SOME CONSIDERATIONS IN THE ADJUSTMENT OF GPS-DERIVED BASELINES IN THE NETWORK MODE

A Thesis

Presented in Partial Fulfillment of the Requirements for
The degree Master of Science in the
Graduate School of the Ohio State University

By

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1991

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To My Wife, Hoda

ACKNOLWEDGMENTS

I wish to express my sincere appreciation to my advisor Professor Clyde C. Goad for suggesting this topic, for his continuous guidance, as well as for all the fruitful discussions throughout this research work. His advice, constructive criticism, and patience have all been greatly appreciated. I am also grateful to Professor Burkhard Schaffrin for his invaluable assistance, suggestions and comments.

My acknowledgments are also given to Professor Mona El-Kady, Director of the Egypt Survey Research Institute, who was first to lead me to the GPS environment before I came to the Ohio State University.

Special thanks go to my colleagues in the Department of Geodetic Science and Surveying who helped me to have an enjoyable stay and rewarding research experiences. In particular, I wish to extend my deep indebtedness to Mr. Jarir Saleh and Mr. Howard Small.

I wish to acknowledge USAID Project No. 263-0132, the Egypt Water Research Center, and Colorado State University for providing the financial assistance and the opportunity for conducting this research>

No graduate study for a married man away from his spouse is possible without the understanding, patience, and unconditional encouragement of his wife. To Hoda, I offer my ultimate thanks and appreciation.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Over the last few years, the Global Positioning System (GPS) has been used with increasingly more accurate results for precise positioning. The GPS observations are analyzed interferometrically to determine baseline components in a geodetic network. When the network geometry is poor, in terms of lack of observations, some configuration defects might exist. A no-check observation, which is one of the most important observation defects, may decrease the reliability of the estimated results since it can not be examined for blunder detection. This study shows that the results of the least-squares adjustment, when examined carefully, reveal important information about the deficiencies in the observation campaign. The concept of redundancy number is a helpful tool used to detect no-check observations.

When using GPS to determine high-precision relative positions, the set-up errors associated with the antenna are critical and must be considered. The error sources of setting-up the antenna are addressed and some precautionary field procedures are reviewed in order to minimize these errors and to help avoid blunders. However, the main objective of this study was detecting such situations by analyzing the adjustment results.

A method was developed and included in an adjustment computer program to detect no-check observations. This method handles two problems: (a) detecting inconsistent observations, for example due to set-up errors, for the stations which have redundant occupations in different observation sessions; and (b) detecting no-check observations in terms of deficiencies in the observation campaign. A full description of this method is presented and the results of processing a test GPS network are provided.

The developed adjustment program is used to estimate three-dimensional coordinates of the stations based on the measurements obtained from processing double differences of phase observables. The fundamentals of the GPS measurements are reviewed with emphasis given to the double difference observables and the correlations among them. It is expected that, under some circumstances, for the GPS-derived baselines in the network mode, the least-squares adjustment suffers from three datum defects due to the lack of the datum origin definition. The least-squares adjustment method in the extended Gauss-Markoff model with pseudo observations was used to overcome the expected rank defects by introducing a fiducial station in the network. The program also

transforms the estimated Cartesian coordinates of the network to geodetic coordinates, relative to a specific reference ellipsoid, along with their standard deviations in both coordinate systems. The FORTRAN code is given in the appendix.

CHAPTER I

INTRODUCTION

Since the early 1980s, many geodetic networks have been established using the Global Positioning System (GPS) measurements to provide baseline vector estimates at better than 1 part per million (ppm) level of precision (Goad 1985b, Remondi 1985). Much researches have been devoted to examining all possible systematic and random errors (e.g. satellite clock errors, receiver clock errors, cycle slips, etc.) in processing the GPS observables. However, errors associated with setting-up antennas during the observing time period or deficiencies in planning the observation campaign can also dangerously decrease the quality of the results.

GPS results are influenced by systematic effects (biases) when the observables are not correctly modeled (Beutler et al. 1989). Several papers have addressed sources of biases in GPS measurements (e.g. Goad 1985a, Georgiadou et al. 1988, Lichtenegger et al. 1989). Generally speaking, the biases that influence the GPS measurements can be grouped under three categories (Well 1986): satellite biases (in both satellite ephemerides and clocks); station biases (such as receiver clock biases); and observation dependent biases (for instance, ambiguity biases inherent in the carrier beat phase observables, multipath, ionospheric and tropospheric delay). Besides the systematic effects, the precision of baseline determinations from GPS observations is dependent on the errors affecting the observations themselves. These error sources include cycle slips, antenna phase center movement, and random observation errors. The major source of biases and errors, roughly speaking, are the satellite ephemerides, the troposphere, the ionosphere, and multipath (Bock et al. 1986b).

The detrimental impact of biases sometimes can be suppressed by modeling them in order to achieve the highest precision of the GPS measurements (Vanicek et al. 1985). However, this task might be difficult to achieve because of some limitations. For example, irregular ionospheric effects are hard to be modeled correctly when only one transmission frequency is observed (Beutler et al. 1989). Besides modeling these biases and errors, the known differencing algorithms can eliminate, or dramatically reduce, some of the common model errors (e.g. clock errors). But, some error sources, such as multipath, cannot be modeled or eliminated through differencing.

The least-squares adjustment technique is the most famous estimation method in the geodetic community. A basic assumption in the least-squares technique is that all gross and systematic errors have not been considered in the functional model, should be eliminated prior to the adjustment (Karvouras 1982). Therefore, committing an undiscovered, and hence unmodeled, error in setting up the antenna relative to the station mark will limit the precision of the least-squares adjustment. These systematic errors are quite probable when the GPS receivers move from one station to another to occupy many stations in the available GPS observation window, i.e. multi-session observations. Detecting such errors is very important as far as the quality of the geodetic networks is concerned.

