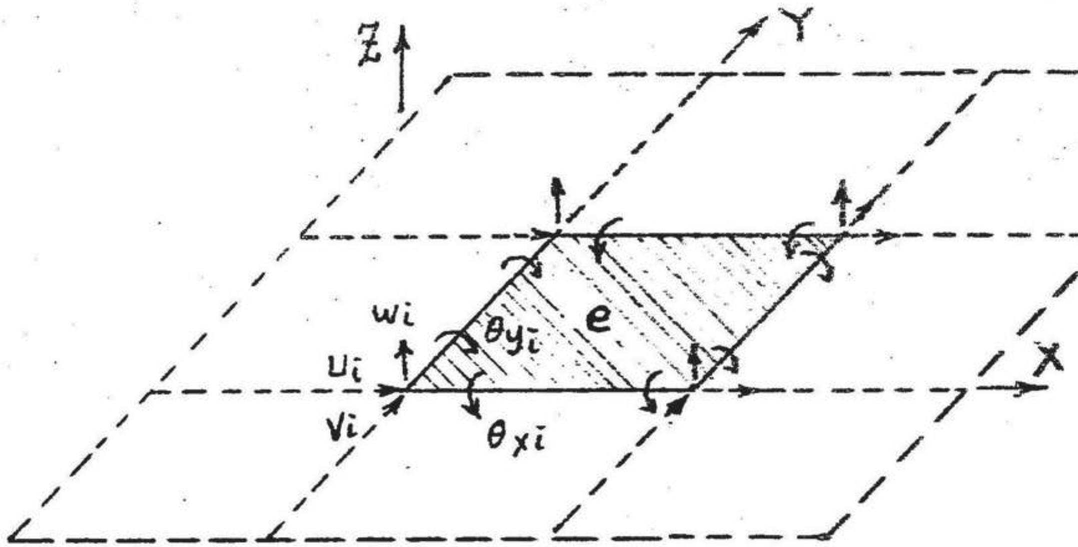


Fig. (4-2)

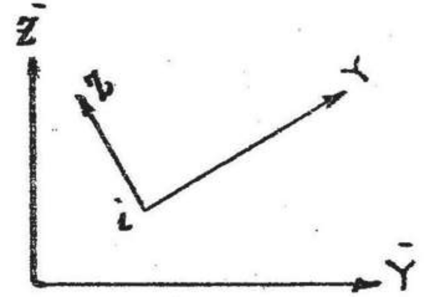


Fig. (4-3)

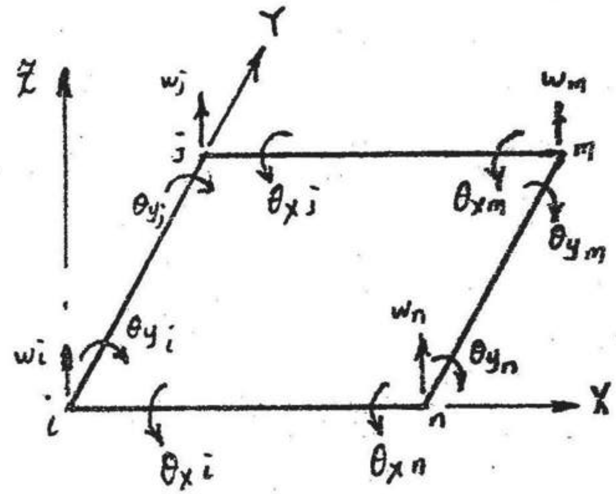


Fig. (4-4)

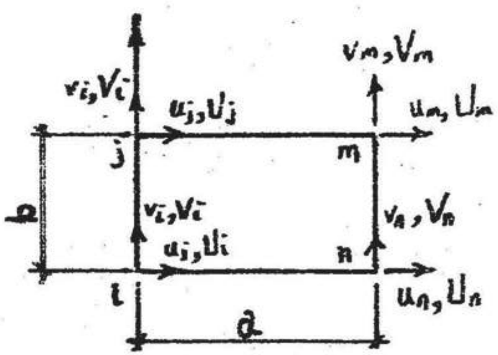


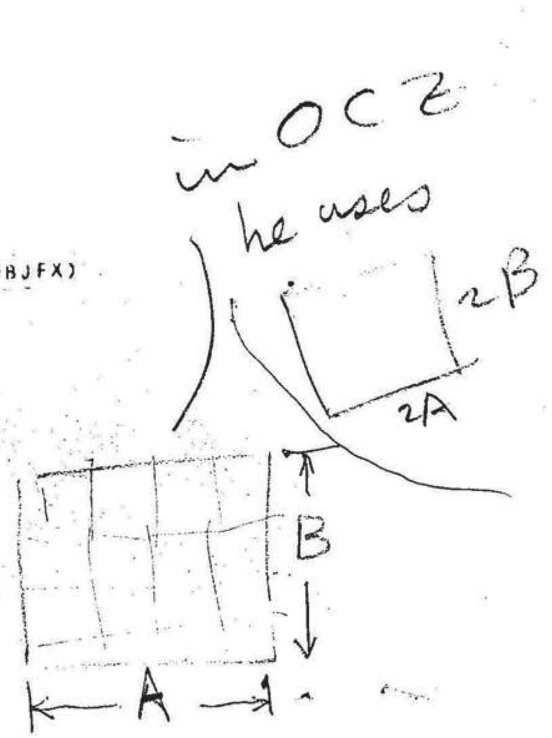
Fig. (4-5)