The errors associated with antenna set-up are significant and influence the final output of the network adjustment. These error sources may be divided into three main types (Minkel 1989): plumbing error; antenna height measurement error; and error in setting of the tripod. Some guidelines for GPS field survey procedures are given (FGCC 1989). However, there is still a chance that one or more of these set-up errors might be committed in any station occupation in a GPS network. Antenna set-up errors fall in the category of those errors which can neither be modeled nor canceled out by differencing.

Deficiencies in planning a GPS observation campaign can result in no-check observations. A no-check observation is an observation that is not checked by any other observation in the network. The estimation of the coordinates of the stations associated with the no-check baselines are not reliable even though they have small residuals. This is due to the fact that no blunder detection algorithm can be applied to test the uncontrolled observations.

The redundancy number concept is used to indicate the reliability of the adjustment of individual observations (Milbert 1985). A redundancy matrix is formed by multiplying the cofactor matrix of the residuals by the weight matrix of the observations. The i-th diagonal element of this matrix product is called the redundancy number of the i-th observation. Leick (1990) finds that the redundancy number of a no-check observation will be zero. Therefore, the careful inspection of the least-squares adjustment results supplies useful information and helps to detect no-check observations.

There is not much literature available on the detection of inconsistent observations and the effects of unmodeled errors, especially the antenna set-up errors in GPS network. Moreover, the detection of no-check observations is very crucial and needs to be employed in any geodetic network adjustment.

The purpose of this study was to develop a method that serves two functions: (1) to detect inconsistent observations for those stations which have redundant occupations in different observation sessions; and (2) to detect no-check observations in a GPS network. The main characteristics of this method is that it is based on analyzing the least-squares adjustment results, which in turn means that the developed procedure can be part of a least-squares program.

The results of implementing the developed method could be useful in the analysis of GPS networks and spotting both configuration defects and suspect stations. When recovering the detected error sources or deficiencies, the quality of the GPS network is improved.

First, a concise, but simple, overview of the GPS basics and observable types is given in Chapter Two. The phase measurements are given much more emphasis in the discussion since they are the most accurate GPS observables. Single, double, and triple differencing algorithms are also addressed along with the principles of the interferometric analysis of the GPS observables.

The use of the Gauss-Markoff model, which is sometimes called observation equations, is presented in Chapter Three since it was applied in the developed adjustment program. The datum defects in GPS networks, along with the principle of the extended Gauss-Markoff model with prior information, are given. The use of the Cholesky factorization algorithm is reviewed also.

Chapter Four, which is the main segment of this thesis, includes a detailed explanation of the sources of observation defects in GPS networks. The proposed method is demonstrated fully and the results of processing a small GPS network are provided.

Finally, a summary and conclusions are presented in Chapter Five. Also, some recommendations for survoyers who work with GPS networks are given.

The FORTRAN code of the developed least-squares adjustment program, NANI, is given in the Appendix.

CHAPTER II

GLOBAL POSITIONING SYSTEM

2.1 An overview of the GPS Fundamentals and Observables

The Navigation Satellite Timing and Ranging (NAVSTAR) Global Positioning System (GPS) is a satellite-based positioning system under development by the US Department of Defense (DoD). In its final constellation, the GPS may consist of 21 operational (Block II) satellites plus 3 in-orbit spares. Each orbit is nearly circular with a 20183 km nominal altitude. Once fully operational, the GPS will provide 24-hour, all-weather navigation and surveying capability.

Each GPS satellite transmits at two frequencies: L1 = 1575.42 MHz; and L2 = 1227.60 MHz modulated with two types of code and a navigation message. The L1 signal is modulated with a precise (P) code, known also as the Precise Positioning Service (PPS), and a coarse acquisition (C/A) code, which is known also as the Standard Positioning Service (SPS); the L2 signal is modulated with only the P code. These two pseudo-random noise (PRN) codes have a period of 37 weeks and 1 millisecond for the P and C/A codes respectively, which means that the P code is more precise than the C/A code since it provides a predictable signal for a long period (Spilker 1978). For national security purposes, the DoD may restrict access of the P code only for military users. The 50 Hz navigation message contains, among other data, the broadcast ephemerides parameters and the satellite clock correction coefficients. This low frequency stream of data informs the user about the health and position of the satellite.

The surveying techniques used to collect GPS observations can be divided, in general, into three groups. In static surveying, GPS receivers are kept fixed over ground marks and observe at least four satellites simultaneously for a period of time ranging from 1 to a few hours. This is the conventional or "classical" GPS surveying. Kinematic GPS surveying, used mainly for navigation, is characterized by the continuous tracking operation of the receivers as they move from one place to another. The third class of GPS surveying techniques is tracked back to the pioneering work of Remonsi (1985), known as pseudo-kinematic, where one receiver is kept fixed over a known station while another receiver moves to survey other stations for few minutes of observations at each point (Kleusberg 1990).

The two basic types of the GPS observables are the pseudorange and the carrier beat phase (simply the "phase"). Traditionally, each observable type is treated separately. However, recent research provides a means of combining both the pseudoranges and phases and, thus, can benefit significantly both static and kinematic positioning (Euler and Goad 1991).