2- COMPUTER PROGRAM

```
FORTRAN COMPILATION BY #XFAI MK 4C     DATE  18/01/75  TIME  13/20/47

          SEND TO (SEMICOMPILED.ZZZZ)
         PRUGRAM(APMD)
         INPUT1=CRU
         OUTPUT 2=LP0/160
         TRACE 0
         END

         MASTER TEST1
         REAL MUA,KK,KP,KB,KE,ZS
         INTEGER H,HH,ST
         INTEGER RL,SOR,D,SUN1,SUN2,SUN3,SUN4,SUM1,SUM2,SUM3,SUM4,SUM5,SUM6
         INTEGERSOZ,SOH,SOL
         INTEGERP,Q,R,S,Z,TP1,TP2,TP3,TP4
         REAL K1
         INTEGERRAW,COL,BAND6
         DIMENSION NBL(7),I1(8,7),I2(8,7),I3(8,7),I4(8,7),X(16),Y(16),X1(16
        1),Y1(16),Z1(16),HH(30),COL(96),RAW(96),KP(10,2,2),KB(10,3,3),KE(10
        2,6,6),A1(6,6),B1(6,6),K1(10,6,6),D1(10,6,6),D2(10,6,6),
        3D3(10,6,6),JL(48),SZ(48,6),RL(30,6),DS(265,1),F(365,1)
         DIMENSION S1(56,1),S2(56,1),S3(56,1),S4(56,1),S5(56,1),S6(56,1)
         COMMON/ARRAY/KK(265,57)/LUD DIS/SS(265,1),ZS(265,1)
         READ(1,1),NBJ,NBT,NBJFX,LP,TP1,TP2,Tp3,TP4
       1 FORMAT(21I0)
         READ(1,2)MUA,E,TH
       2 FORMAT(12F0.0)
         READ(1,3)
```




```
      3 FORMAT(20I0)
        READ(1,1)((HH(S),(RL(S,I),I=1,6)),S=1,NBJFX)
        DO 6 I=1,NBJ
      6 RAW(I)=5
        RAW(2),RAW(3),RAW(6),RAW(7)=6
        DO 7003 I=8,40,8
   7003 RAW(3+I),RAW(7+I),RAW(2+I),RAW(6+I)=6
        SOR=0
        DO 12 I=1,NBJ
     12 SOR=SOR+RAW(I)
        BAND6=56
        DO 14 I=1,SOR
        F(I,1),ZS(I,1)=0.0
        DO 13 J=1,BAND6
     13 KK(I,J)=0.0
     14 CONTINUE
        VAL=0.0
   1000 DO 2000 Z=1,NBJ
        P=I1(1,Z)
        Q=I2(1,Z)
        R=I3(1,Z)
        S=I4(1,Z)
        READ(1,4)X(P),Y(P),X(Q),Y(Q),X(R),Y(R),X(S),Y(S)
      4 FORMAT(8F0.0)
        READ(1,2)X1(P),Y1(P),Z1(P),X1(Q),Y1(Q),Z1(Q),X1(R),Y1(R),Z1(R),
       1X1(S),Y1(S),Z1(S)
        BB=Y(Q)-Y(P)
        AA=X(S)-X(P)
        B=BB/AA
        BP=AA/BB
        A=(E*TH)/(12.*(1-(MUA**2)))
        AB=(E*(TH**3))/(12.*(1.-(MUA**2))*AA*BB)
        AS=B/(AA*BB*(1.-(MUA**2.)))
        DX,DY=((E*(TH**3))/(12.*(1.-(MUA**2))))
        DXY=((1.-MUA)/2.)*DX
        D01=MUA*DX
        IF(VAL.EQ.1.0) GO TO 6000
   6000 CONTINUE
        KP(1,1,1),KP(5,1,1),KP(8,1,1),KP(10,1,1)=4.*B*A+((2./B)*(1.-MUA))
       1*A
        KP(1,1,2),KP(1,2,1),KP(7,1,2),KP(7,2,1),KP(8,1,2),KP(8,2,1)=
       1(3./2.)*(1.+MUA)*A
        KP(1,2,2),KP(5,2,2),KP(8,2,2),KP(10,2,2)=((4./B)+(2.*B*
       1(1.-MUA)))*A
        KP(2,1,1),KP(9,1,1)=((2.*B)-((2./B)*(1-MUA)))*A
        KP(2,1,2),KP(4,2,1),KP(6,1,2),KP(9,1,2)=(-3./2.)*(1.-3.*MUA)*A
        KP(2,2,1),KP(4,1,2),KP(6,2,1),KP(9,2,1)=(3./2.)*(1.-3.*MUA)*A
        KP(2,2,2),KP(9,2,2)=((-4./B)+(1.-MUA)*B)*A
        KP(3,1,1),KP(7,1,1)=(-(2.*B)-((1.-MUA)/B))*A
        KP(3,1,2),KP(3,2,1),KP(5,1,2),KP(5,2,1),KP(10,1,2),KP(10,2,1)
       1=((-3./2.)*(1.+MUA))*A
        KP(3,2,2),KP(7,2,2)=(-(2./B)-((1.-MUA)*B))*A
        KP(4,1,1),KP(6,1,1)=(-(4.*B)+((1.-MUA)/B))*A
        KP(4,2,2),KP(6,2,2)=((2./B)-(2.*B*(1.-MUA)))*A
        KB(1,1,1),KB(5,1,1),KB(8,1,1),KB(10,1,1)=
       1(AB*(4.*(B**2+(1./(B**2)))+(1./5.)*(14.-(4.*MUA))))
        KB(1,2,1),KB(1,1,2),KB(10,2,1),KB(10,1,2)=
       1((2./(B**2))+(1./5.)*(1.+(4.*MUA)))*AB*BB
        KB(1,3,1),KB(1,1,3),KB(5,3,1),KB(5,1,3)=
       1(-AA)*AB*((2.*(B**2))+(1./5.)*(1.+4.*MUA))
        KB(1,2,2),KB(5,2,2),KB(8,2,2),KB(10,2,2)=
       1(BB**2)*AB*((4.0/3.)*(1./(B**2))+(4./15.)*(1.-MUA))
        KB(1,3,2),KB(1,2,3),KB(8,3,2),KB(8,2,3)=((-MUA)*AB*AA*BB)
        KB(1,3,3),KB(5,3,3),KB(8,3,3),KB(10,3,3)=(AA**2)*AB*((4./3.)*
```

$$\left(\frac{Et^3}{12(1-\nu^2)ab}\right) = \frac{D}{ab}$$

Kp — plane stress

KB — plate

```
     1(B**2)+(4./15.)*(1.-MUA))
      KB(2,1,1),KB(9,1,1)=AB*(2.*((B**2)-2./(B**2))-
     1(1./5.)*(14.-(4.*MUA)))
      KB(2,1,2),KB(9,2,1)=(-BB)*AB*((2./(B**2))+
     1(1./5.)*(1.-MUA))
      KB(2,1,3),KB(2,3,1)=AA*AB*(-(B**2)+
     1(1./5.)*(1.+(4.*MUA)))
      KB(2,2,1),KB(9,1,2)=BB*((2./(B**2))+(1./5.)*
     1(1.-MUA))*AB
      KB(2,2,2),KB(9,2,2)=((BB**2)*AB*((2./3.)*(1./(B**2))-(1./15.)*
     1(1.-MUA)))
      KB(2,2,3),KB(2,3,2),KB(3,3,2),KB(3,2,3),KB(4,2,3),
     1KB(4,3,2),KB(6,2,3),KB(6,3,2),KB(7,2,3),KB(7,3,2),
     2KB(9,2,3),KB(9,3,2)=0.0
      KB(5,2,1),KB(5,1,2),KB(8,2,1),KB(8,1,2)=(-AB)*BB*
     1((2./(B**2))+(1./5.)*(1.+(4.*MUA)))
      KB(2,3,3),KB(9,3,3)=AB*(AA**2)*((2./3.)*(B**2)-(4./15.)
     1*(1-MUA))
      KB(5,3,2),KB(5,2,3),KB(10,3,2),KB(10,2,3)=MUA*AA*BB*AB
      KB(5,1,1),KB(7,1,1)=AB*(-(2.*((B**2)+(1./(B**2))))+
     1(1./5.)*(14.-(4.*MUA)))
      KB(3,1,2),KB(7,2,1)=BB*AB*((-1./(B**2))+(1./5.)*(1.-MUA))
      KB(3,1,3),KB(7,1,3)=AA*AB*((B**2)-(1./5.)*(1.-MUA))
      KB(6,1,1),KB(4,1,1)=AB*(-(2.)*(2.*(B**2)-(1./(B**2)))
     1-(1./5.)*(14.-(4.*MUA)))
      KB(6,1,2),KB(6,2,1)=BB*AB*(-(1./(B**2))+((1./5.)*(1.+4.*MUA)))
      KB(6,1,3),KB(4,1,3)=AA*AB*(2.*(B**2)+(1./5.)*(1.-MUA))
      KB(3,2,1),KB(7,1,2)=BB*AB*(1./(B**2)-(1./5.)*(1.-MUA))
      KB(3,2,2),KB(7,2,2)=(BB**2)*AB*((1./(3.*(B**2)))
     1+(1./15.)*(1.-MUA))
      KB(6,2,2),KB(4,2,2)=(BB**2)*AB*((2./3.)*(1./(B**2))-
     1(4./15.)*(1.-MUA))
      KB(3,3,1),KB(7,3,1)=AA*AB*(-(B**2)+(1./5.)*(1.-MUA))
      KB(3,3,3),KB(7,3,3)=(AA**2)*AB*(((B**2)/3.)+
     1(1./15.)*(1.-MUA))
      KB(6,3,1),KB(4,3,1)=(-AA)*AB*(2.*(B**2)+(1./5.)*(1.-MUA))
      KB(6,3,3),KB(4,3,3)=(AA**2)*AB*((2./3.)*(B**2)-(1./15.)*(1.-MUA))
      KB(8,3,1),KB(8,1,3),KB(10,1,3),KB(10,3,1)=AA*AB*(2.*(
     1B**2)+(1./5.)*(1.+(4.*MUA)))
      KB(4,1,2),KB(4,2,1)=BB*AB*((1./(B**2)-(1./5.)*(1.+4.*MUA)))
      KB(9,1,3),KB(9,3,1)=AA*AB*((B**2)-(1./5.)*(1.+(4.*MUA)))
      IF(VAL.EQ.0) GO TO 6001
 6001 DO 19 I=1,10
      DO 18 J=1,6
      DO 17 L=1,6
   17 KE(I,J,L),K1(I,J,L)=0.0
   18 CONTINUE
   19 CONTINUE
      DO 20 I=1,10
      KE(I,1,1)=KP(I,1,1)
      KE(I,1,2)=KP(I,1,2)
      KE(I,2,1)=KP(I,2,1)
      KE(I,2,2)=KP(I,2,2)
      KE(I,3,3)=KB(I,1,1)
      KE(I,3,4)=KB(I,1,2)
      KE(I,3,5)=KB(I,1,3)
      KE(I,4,3)=KB(I,2,1)
      KE(I,4,4)=KB(I,2,2)
      KE(I,4,5)=KB(I,2,3)
      KE(I,5,3)=KB(I,3,1)
      KE(I,5,4)=KB(I,3,2)
      KE(I,5,5)=KB(I,3,3)
 1010 FORMAT(6(5X,F13.7))
   20 CONTINUE
 6002 CONTINUE
      DD=SQRT((Y1(Q)-Y1(P))**2+(Z1(Q)-Z1(P))**2)
```