Pseudoranges are essentially distance measurements between the satellites and the receivers at the epochs of transmission and reception of the signals (Figure 1). The pseudorange equation takes the form:

$$P^i_j = \rho^i_j + c\ .\ (dt - dT\ ) + d_{ion} + d_{trop} + \varepsilon \qquad (2\text{-}1)$$

where

P^i_j	is the measures pseudorange
c	is speed of light
Dt	is the offset of the i-th satellite clock from GPS time
dT	is the offset of the j-th receiver clock from the GPS time
d_{ion}	is the ionospheric delay
d_{trop}	is the tropospheric delay
ε	is the effect of the (assumed random) measurements noise and unmodeled influences.
ρ^i_j	is the geometrical or topocentric distance between the satellite and the receiver (at epoch t_k of the receiver time):

$$\rho^i_j \quad = [\ (X^i - X_j\ )^2 + (\ Y^i - Y_j\ )^2 + (\ Z^i - Z_j)^2\ ]^{\ 1/2} \qquad (2\text{-}2)$$

where (X^i, Y^i, Z^i) and (X_j, Y^i, Z_j) denote the Cartesian coordinates of the satellite i and the receiver j in the WGS-84 coordinate system (as in most GPS literature, the superscripts refer to a particular satellite while subscripts refer to a particular ground receiver).

In the navigation solution, i.e. kinematic surveying, the satellite clock offsets are approximated by polynomials in time, whose coefficients are included in the broadcast message, and both the ionospheric and tropospheric delays are computed from some models. That leaves only four unknowns in equation (1), the three-dimensional coordinates of the receiver and the offset of the receiver clock. Therefore, observing at least four satellites simultaneously (assuming $\varepsilon = 0$) gives a solvable system of equations.

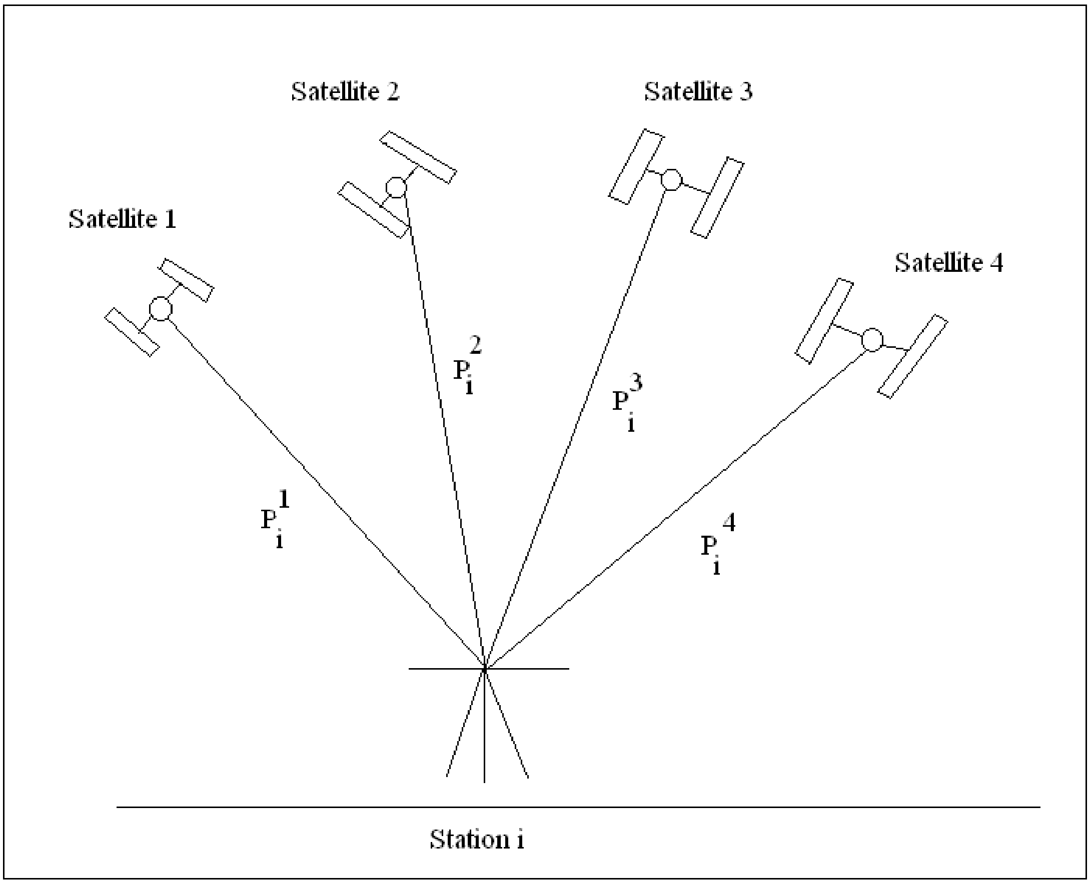


Figure 1: Pseudorange Observables

The precision of pseudorange measurements is about 1 % of the period between successive code phase (Wells 1986). For the C/A code, successive epochs are 1 millisecond apart, while they are 0.1 millisecond apart for the P code. This implies that the range measurement precision is ± 3 m and ± 0.3 m for the C/A and P code respectively.

The phase observables is the phase difference between the incoming Doppler-shifted carrier signal transmitted at the satellite and the integral of the nominally-constant reference frequency generated in the receiver. Following the notations of Goad (1985), the phase observable could be written (in units of cycles) as:

$$\phi^i_j\,(t_k) = \phi^i\,(t_k) - f / c\ \rho^i_j - \phi_j\,(t_k) + N^i_j + + d_{ion} + d_{trop} + \varepsilon \qquad (2\text{-}3)$$

where,

ϕ^i	denotes the received phase of satellite i as measured at the receiver j in the received time t_k
ϕ_j	denotes the receiver phase at time t_k
f	is the receiver oscillator frequency
c	is the speed of light
ρ^i_j	is the distance between the satellite i and the receiver j at time t_k
d_{ion}	is the ionospheric delay
d_{trop}	is the tropospheric delay
ε	is the effect of the (assumed random) measurements noise and unmodeled influences
N^i_j	is an integer bias representing the ambiguity of the first phase measurements, i.e. it denotes the unknown number of integer cycles at the initial epoch. Therefore, if lock is maintained, N^i_j will be the same for all phase observables between the receiver j and the satellite i.