DØ

```
      YY=Y1(Q)-Y1(P)
      ZZ=Z1(Q)-Z1(P)
      DO 24 I=1,6
      DO 23 J=1,6
   23 A1(I,J),B1(I,J)=0.0
   24 CONTINUE
      A1(1,1),A1(4,4),B1(1,1),B1(4,4)=1.0
      B1(2,2),B1(5,5),A1(2,2),A1(5,5)=YY/DD
      B1(2,3),B1(5,6),A1(3,2),A1(6,5)=ZZ/DD
      B1(3,2),B1(6,5),A1(2,3),A1(5,6)=-(ZZ/DD)
      B1(3,3),B1(6,6),A1(3,3),A1(6,6)=+(YY/DD)
      IF(Z.LE.TP1) GO TO 25
      IF(Z.LE.TP2) GO TO 42
      IF(Z.LE.TP3) GO TO 58
      IF(Z.LE.TP4) GO TO 300
      GO TO 5000
   25 DO 28 NK=1,6
      DO 27 M=1,5
      V,V1=0.0
      DO26L=1,6
      V=V+A1(NK,L)*KE(2,L,M)
   26 V1=V1+A1(NK,L)*KE(3,L,M)
      K1(2,NK,M)=V
   27 K1(3,NK,M)=V1
   28 CONTINUE
      DO 31 NK=1,6
      DO30M=1,6
      R12,R1,R2=0.0
      DO 29 L=1,6
      R12=R12+A1(NK,L)*KE(5,L,M)
      R1=R1+A1(NK,L)*KE(6,L,M)
   29 R2=R2+A1(NK,L)*KE(8,L,M)
      D3(5,NK,M)=R12
      D1(6,NK,M)=R1
   30 D2(8,NK,M)=R2
   31 CONTINUE
      DO34 NK=1,5
      DO 33 M=1,6
      W,W1=0.0
      DO 32 L=1,6
      W=W+KE(7,NK,L)*B1(L,M)
   32 W1=W1+KE(9,NK,L)*B1(L,M)
      K1(7,NK,M)=W
   33 K1(9,NK,M)=W1
   34 CONTINUE
      DO37 NK=1,6
      DO 36 M=1,6
      R12,R1,R2=0.0
      DO 35 L=1,6
      R12=R12+D3(5,NK,L)*B1(L,M)
      R1=R1+D1(6,NK,L)*B1(L,M)
   35 R2=R2+D2(8,NK,L)*B1(L,M)
      K1(5,NK,M)=R12
      K1(6,NK,M)=R1
   36 K1(8,NK,M)=R2
   37 CONTINUE
      DO 39 I=1,5
      DO 38J=1,5
      K1(1,I,J)=KE(1,I,J)
      K1(4,I,J)=KE(4,I,J)
   38 K1(10,I,J)=KE(10,I,J)
   39 CONTINUE
      GO TO 1700
   42 DO 45 NK=1,6
      DO44 M=1,5
      V,V1=0.0
```

```
      DO 43 L=1,6
      V=V+A1(NK,L)*KE(7,L,M)
   43 V1=V1+A1(NK,L)*KE(9,L,M)
      K1(7,NK,M)=V
   44 K1(9,NK,M)=V1
   45 CONTINUE
      DO48NK=1,6
      DO47 M=1,6
      R12,R1,R2=0.0
      DO46 L=1,6
      R12=R12+A1(NK,L)*KE(1,L,M)
      R1=R1+A1(NK,L)*KE(4,L,M)
   46 R2=R2+A1(NK,L)*KE(10,L,M)
      D3(1,NK,M)=R12
      D1(4,NK,M)=R1
   47 D2(10,NK,M)=R2
   48 CONTINUE
      DO51NK=1,5
      DO50 M=1,6
      W,W1=0.0
      DO 49 L=1,6
      W=W+KE(2,NK,L)*B1(L,M)
   49 W1=W1+KE(5,NK,L)*B1(L,M)
      K1(2,NK,M)=W
   50 K1(5,NK,M)=W1
   51 CONTINUEE
      DO 54 NK=1,6
      DO 53 M=1,6
      R12,R1,R2=0.0
      DO 52 L=1,6
      R12=R12+D3(1,NK,L)*B1(L,M)
      R1=R1+D1(4,NK,L)*B1(L,M)
   52 R2=R2+D2(10,NK,L)*B1(L,M)
      K1(1,NK,M)=R12
      K1(4,NK,M)=R1
   53 K1(10,NK,M)=R2
   54 CONTINUE
      DO 56 I=1,5
      DO 55 J=1,5
      K1(5,I,J)=KE(5,I,J)
      K1(8,I,J)=KE(8,I,J)
   55 K1(6,I,J)=KE(6,I,J)
   56 CONTINUE
   57 FORMAT(5X,'STIF.MAT.TYPE2')
      GO TO 1700
   58 DO 61 L=1,10
      DO 60 I=1,5
      DO 59 J=1,5
   59 K1(L,I,J)=KE(L,I,J)
   60 CONTINUE
   61 CONTINUE
      GO TO 1700
  300 DO304L=1,10
      DO303N=1,6
      DO 302I=1,6
      R12=0.0
      DO301J=1,6
  301 R12=R12+A1(N,J)*KE(L,J,I)
  302 D1(L,N,I)=R12
  303 CONTINUE
  304 CONTINUE
      DO308L=1,10
      DO 307N=1,6
      DO 306I=1,6
      R12=0.0
      DO 305J=1,6
```

```
  305 R12=R12+D1(L,N,J)*B1(J,I)
  306 K1(L,N,I)=R12
  307 CONTINUE
  308 CONTINUE
 1700 CONTINUE
      DO 1900 D=1,NBL(Z)
      P=I1(D,Z)
      Q=I2(D,Z)
      R=I3(D,Z)
      S=I4(D,Z)
      SUN1,SUN2,SUN3,SUN4,SUM1,SUM2,SUM3,SUM4,SUM5,SUM6=0
      IF(P.EQ.1) GO TO 1006
      DO 65 I=1,P-1
   65 SUN1=RAW(I)+SUN1
 1006 CONTINUE
      DO 66 I=1,Q-1
   66 SUN2=RAW(I)+SUN2
      DO 67 I=1,R-1
   67 SUN3=RAW(I)+SUN3
      DO 68 I=1,S-1
   68 SUN4=RAW(I)+SUN4
      DO 69 I=1,Q-P
   69 SUM1=RAW(I+P-1)+SUM1
      DO 70 I=1,R-P
   70 SUM2=RAW(I+P-1)+SUM2
      DO 71 I=1,S-P
   71 SUM3=RAW(I+P-1)+SUM3
      DO 72 I=1,R-Q
   72 SUM4=RAW(I+Q-1)+SUM4
      DO 73 I=1,S-Q
   73 SUM5=RAW(I+Q-1)+SUM5
      DO 74 I=1,R-S
   74 SUM6=RAW(I+S-1)+SUM6
      IF(VAL.EQ.1.0) GO TO 1600
      IF(Z.LE.TP1) GO TO 76
      IF(Z.LE.TP2) GO TO 85
      IF(Z.LE.TP3) GO TO 95
      IF(Z.LE.TP4) NO1=6
      GO TO 700
   76 NO1=5
      DO 80 I=1,NO1
      DO 76 J=1,NO1
      KK(SUN1+I,SUM5+J-I+1)=KK(SUN1+I,SUM5+J-I+1)+K1(4,J,I)
      KO=J-I+1
      IF(KO)78,78,77
   77 KK(SUN1+I,KO)=KK(SUN1+I,KO)+K1(1,I,J)
      KK(SUN4+I,KO)=KK(SUN4+I,KO)+K1(10,I,J)
   78 CONTINUE
      DO 79 J=1,6
      KK(SUN1+I,SUM1+J-I+1)=KK(SUN1+I,SUM1+J-I+1)+K1(2,J,I)
      KK(SUN1+I,SUM2+J-I+1)=KK(SUN1+I,SUM2+J-I+1)+K1(3,J,I)
   79 KK(SUN4+I,SUM6+J-I+1)=KK(SUN4+I,SUM6+J-I+1)+K1(9,I,J)
   80 CONTINUE
      DO 84 I=1,6
      DO 81 J=1,NO1
   81 KK(SUN2+I,SUM5+J-I+1)=KK(SUN2+I,SUM5+J-I+1)+K1(7,J,I)
      DO 83 J=1,6
      KO=J-I+1
      IF(KO)83,83,82
   82 KK(SUN2+I,KO)=KK(SUN2+I,KO)+K1(5,I,J)
      KK(SUN3+I,KO)=KK(SUN3+I,KO)+K1(8,I,J)
   83 KK(SUN2+I,SUM4+J-I+1)=KK(SUN2+I,SUM4+J-I+1)+K1(6,J,I)
   84 CONTINUE
      GO TO 100
   85 NO1=5
      DO 89 I=1,NO1
```

```
      DO 87 J=1,N01
      KO=J-I+1
      IF(KO)87,87,86
   86 KK(SUN2+I,KO)=KK(SUN2+I,KO)+K1(5,I,J)
      KK(SUN3+I,KO)=KK(SUN3+I,KO)+K1(8,I,J)
   87 KK(SUN2+I,SUM4+J-I+1)=KK(SUN2+I,SUM4+J-I+1)+K1(6,J,I)
      DO 88 J=1,6
88    KK(SUN2+I,SUM5+J-I+1)=KK(SUN2+I,SUM5+J-I+1)+K1(7,J,I)
   89 CONTINUE
      DO 93 I=1,6
      DO 90 J=1,N01
      KK(SUN1+I,SUM1+J-I+1)=KK(SUN1+I,SUM1+J-I+1)+K1(2,J,I)
      KK(SUN1+I,SUM2+J-I+1)=KK(SUN1+I,SUM2+J-I+1)+K1(3,J,I)
   90 KK(SUN4+I,SUM6+J-I+1)=KK(SUN4+I,SUM6+J-I+1)+K1(9,I,J)
      DO 92 J=1,6
      KO=J-I+1
      IF(KO)92,92,91
   91 KK(SUN1+I,KO)=KK(SUN1+I,KO)+K1(1,I,J)
      KK(SUN4+I,KO)=KK(SUN4+I,KO)+K1(10,I,J)
   92 KK(SUN1+I,SUM3+J-I+1)=KK(SUN1+I,SUM3+J-I+1)+K1(4,J,I)
   93 CONTINUE
      GO TO 100
   95 N01=5
  700 CONTINUE
      DO 99 I=1,N01
      DO 98 J=1,N01
      KO=J-I+1
      IF(KO)97,97,96
   96 KK(SUN1+I,KO)=KK(SUN1+I,KO)+K1(1,I,J)
      KK(SUN2+I,KO)=KK(SUN2+I,KO)+K1(5,I,J)
      KK(SUN3+I,KO)=KK(SUN3+I,KO)+K1(8,I,J)
      KK(SUN4+I,KO)=KK(SUN4+I,KO)+K1(10,I,J)
   97 KK(SUN1+I,SUM1+J-I+1)=KK(SUN1+I,SUM1+J-I+1)+K1(2,J,I)
      KK(SUN1+I,SUM2+J-I+1)=KK(SUN1+I,SUM2+J-I+1)+K1(3,J,I)
      KK(SUN2+I,SUM4+J-I+1)=KK(SUN2+I,SUM4+J-I+1)+K1(6,J,I)
      KK(SUN2+I,SUM5+J-I+1)=KK(SUN2+I,SUM5+J-I+1)+K1(7,J,I)
      KK(SUN1+I,SUM3+J-I+1)=KK(SUN1+I,SUM3+J-I+1)+K1(4,J,I)
   98 KK(SUN4+I,SUM6+J-I+1)=KK(SUN4+I,SUM6+J-I+1)+K1(9,I,J)
   99 CONTINUE
  100 IF(VAL.EQ.0.0) GO TO 1900
 1600 DO 1800 L=1,LP
      DO 111 I=1,5
  111 DS((P-1)*5+I,L),DS((Q-1)*5+I,L),DS((R-1)*5+I,L),DS((S-1)*5+I,L)=0
      IF(Z.LE.TP1) GO TO 101
      IF(Z.LE.TP2) GO TO 104
      IF(Z.LE.TP3) GO TO 107
      IF(Z.LE.TP4) GO TO 380
  101 N01=5
      DO 103 I=1,N01
      DS((P-1)*5+I,L)=ZS(SUN1+I,L)
      DS((S-1)*5+I,L)=ZS(SUN4+I,L)
      C,O=0.0
      DO 102 J=1,6
      C=C+B1(I,J)*ZS(SUN2+J,L)
  102 O=O+B1(I,J)*ZS(SUN3+J,L)
      DS((Q-1)*5+I,L)=C
  103 DS((R-1)*5+I,L)=O
      GO TO 110
  104 DO 106 I=1,5
      DS((Q-1)*5+I,L)=ZS(SUN2+I,L)
      DS((R-1)*5+I,L)=ZS(SUN3+I,L)
      C,O=0.0
      DO 105 J=1,6
      C=C+B1(I,J)*ZS(SUN1+J,L)
  105 O=O+B1(I,J)*ZS(SUN4+J,L)
      DS((P-1)*5+I,L)=C
```

```
  106 DS((S-1)*5+1,L)=0
      GO TO 110
  107 NO1=5
      DO 108 I=1,NO1
      DS((P-1)*5+I,L)=ZS(SUN1+I,L)
      DS((Q-1)*5+I,L)=ZS(SUN2+I,L)
      DS((R-1)*5+I,L)=ZS(SUN3+I,L)
  108 DS((S-1)*5+I,L)=ZS(SUN4+I,L)
      GO TO 110
  380 DO 400 I=1,5
      C,O,CC,OO=0.0
      DO 390 J=1,6
      C=C+B1(I,J)*ZS(SUN1+J,L)
      O=O+B1(I,J)*ZS(SUN2+J,L)
      CC=CC+B1(I,J)*ZS(SUN3+J,L)
  390 OO=OO+B1(I,J)*ZS(SUN4+J,L)
      DS((P-1)*5+I,L)=C
      DS((Q-1)*5+I,L)=O
      DS((R-1)*5+I,L)=CC
  400 DS((S-1)*5+I,L)=OO
  110 CONTINUE
      S1(P,L),S2(P,L),S3(P,L),S4(P,L),S5(P,L),S6(P,L),
     1S1(Q,L),S2(Q,L),S3(Q,L),S4(Q,L),S5(Q,L),S6(Q,L),S1(R,L),S2(R,L),S3(
     2(R,L),S4(R,L),S5(R,L),S6(R,L),S1(S,L),S2(S,L),S3(S,L),
     3S4(S,L),S5(S,L),S6(S,L)=0.
      P1=DS((P-1)*5+1,L)
      P2=DS((P-1)*5+2,L)
      P3=DS((Q-1)*5+1,L)
      P4=DS((Q-1)*5+2,L)
      P5=DS((R-1)*5+1,L)
      P6=DS((R-1)*5+2,L)
      P7=DS((S-1)*5+1,L)
      P8=DS((S-1)*5+2,L)
      BE1=DS((P-1)*5+3,L)
      BE2=DS((P-1)*5+4,L)
      BE3=DS((P-1)*5+5,L)
      BE4=DS((Q-1)*5+3,L)
      BE5=DS((Q-1)*5+4,L)
      BE6=DS((Q-1)*5+5,L)
      BE7=DS((R-1)*5+3,L)
      BE8=DS((R-1)*5+4,L)
      BE9=DS((R-1)*5+5,L)
      BE10=DS((S-1)*5+3,L)
      BE11=DS((S-1)*5+4,L)
      BE12=DS((S-1)*5+5,L)