The phase measurements could be made at 1 % of the carrier signal wavelength. For the L1, whose wavelength is 19 cm, the phase measurement precision is about ± 3 mm. Consequently, the phase ambiguous observables are more precise than the pseudoranges.

2.2. Processing of GPS Measurements

Like any other measurement techniques, the potential of the GPS is affected by both random and systematic errors. Various linear combinations of the phase observables have been used to reduce or eliminate the effects of some of the biases in the phase observation equations (equation 3). These bias elimination procedures are illustrated, geometrically, in Figure 2.

The single difference observable is formed by differencing the phases of two receivers, j and l, to the same satellite, i, at the same epoch k. This linear combination removes the satellite clock error. The single difference equation (neglecting d_{ion} and d_{trop} for a moment) is:

$$\phi^i_{j,l}\,(t_k) = \phi_j\,(t_k) - \phi_l\,(t_k) + f/c\,(\,\rho^i_j - \rho^i_l\,) + N^i_j + N^i_l + \delta\varepsilon_1 \qquad (2\text{-}4)$$

The double difference equation,

$$\phi^{i,m}_{j,l}\,(t_k) = f/c\;(\,\rho^i_j - \rho^i_l - \rho^m_j + \rho^m_l\,) + N^m_j - N^m_l - N^i_j + N^i_l + \delta\varepsilon_2 \qquad (2\text{-}5)$$

is obtained by differencing two single-difference observables of the two receivers, j and l, with respect to two satellites, i and m, at the same epoch k. The double difference does not explicitly include the satellite and the receiver clock errors.

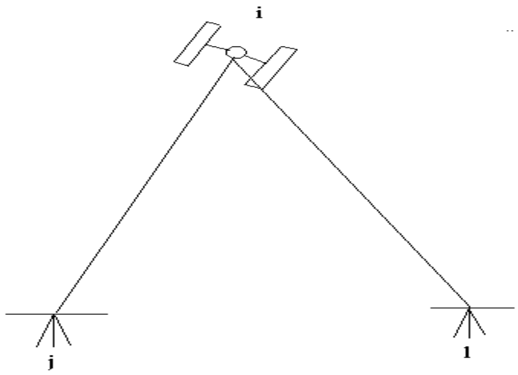
The triple difference is the difference of two double-differences for two different epochs, t_k and t_{k+1} :

$$\phi^{i,m}_{j,l}\,(t_{k+1}\,,\,t_k\,) = f/c\;[\;\rho^i_l\,(t_{k+1}) - \rho^i_j\,(t_{k+1}) - \rho^m_l\,(t_{k+1}) + \rho^m_j\,(t_{k+1})\; - \rho^i_l\,(t_k) + \rho^m_l\,(t_k) - \rho^m_j\,(t_k) + \delta\varepsilon_3 \qquad (2\text{-}6)$$

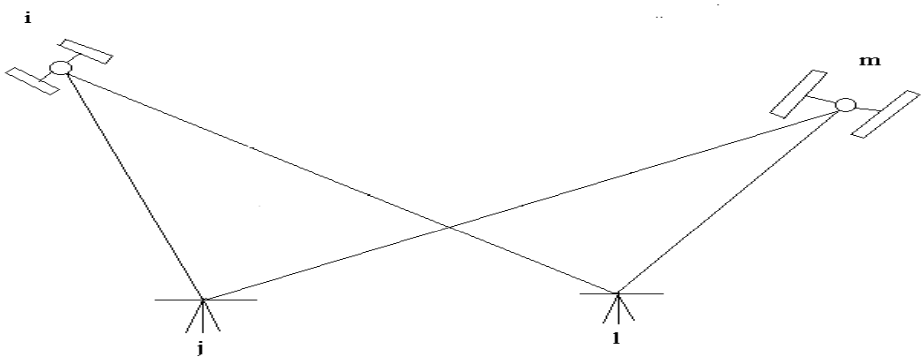
where $\delta\varepsilon_1$, $\delta\varepsilon_2$, and $\delta\varepsilon_3$ are the linear combinations of the effects of the measurements noise in the single, double, and triple differences respectively. It has been noticed that the ionospheric and tropospheric delays have not been considered in this section since there are other bias estimation procedures, for example wide-lane and narrow-lane, used to eliminate or reduce the atmospheric effects (King et al. 1985).

The main advantage of he triple difference is that it does not include the integer-bias ambiguities and, therefore, it can be used in an automatic detection procedure for the occurrence of cycle slips (Goad and Remondi 1984). Cycle slips occur when the satellite signals are lost, i.e. obstructed by buildings, trees, .. etc, and, consequently, when acquiring the phases the ambiguity takes on a different value. It is known that the presence of cycle slips is one of the most prevalent problems in processing the GPS phase measurements. On the other hand, disadvantages of the triple differences are the high correlation between the observables and inability to use the integer bias values if they are known. Although some procedures were suggested, for example by Eren (1986), to use the triple difference observables in network adjustment, the double differences, or their equivalences, are the most powerful GPS measurements.