      BE2=-BE2
      BE3=-BE3
      BE5=-BE5
      BE6=-BE6
      BE8=-BE8
      BE9=-BE9
      BE11=-BE11
      BE12=-BE12
      S1(P,L)=(-BB*P1-MUA*AA*P2+MUA*AA*P4+BB*P7)*AS+S1(P,L)
      S2(P,L)=(-MUA*BB*P1-AA*P2+AA*P4+MUA*BB*P7)*AS+S2(P,L)
      S3(P,L)=(-(1.-MUA)*(AA/2.)*P1-(1.-MUA)*(BB/2.)*P2+(1.-MUA)
     1*(AA/2.)*P3+(1.-MUA)*(BB/2.)*P8)*AS+S3(P,L)
      S1(Q,L)=(-(MUA*AA*P2)-(BB*P3)+(MUA*AA*P4)+(BB*P5))*AS+S1(Q,L)
      S2(Q,L)=(-AA*P2-MUA*BB*P3+AA*P4+MUA*BB*P5)*AS+S2(Q,L)
      S3(Q,L)=(-(1.-MUA)*(AA/2.)*P1+(1.-MUA)*(AA/2.)*P3-(1.-MUA)*
     1(BB/2.)*P4+(1.-MUA)*(BB/2.)*P6)*AS+S3(Q,L)
      S1(R,L)=(-BB*P5+BB*P5+MUA*AA*P6-MUA*AA*P8)*AS+S1(R,L)
      S2(R,L)=(-MUA*BB*P3+MUA*BB*P5+AA*P6-AA*P8)*AS+S2(R,L)
      S3(R,L)=(-(1.-MUA)*P4*(BB/2.)+(1.-MUA)*P5*(AA/2.)+(1.-MUA)
     1*P6*(BB/2.)-(1.-MUA)*P7*(AA/2.))*AS+S3(R,L)
      S1(S,L)=(-BB*P7+MUA*AA*P6-BB*P7-MUA*AA*P8)*AS+S1(S,L)
```

```
      S2(S,L)=(-MUA*BB*P1+AA*P6+MUA*BB*P7-AA*P8)*AS+S2(S,L)
      S3(S,L)=(-(1.-MUA)*P2*(BB/2.)+(1.-MUA)*P5*(AA/2.)-(1.-MUA)
     1*P7*(AA/2.)+(1.-MUA)*P8*(BB/2.))*AS+S3(S,L)
      S4(P,L)=((6.*B*DX+(6.*D01/B))*BE1-4.*AA*D01*BE2+
     14.*BB*DX*BE3-(6.*D01/B)*BE4-2.*AA*D01*BE5-
     26.*B*DX*BE10+2.*BB*DX*BE12)*(1./AA*BB)+S4(P,L)
      S5(P,L)=(((6.*DY/B)+6.*B*D01)*BE1-4.*AA*DY*BE2+
     14.*BB*D01*BE3-(6.*DY*BE4/B)-2.*AA*DY*BE5
     2-6.*B*D01*BE10+2.*BB*D01*BE12)*(1./AA*BB)+S5(P,L)
      S6(P,L)=(-2.*DXY*BE1+2.*BB*DXY*BE2-2.*AA*DXY*
     1BE3+2.*DXY*BE4+2.*AA*DXY*BE6-2.*DXY*BE7
     2+2.*DXY*BE10-2.*BB*DXY*BE11)*(1/AA*BB)+S6(P,L)
      S4(Q,L)=(-(6.*D01*BE1/B)+2.*AA*D01*BE2+(6.*B*DX+(6.
     1*D01/B))*BE4+4.*AA*D01*BE5+4.*BB*DX*BE6-6.
     2*B*DX*BE7+2.*BB*DX*BE9)*(1./AA*BB)+S4(Q,L)
      S5(Q,L)=(-(6.*DY/B)*BE1+2.*AA*DY*BE2+((6.*DY/B)+
     16.*B*D01)*BE4+4.*AA*DY*BE5+4.*BB*D01*BE6
     2-6.*B*D01*BE7+2.*BB*D01*BE9)*(1./AA*BB)+S5(Q,L)
      S6(Q,L)=(-2.*DXY*BE1-6.*AA*DXY*BE3+2.*DXY*BE4+
     12.*BB*DXY*BE5+2.*AA*DXY*BE6-2.*DXY*BE7
     2-2.*BB*DXY*BE8+2.*DXY*BE10)*(1./AA*BB)+S6(Q,L)
      S4(R,L)=(-6.*B*DX*BE4-2.*BB*DX*BE6+6.*(B*DX+(D01
     1/B))*BE7+6.*AA*D01*BE8-4.*BB*DX*BE9-(6.*D01
     2/B)*BE10+2.*AA*D01*BE11)*(1./AA*BB)+S4(R,L)
      S5(R,L)=(-6.*B*D01*BE4-2.*BB*D01*BE6+6.*((DY/
     1B)+B*D01)*BE7+4.*AA*DY*BE8-4.*BB*D01*BE9
     2-6.*(DY/B)*BE10+2.*AA*DY*BE11)*(1./AA*BB)+S5(R,L)
      S6(R,L)=(-2.*DXY*BE1+2.*DXY*BE4+2.*BB*DXY*BE5
     1-2.*DXY*BE7-2.*BB*DXY*BE8+2.*AA*DXY*BE9+2.
     2*DXY*BE10-2.*AA*DXY*BE12)*(1./AA*BB)+S6(R,L)
      S4(S,L)=(-6.*B*DX*BE1-2.*BB*DX*BE3-(6.*D01*BE7
     1/B)-2.*AA*D01*BE8+6.*BE10*(B*DX+D01/B)-4.
     2*AA*D01*BE11-4.*BB*DX*BE12)*(1./AA*BB)+S4(S,L)
      S5(S,L)=(-6.*B*D01*BE1-2.*BB*D01*BE3-6.*(DY/B)
     1*BE7-2.*AA*DY*BE8+6.*BE10*((DY/B)+B*D01)-4.*
     6AA*DY*BE11-4.*BB*D01*BE12)*(1./AA*BB)+S5(S,L)
      S6(S,L)=(-2.*DXY*BE1+2.*DXY*BB*BE2+2.*DXY*BE4-2.*DY
     1*BE7+2.*DXY*AA*BE9+2.*DXY*BE10-2.*BB*DXY*BE11-2.
     2*AA*DXY*BE12)*(1./AA*BB)+S6(S,L)
      WRITE(2,109)P,Z,D,S1(P,L),S2(P,L),S3(P,L),S4(P,L),S5(P,L),S6(P,L)
      WRITE(2,109)Q,Z,D,S1(Q,L),S2(Q,L),S3(Q,L),S4(Q,L),S5(Q,L),S6(Q,L)
      WRITE(2,109)R,Z,D,S1(R,L),S2(R,L),S3(R,L),S4(R,L),S5(R,L),S6(R,L)
      WRITE(2,109)S,Z,D,S1(S,L),S2(S,L),S3(S,L),S4(S,L),S5(S,L),S6(S,L)
  109 FORMAT(3(5X,I2),6(5X,F13.7))
      DO 6007 I=1,6
      Y5,Y2,Y3,Y4=0.
      DO 6008 J=1,6
      Y5=Y5+(K1(1,I,J)*ZS(SUN1+J,L)+K1(2,J,I)*ZS(SUN2+J,L)+K1(3,J,I)*ZS
     1(SUN3+J,L)+K1(4,J,I)*ZS(SUN4+J,L))
      Y2=Y2+(K1(2,I,J)*ZS(SUN1+J,L)+K1(5,I,J)*ZS(SUN2+J,L)
     1+K1(6,J,I)*ZS(SUN3+J,L)+K1(7,J,I)*ZS(SUN4+J,L))
      Y3=Y3+(K1(3,I,J)*ZS(SUN1+J,L)+K1(6,I,J)*ZS(SUN2+J,L)
     1+K1(8,I,J)*ZS(SUN3+J,L)+K1(9,J,I)*ZS(SUN4+J,L))
      Y4=Y4+(+K1(4,I,J)*ZS(SUN1+J,L)+K1(7,I,J)*
     1ZS(SUN2+J,L)+K1(9,I,J)*ZS(SUN3+J,L)+K1(10,I,J)*ZS(SUN4+J,L))
 6008 CONTINUE
      F(SUN1+I,L)=Y5+F(SUN1+I,L)
      F(SUN2+I,L)=Y2+F(SUN2+I,L)
      F(SUN3+I,L)=Y3+F(SUN3+I,L)
 6007 F(SUN4+I,L)=Y4+F(SUN4+I,L)
 1800 CONTINUE
 1900 CONTINUE
 2000 CONTINUE
      IF(VAL.EQ.0.0) GO TO 150
      ISUN=0
```

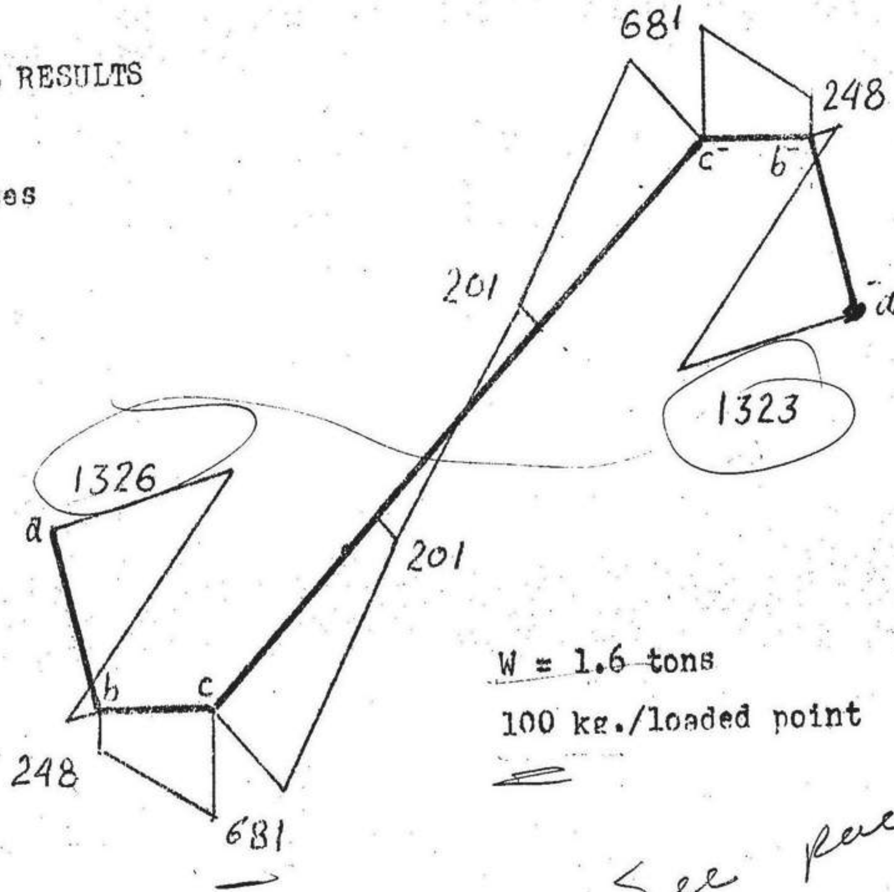
```
      IF(I.EQ.1) GO TO 15
      ISUN=RAW(I-1)+ISUN
   15 WRITE(2,16)I,(ZS(ISUN+J,1),J=1,RAW(I))
   16 FORMAT(5X,I2,6(3X,F12.7))
      ISUN=0
      DO 21 I=1,NBJ
      IF(I.EQ.1) GO TO 21
      ISUN=RAW(I-1)+ISUN
   21 WRITE(2,16)I,(F(ISUN+J,1),J=1,RAW(I))
      GO TO 5000
  130 CONTINUE
      DO 137 S=1,NBJFX
      H=HH(S)
      IF(H.EQ.1) GO TO 132
      SOH=0
      DO 133 I=1,H-1
  133 SOH=SOH+RAW(I)
  132 DO 136 I=1,RAW(H)
      IF(RL(S,I))134,136,134
  134 Z=SOH+I
      DO 135 J=1,BAND6
      KK(Z,J)=0.0
      IF(J.GT.Z) GO TO 135
      KK(Z-J+1,J)=0.0
  135 CONTINUE
      KK(Z,1)=1.0
  136 CONTINUE
  137 CONTINUE
      DO 140 S=1,NBJFX
      H=HH(S)
  140 WRITE(2,139)H,(RL(S,I),I=1,RAW(H))
  139 FORMAT(12X,I2,9X,2(I1,2X),I1,4X,I1,6X,I1,6X,I1)
      DO 143 J=1,LP
      DO 142 I=1,SOR
  142 SS(I,J)=0.0
  143 CONTINUE
      DO 147 K=1,LP
      READ(1,1)NLJ
      READ(1,3)(JL(I),I=1,NLJ)
      READ(1,2)((SZ(I,J),J=1,6),I=1,NLJ)
      DO 146 I=1,NLJ
      ST=JL(I)
      SUL=0
      DO 144 L=1,ST-1
  144 SUL=SUL+RAW(L)
      DO 145 J=1,RAW(ST)
  145 SS(SUL+J,K)=SZ(I,J)
  146 CONTINUE
  147 CONTINUE
  148 FORMAT(1H1,10(2X,F7.5))
      WRITE(2,148)((SS(I,J),I=1,SOR),J=1,LP)
      CALL BAND MAT(BAND6,SOR,LP)
      VAL=1.0
      GO TO 1000
 5000 CONTINUE
      STOP
      END
```