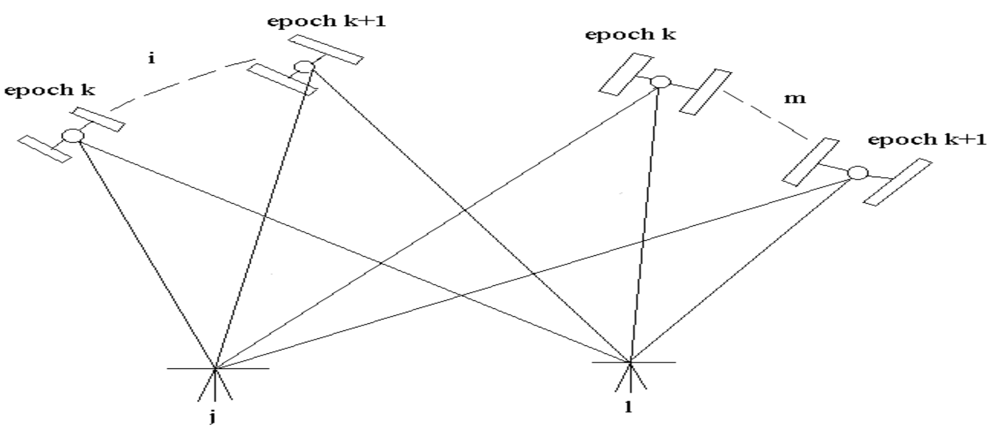
In practice, the non-difference (raw) phases are assumed to be uncorrelated and have equal variance, say σ^2. However, as linear combinations are usually formed, the mathematical correlations between the differenced observables should be considered. To demonstrate the correlations between the double difference observables, let us look at two double difference equations between two receivers and three satellites. In a simple form, these two equations could be written as:



Single Difference: Two receivers observe the same satellite at the same epoch



Double Difference: Two receivers observe two satellites at the same epoch



Triple Difference: Two receivers observe two satellites from one epoch to the next

Figure 2: Differencing Procedures

$$\phi^{2,1}{}_{2,1} = \phi^{2}{}_{2,1} - \phi^{1}{}_{2,1} = \phi^{2}{}_{2} - \phi^{2}{}_{1} - \phi^{1}{}_{2} + \phi^{1}{}_{1}$$
$$\phi^{3,1}{}_{2,1} = \phi^{3}{}_{2,1} - \phi^{1}{}_{2,1} = \phi^{3}{}_{2} - \phi^{3}{}_{1} - \phi^{1}{}_{2} + \phi^{1}{}_{1} \qquad (2\text{-}7)$$

In matrix notations:

$$D = B \, . \, \varphi \qquad (2\text{-}8)$$

where,

$$D = [\ \phi^{2,1}{}_{2,1} \quad \phi^{3,1}{}_{2,1}\ ]^{T}$$

$$B = \begin{bmatrix} 1 & -1 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

and,

$$\varphi = (\ \phi^{1}{}_{1}\ ,\quad \phi^{2}{}_{1}\ ,\quad \phi^{3}{}_{1}\ ,\quad \phi^{1}{}_{2}\ ,\quad \phi^{2}{}_{2}\ ,\quad \phi^{3}{}_{2}\ )^{T}$$

Applying the error-propagation law, the covariance matrix of the two double-difference observables, Σ_{DD}, is:

$$\Sigma_{DD} = B\,(\sigma^{2} I\,)\,B^{T} = \sigma^{2}\ W \qquad (2\text{-}9)$$

where,

$$W = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

Therefore, the mathematical correlations between the differenced observables should be modeled in the least-squares adjustment algorithm when processing the GPS phase measurements. However, the correlation between the double differences is more complicated than as it appears in the previous simple example. Since the object of this section in the thesis is just to highlight some of the GPS issues, the subject of the mathematical correlations will not be discussed, here, in full details (the interested reader can refer to, for example, Schafrrin et al. 1989, Bock et al. 1986a, Goad and Mueller 1988, or Beutler et al. 1987).

2.3 Interferometric Analysis of GPS Observations

Interferometry with the GPS is a technique by which the baseline components (and possibly absolute coordinates) in a GPS network can be determined in three dimensions with respect to an earth-fixed coordinate system. Bock et al. (1986a) present a processing procedure that maps phase observations into double differences. The main advantage of this procedure is that it extracts the maximum relative positioning information available from the raw phases. It, also, takes into account the mathematical correlation that is introduced in the differencing process. At the heart of this algorithm is a double difference operator, D, which constructs an independent set of double differences from all phases collected at a certain epoch. That double difference operator is the same as the matrix B in the simple example of section 2.2.

Goad and Mueller (1988) suggest an automated procedure to generate the optimum set of independent double differences while processing the phase measurements interferometrically. It is known that when R stations simultaneously observe S satellites, [R! / 2 (R-2)!) (S!/2(S-2)!)] = (RS/4) (R-1) (S-1) possible double differences could be obtained. But, among them only (R-1) (S-1) double differences are linearly independent (Bock et al. 1986a). Goad and Mueller's algorithm depends on three features: (a) selecting the shortest possible baselines to enhance the recovery of the integer bias value of the double differences in the multiple baseline mode; (b) the use of the Cholesky decomposition procedure to detect any dependent double differences. That is due to the fact that a zero on the diagonal of the Cholesky factor reveals a situation of linear dependence; and (c) the generation of the Gram matrix, which is a matrix of linear (dot) products used to determine the linear dependence or independence of a set of vectors. A similar algorithm is presented by Dong and Bock (1989) which generate the double difference ambiguities from the original raw phase ambiguities by means of a mapping based primarily on baseline length.

Following the notations of Bock et al. (1986a), the linearized observation equations for the raw phases, in matrix form, (and neglecting terms which are removed under the double difference operator) are:

$$L = A\,x + \varepsilon \qquad (2\text{-}10)$$

where,

L   is the residual vector (observed minus computed) of the phases

A	is the design matrix or partial derivatives of the phases with respect to the parameters
x	is the correction vector of the parameters of interest
ε	is the vector of the observation errors.