Where is BAND

ND OF SEGMENT, LENGTH 8290, NAME TEST1

3- FINIAL RESULTS

a. Stresses



W = 1.6 tons

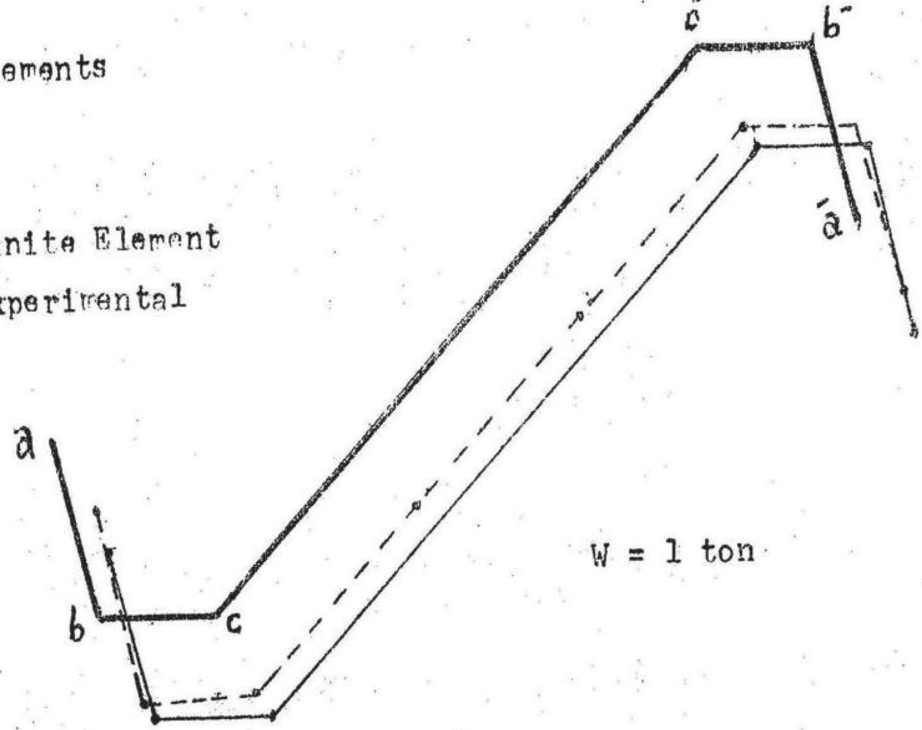
100 kg./loaded point

See page 52

b. Displacements

...... Finite Element

—— Experimental



W = 1 ton

1	1	-229.8205530	-76.6087746	33.3564911	-0.1810942	-0.0603647	-2.4206059
1	1	32.4498818	10.8166106	66.6810220	0.9252712	0.0603647	-4.1305386
1	1	52.8256856	71.9440889	182.2091452	-4.5045790	4.3008502	-3.8910138
1	1	-209.4507498	-15.4813022	169.0846142	-120.1090107	-0.5154683	145.6223942
1	2	-675.4796662	-170.8261123	-106.5105954	-5.1092738	-0.4263432	-2.7814267
1	2	112.6357027	91.8807412	-85.6382721	-6.7643372	6.2117251	-1.8769178
1	2	145.0399358	189.0935381	322.2116248	-16.9539375	-8.2378353	-1.9910382
1	2	-643.0734128	-73.0115135	301.3393019	-253.2770629	0.8063437	277.8558735
1	3	-987.9131181	-188.5571022	-281.4824934	-15.7507235	0.6707480	-2.1124960
1	3	192.8701463	205.0389456	-252.1402288	-25.0653284	-8.1022397	-5.2807092
1	3	191.8550136	201.9755402	358.9185210	-12.5249089	8.8462450	-4.8567909
1	3	-938.9342508	-191.6205037	359.5762565	-310.6134285	-0.8732519	381.8308953
1	4	-1219.2161640	-268.3810646	-326.7344863	-13.4370542	-1.0493761	-2.8376173
1	4	222.4101145	212.1605476	-341.5868360	-17.9804997	9.0223691	-0.9578890
1	4	245.9268065	282.7106939	434.6551361	-22.1776478	-6.2899793	0.1649426
1	4	-1195.6994720	-197.8309182	419.3074859	-368.6345541	0.9857496	447.7462599
1	5	-1310.7106209	-236.1679243	-402.8071674	-19.9663261	0.9105187	-0.8725929
1	5	261.3131313	287.8594640	-410.8647609	-33.4363658	-6.2147486	-3.3061902
1	5	268.8057358	290.3112400	402.6579376	-15.2449136	9.3560350	-4.8848647
1	5	-1323.2200184	-273.0961535	410.7155312	-351.8085883	-0.9808163	471.9806119
2	1	-62.3467671	-14.1155749	-28.8672283	-0.9938699	-0.3312896	0.8993922
2	1	-125.9431365	-41.1816709	-9.5737938	2.9814599	0.6625793	0.6648049
2	1	-104.3732085	16.3281700	-36.4667071	33.8570373	-2.2956918	10.7095783
2	1	-23.1768392	43.3942600	-55.7601417	418.3478136	16.9389635	-727.6188774
2	2	-89.9673373	22.4641143	5.0657202	56.2030344	17.5695721	4.9922708
2	2	-348.2531053	-64.9643010	5.8217909	50.0873538	-3.5567089	2.1911204
2	2	-347.5018746	-62.7106866	-81.0472923	157.0346480	71.7854122	-1.8440166
2	2	-85.2101266	24.7178087	-81.8053630	801.4259285	7.5944484	-1398.9366099
2	3	-168.1187602	3.7502851	38.6299746	103.3165107	7.2808595	1.8938011
2	3	-532.8596341	-124.4965446	66.3583532	158.9055607	72.4125900	16.0629777
2	3	-505.3007565	-61.8438291	-61.0678860	87.6915233	-4.5531952	16.2458138
2	3	-120.3678826	86.4030006	-88.7962666	1079.5591219	37.8963794	-1900.7383203
2	4	-161.7108682	72.0888858	60.1633303	134.6465056	38.0383279	1.2077280
2	4	-628.6185239	-82.9470437	50.7937027	87.2916902	-4.8368921	-2.8768102
2	4	-637.9283901	-110.8766714	-103.8461900	211.0317873	76.3743930	46.9101795
2	4	-171.0207348	46.7590380	-94.4763423	1204.7824302	15.0634356	-2238.9239376
2	5	-191.9768019	37.9071094	80.1701822	132.7304028	15.0811364	2.1771833
2	5	-699.7410121	-131.4808381	98.7310460	210.3821007	76.3389917	10.6643349
2	5	-681.2989654	-76.1966026	-69.5730909	110.6021341	-3.5372858	8.6041679
2	5	-173.1345540	93.8333069	-88.1339340	1340.9164356	41.4263716	-2339.7872189
3	1	125.9431360	41.1816708	-9.5737933	1.9877399	0.6625793	-6.8648051
3	1	62.3467675	14.1155751	-28.8872280	0.0189321	-0.3312896	-0.8993944
3	1	23.1768305	-43.3942063	-55.7601411	34.3112074	16.9389648	-5.1661670
3	1	104.3732080	-16.3281707	-36.4667063	393.9321519	-2.2956872	-735.4989970
3	2	348.2531056	64.9643805	5.8217915	50.0733836	-3.5569043	-2.1911207
3	2	89.9673385	-22.4641149	5.0657208	56.8497768	17.5695735	-4.9922728
3	2	85.2101275	-24.7178091	-81.8053622	106.2572796	7.5944504	-0.7571337
3	2	347.5018768	62.7106899	-81.0472916	840.0568221	71.7854219	-1373.1999686
3	3	532.8596344	124.4965440	66.3583538	158.9161878	72.4125997	-14.0629796
3	3	168.1187622	-3.7502852	38.6299754	105.6489276	7.2808816	-1.8938025
3	3	120.3678847	-86.4030002	-88.7962632	136.2212604	37.8965831	-2.0766406
3	3	505.3007570	41.8438290	-61.0678848	1021.4479208	-4.5531897	-1893.7890894
3	4	628.6185261	82.9470462	50.7937038	86.8410319	-4.8366868	2.8768099
3	4	161.7108709	-72.0888652	60.1639512	134.7912103	38.0383318	-1.2077296
3	4	-171.0207374	-44.7590380	-94.4763445	132.6973026	15.0634377	0.4256899
3	4	637.9283921	110.8766718	-103.8461887	1504.4894868	76.3744059	-2196.4932478
3	5	699.7410164	131.4808948	98.7310478	210.9253895	76.3390041	-10.6643397
3	5	191.9766068	-37.9071092	80.1701835	132.7792427	15.0811386	-2.1771867
3	5	173.1345571	-93.2355099	-88.1339547	128.3935277	41.4263748	-0.3170166
3	5	681.2989717	76.1966020	-69.5730906	1257.9647103	-3.5372720	-2318.1118797
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4	4	-245.9268051	-282.7106901	434.6551298	-354.3499691	-6.2899798	427.7545683
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45	-692.1077719	-0.0000003	0.0000000	0.0000000	43.9070673	
46	-1287.6388094	-0.0000014	0.0000008	-0.0000000	9.8473777	-7.0934046
47	-21.2942302	0.0000010	0.0000006	-0.0000000	3.2713950	-1.9730273
48	1003.3405437	0.0000004	0.0000000		-1.8406641	

CHAPTER

V

C O N C L U S I O N S

The results presented in this thesis illustrate the importance of careful study of the experimental tests before developing an analytical method of solution. The experimental studies provided the following conclusions for the Saw-Tooth folded plate structures:

1. Both Aluminum as well as reinforced concrete models are required for studying the structural action of hipped plate roofs, Aluminum is caracterized by straight line stress-strain relationship while the reinforced concrete model indicates the behaviour of the real structures .

2. Due to the presence of wide plates in the Saw-Tooth folded plate structure, the behaviour is not far from that of the beam analysis .

3. The effect of the transverse deformations should be taken into consideration specially when the resultant of loads are far from the shear center of the cross section.

4. The theoretical strains based on Gaafar's analysis are very close to the experimental results for both aluminum and reinforced concrete models all over the domain. The tensile strains which occur

In the reinforced concrete models are much bigger than the theoretical .

5. The deflected corss sections under loads are similar to that concluded by Gaafar's method. The aluminum model gives closer results .

6. Such roofs are very sensitive to the cross sectional dimensions , small change in this section causes great changes in the analytical results .

7. For thin walled sections, Gaafar's Method provides a good tool to determine the stresses due to both symmetrical or unsymmetrical loadings.

8. The finite element method offers a coloured method for researchers . Designers may benefit the results after checking it using any pre-tested model to have an idea how much the results deviate from the true values .

Suggested Further Studies on Saw-Tooth Folded Plate Structures :

1. Similar shapes with hollow blocks intermediate plate .
2. Prestressed models
3. Similar Shapes with variable intermediate plate length .
4. Other shapesof the Saw-Tooth folded plates .
5. Different span/ width ratios
6. Continuous structures .

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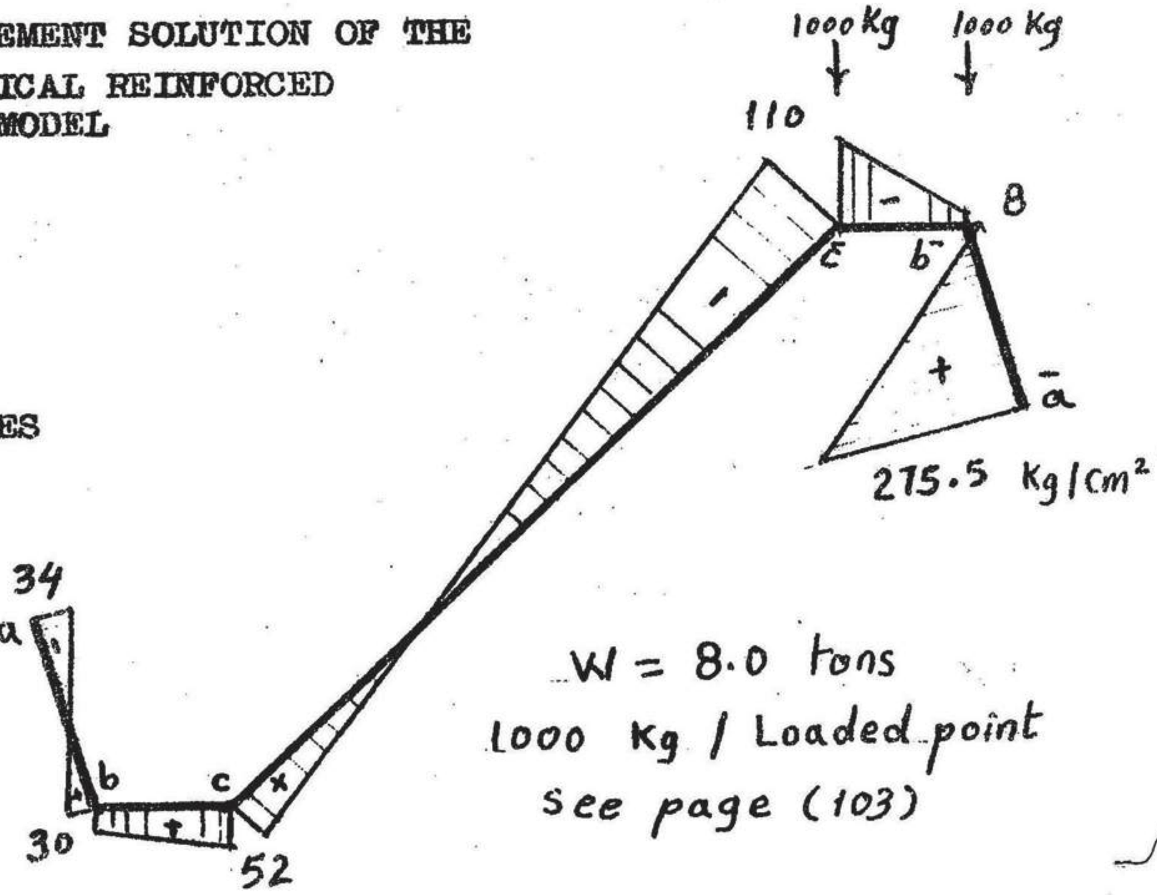
APPENDIX

INITE ELEMENT SOLUTION OF THE NSYMMETRICAL REINFORCED ONCRETE MODEL

a–STRESSES



b–DISPLACEMENTS