Applying the double difference operator, D, yields:

$$D\,L = D\,A\,x + D\,\varepsilon \tag{2-11}$$

Assuming uncorrelated phases,

$$L = (\,0\,,\,\sigma^2 I\,) \tag{2-12}$$

The covariance matrix of the double differences is:

$$E = \{\,D\,v\,v^T D^T\,\} = D\,(\sigma^2\,I\,)\,D^T = \sigma^2\,D\,D^T \tag{2-13}$$

with

$$E\,\{\,D\,\varepsilon\,\} = 0 \tag{2-14}$$

where E denotes the expectation operator.

The least-squares estimate of x is:

$$x^{\wedge} = (A^T D^T (DD^T)^{-1} DA)^{-1} A^T D^T (DD^T)^{-1} DL \tag{2-15}$$

and the covariance matrix of the estimate is:

$$\sum_x{}^{\wedge} = \sigma^2\,(\,A^T D^T (DD^T)^{-1} DA)^{-1} \tag{2-16}$$

The Cholesky decomposition procedure is used in Goad and Mueller's algorithm to factor and then invert the matrix ($D\,D^T$) by:

$$(\,D\,D^T\,)^{-1} = (\,S\,S^T\,)^{-1} = (\,S^T\,)^{-1}\,S^{-1} = (\,S^{-1})^T\,S^{-1} \tag{2-17}$$

where S is the lower-triangle Cholesky factor.

Therefore, equation 15 could be written as:

$$x^{\wedge} = [\,(S^{-1}\,DA\,)^T\,(S^{-1}\,DA)\,]^{-1}\,(S^{-1}\,DA)^T\,S^{-1}\,DL \tag{2-18}$$

In the decomposition process, DD^T is the Gram matrix of the row vector of D and, thus, during factorization generating a zero on the diagonal of the Cholesky factor, S, will indicate dependence of the corresponding double difference which, in this case, should be rejected (Goad and Mueller 1988). The process of testing the independence of the double differences continues until the maximum number is reached. By this independence check, the existence of $(DD^T)^{-1}$ is assured.

Schaffrin and Bock (1988) propose another processing scheme using dual-frequency (L1 and L2) phase observations. In their algorithm, they allow for the incorporation of full covariance matrices for both the phase measurements and the weighted ionospheric constraints. This algorithm depends on constructing the ionospheric-free linear combinations.

In three-dimensional relative geodetic applications, a minimum of 3(R-1) station occupations are to be adjusted and (R-1)(S-1) phase-bias parameters to be estimated, where R is the number of stations and S is the number of satellites (Bock et al. 1985). This is the so-called ambiguity-free solution. Under certain operational conditions, the obtained ambiguity parameters may be resolved to their nearest (theoretical) integer value and, thus, held fixed in the so-called ambiguity-fixed solution. The sequential adjustment provides a helpful tool in searching for the correct integer-bias values over all possible near-integer ambiguity estimates obtained from the free solutions.

The ambiguity resolution, i.e. elimination, is an important step in processing the GPS phases since it converts the solution from one which involves ambiguous ranges to one based on precise unambiguous range measurements, and therefore increases the precision of the stations position estimation.

CHAPTER III

A GPS NETWORK ADJUSTMENT PROGRAM

3.1 The Use of the Least-Squares Adjustment in a Gauss-Markoff Model

Since the proposed method for detecting no-check and inconsistent observations in GPS networks is based on analyzing the least-squares adjustment results, a computer program was developed to satisfy this purpose. The least-squares estimation method is quite well known to the geodetic community. A review of the use of the Gauss-Markoff model, known also as the observation equations technique, is provided.

In this chapter, it is assumed that the phases have been processed and the baselines components are obtained. The basic observation equations for the GPS baseline components in the earth-centered body-fixed (EFC) World Geodetic Cartesian System 1984 (WGS-84) are:

$$\begin{aligned} \Delta X_{ij} - \varepsilon_{\Delta ij} &= X_j - X_i \\ \Delta Y_{ij} - \varepsilon_{\Delta ij} &= Y_j - Y_i \\ \Delta Z_{ij} - \varepsilon_{\Delta ij} &= Z_j - Z_i \end{aligned} \tag{3-1}$$

which gives the observation equations model

$$Y = A\,X + \varepsilon \tag{3-2}$$

where

- Y is the n x 1 vector of observations,
- ε is the n x 1 vector of observation errors, with $E\,\{\varepsilon\} = 0$ and $E\,\{Y\} = Y - \varepsilon = A\,X$, where E demotes the expectation operator,
- A is the n x m design (or coefficients) matrix of partial derivatives of the observations with respect to the parameters,
- X is the m x 1 vector of unknown parameters,
- n is the number of the observations = 3 x number of the observed baselines,

m is the number of unknown parameters = 3 x number of the stations in the network, assuming $m \leq n$.

The least-squares adjustment in the linear model (3-2) leads to the normal equations:

$$(A^T \Sigma^{-1} A) X^{\wedge} - (A^T \Sigma^{-1} Y) = \sigma_o^2 (N X^{\wedge} - C) = 0 \qquad (3\text{-}3)$$

where

$\Sigma = E\{\varepsilon \varepsilon^T\} = \sigma_o^2 P^{-1}$ is the n x n positive definite covariance matrix of the observations, and

$P = \sigma_o^2 \Sigma^{-1}$ is the weight matrix, with σo2 being the unknown variance of unite weight.

The solution of (3-3) for the parameters is:

$$X^{\wedge} = (A^T P A)^{-1} A^T P Y = N^{-1} C \qquad (3\text{-}4)$$

With the covariance matrix

$$\Sigma_X{}^{\wedge} = (A^T \Sigma^{-1} A)^{-1} = \sigma_o^2 (A^T P A)^{-1} = \sigma_o^2 N^{-1} \qquad (3\text{-}5)$$

The adjusted observations are:

$$Y^{\wedge} = A X^{\wedge} = A (A^T \Sigma^{-1} A)^{-1} A^T \Sigma^{-1} Y \qquad (3\text{-}6)$$

With the covariance matrix

$$\Sigma_Y{}^{\wedge} = \sigma_o^2 (A N^{-1} A^T) \qquad (3\text{-}7)$$

So that the residual vector is:

$$\varepsilon^{\wedge} = Y - A X^{\wedge} = (I - A N^{-1} A^T P) Y \qquad (3\text{-}8)$$

The estimated variance of unite weight, or the reference factor, is computed as:

$$\sigma_o^{2\,\wedge} = \varepsilon^{\wedge T} P \varepsilon^{\wedge} \; / \; (n - m) \qquad (3\text{-}9)$$

where (n – m) is the degree of freedom.

Therefore, the estimated covariance matrix of the adjusted parameters is:

$$\hat{\Sigma}_{\hat{x}} = \hat{\sigma}_o^2 \; (A^T P A)^{-1} \qquad (3\text{-}10)$$

If the covariance matrix of the observations, Σ, is a diagonal matrix, i.e., the observations are uncorrelated, both the N matrix and the C vector can be formed by summing, or accumulating the contribution of the observations (one by one) without storing A, Σ, or Y in computer memory (Mikhail 1976).

This can be done by:

$$N = \sum_{i=1}^{m} (A^T P A) \qquad (3\text{-}11)$$

$$C = \sum_{i=1}^{m} (A^T P Y) \qquad (3\text{-}12)$$

i.e.,

$$N = (\Sigma_r \; a_{ir} \; P_r \; a_{rj}) \, , C = (\Sigma_r \; a_{ir} \; p_r \, , y_r)$$

where $A = [a_{rj}]$, $Y = [y_r]$, and $P = \text{diagonal} [p_r]$

It has been noticed that N^{-1} exists if

$$\text{rank} [N] = \text{rank} [A] = m \qquad (3\text{-}13)$$

holds. However, this is not the case in most of the geodetic applications due to the rank defects.

3.2 Overcoming the Datum Defects in GPS Networks

From a mathematical point of view, the rank of the square matrix N should equal its dimension in order to invert it and obtain a unique solution for the normal equation system (3-13). However, in most of the geodetic networks a defect exists in defining a geodetic reference system. For some geodetic networks, the datum defects are shown in table 1 (Funcke et al. 1981). For GPS networks, it is necessary to define only an origin for the datum since the orientation and scale are implicitly known from the phase observations since the coordinates of the GPS satellites are assumed to be known. Therefore, the datum defects of the coordinate differences count 3 (Banyai 1991).

Table 1: Datum Defects in Geodetic Networks

Kind of Network	Datum Defects	
	No.	Name
Height net	1	1 translation
2D Trialateration net	3	2 translations
		1 rotation
2D Triangulation net	4	2 translations
		1 rotation
		1 scale
3D net	6 (7)	3 translations
		3 rotations
		(1 scale)

The inner constraints technique, known also as free-network adjustment, may be used to detect the internal precision and consistency of the field observations. The centroid of the unadjusted coordinates is used to control the coordinate translations. Therefore, no external control point coordinates need to be held foxed. However, in the final adjustment, or fully constrained solution, some stations are considered fiducial points in order to merge the GPS network into existing control networks. It is known that there are no translation parameters between the NAD83 and WGS-84 (DMA 1986). Consequently, one station with known coordinates in the North American Datum (NAD83) is enough to overcome the three datum defects in the GPS network.

The model (3-2) has to be extended to accommodate the pseudo observations (i.e. the prior information about the coordinates of the fiducial point) by adding the equation:

$$O = [\, I \;\; 0 \,]\,[\, X_1 \; X_2 \,]^T + \varepsilon_O \qquad (3\text{-}14)$$

where,

O	is the d x a vector of pseudo observations,
I	is the d x d identity matrix,
X_1 (dx1)	is the vector of the parameters for which the prior information is known,
X_2 (qx1)	contains the rest of the parameters,
ε_O	is the d x 1 random error vector of the prior information,

$\varepsilon_O \sim \{\, 0 \,,\, \Sigma_o \,\}$ where Σ_o is the d x d positive definite covariance matrix of the prior information, assuming no correlation between ε and ε_O ,

d is, in our case, the number of rank defects, i.e.,

$d = m - \text{rank}\,(\,A\,) = m - q.$

Combining equations (3-2) and (3-14) yields the extended Gauss-Markoff model:

$$\begin{bmatrix} Y \\ O \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \varepsilon \\ \varepsilon_o \end{bmatrix} , \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\begin{bmatrix} \varepsilon \\ \varepsilon_o \end{bmatrix} \sim \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} , \begin{bmatrix} \Sigma & 0 \\ 0 & \Sigma_o \end{bmatrix} \right) \qquad (3\text{-}15)$$

where the design matrix A(n x m) is partitioned into two sub-matrices A_1 (n x d) and A_2 (n x q). The normal equations system of the extended model (3-15) can be written as:

$$N_e \;X = \; C_e \qquad (3\text{-}16)$$

where

$$N_e = A_e^T \, P_e \, A_e \qquad (3\text{-}17)$$

$$C_e = A_e^T \, P_e \, Y_e \qquad (3\text{-}18)$$

$$A_e = \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix} \qquad (3\text{-}19)$$

$$P_e = \begin{bmatrix} \Sigma & 0 \\ 0 & \Sigma_e \end{bmatrix}^{-1} = \begin{bmatrix} \Sigma^{-1} & 0 \\ 0 & \Sigma_e^{1} \end{bmatrix} = \begin{bmatrix} P & 0 \\ 0 & P_e \end{bmatrix} \qquad (3\text{-}20)$$

$$P_e = \sigma_0^2 \, \Sigma_o^{-1}$$

$$Y_e = \begin{bmatrix} Y \\ 0 \end{bmatrix} \tag{3-21}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{3-22}$$

Therefore, N_e and C_e can be written as:

$$N_e = \begin{bmatrix} A_1^T P A_1 & A_1^T P A_2 \\ A_2^T P A_1 & A_2^T P A_2 \end{bmatrix} = \begin{bmatrix} N_{11} + P_o & N_{12} \\ N_{21} & N_{22} \end{bmatrix} = N + P'_o \tag{3-23}$$

$$C_e = \begin{bmatrix} A_1^T P Y + O P_o \\ A_2^T P Y \end{bmatrix} = C + C_o \tag{3-24}$$

where,

$$P'_o = \begin{bmatrix} P_o & 0 \\ 0 & 0 \end{bmatrix} \tag{3-25}$$

$$C_o = \begin{bmatrix} O P_o \\ 0 \end{bmatrix} \tag{3-26}$$

N and C are previously defined (equation 3-3).

The solution of equation (3-16) is:

$$X^{\wedge} = N_e^{-1} C \tag{3-27}$$

where,

$$N_e^{-1} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \qquad (3\text{-}28)$$

$$H_{11} = (S_1 + P_o)^{-1} \qquad (3\text{-}29)$$

$$H_{12} = - (S_1 + P_o)^{-1} N_{12} N_{22}^{-1} \qquad (3\text{-}30)$$

$$H_{21} = - N_{22}^{-1} N_{21} (S_1 + P_o)^{-1} \qquad (3\text{-}31)$$

$$H_{22} = N_{22}^{-1} N_{21} (S_1 + P_o)^{-1} N_{12} N_{22}^{-1} + N_{22}^{-1} \qquad (3\text{-}32)$$

$$S_1 = N_{11} – N_{12} N_{22}^{-1} N_{21} \qquad (3\text{-}33)$$

S_1 is called the first Schur complement.

The dispersion of the solution, $X^{\wedge}$, is:

$$D \{ X^{\wedge} \} = \begin{bmatrix} N_{11} + P_o & N_{12} \\ N_{21} & N_{22} \end{bmatrix}^{-1} \qquad (3\text{-}34)$$

The residual vector, ε_e, is:

$$\varepsilon^{\wedge}_e = \begin{bmatrix} \varepsilon^{\wedge} \\ \varepsilon^{\wedge}_o \end{bmatrix} = Y_e – A_e X^{\wedge} \qquad (3\text{-}35)$$

which can be written as:

$$\varepsilon^{\wedge}_e = [I - A_e N_e^{-1} A_e^T P_e] \; Y_e \qquad (3\text{-}36)$$

with a dispersion matrix:

$$D \{ \varepsilon^{\wedge}_e \} = D \{ Y_e \} – D \{ A_e X^{\wedge} \} \qquad (3\text{-}37)$$

The estimated variance component is:

$$\hat{\sigma}_o^{\,2} = \hat{\varepsilon}_e^{\,T} \; P_e \; \hat{\varepsilon}_e \; / \; d.f \qquad (3\text{-}38)$$

where d.f = degree of freedom = (n + d – m) (3-39)

From a computational point of view, some of the above equations can be simplified in certain cases. If the solution, $X^{\wedge}$, is the correction to be added to a vector of approximate values of the parameters, the vector of prior information might be a "random" zero vector. Furthermore, if the pseudo observations are given relatively large values (i.e. $\sum_o$ is almost zero), the inverse of (S_1+P_o) may be considered zero. Under these assumptions, equation (3-27) becomes:

$$X^{\wedge} = (N + P'_o)^{-1} C \qquad (3\text{-}40)$$

This special case is known as the observation equations with weighted parameters and can be found, in details, in many adjustment texts (e.g. Uotila 1986, pp. 104). Nevertheless, it is just a simplified least-squares adjustment in the extended Gauss-Markoff model.

The developed adjustment program also converts the estimated parameters, i.e. the Cartesian coordinates of the network stations, to a geodetic coordinate system (latitude, longitude, and height) along with their covariance matrices. The implemented equations are given by Rapp (1976, pp. 56)

3.3 The Use of the Cholesky Factorization Algorithm

The Cholesky factorization algorithm for solving a set of linear equations has been applied in geodesy for many years (Schmid 1973). It is the LU decomposition technique in which the upper triangular matrix, U, is chosen to be the transpose of the lower triangular matrix, L. In terms of execution operations, the LU method requires three times fewer operations (inner loops) than any other similar procedures, for example the Gauss-Jordan routine (Press et al. 1989).

The Cholesky factorization is used, among other functions, to invert the symmetric positive-definite normal equation matrix N (or N_e in the extended Gauss-Markoff model). N is decomposed to the product of a lower triangular matrix, S, and its transpose:

$$N = S\, S^T \qquad (3\text{-}41)$$

The inverse of N could be written as:

$$N^{-1} = (S\,S^T)^{-1} = (S^T)^{-1}\,S^{-1} = (S^{-1})^T\,S^{-1} \qquad (3\text{-}42)$$

To invert the lower triangular matrix S, less execution time is required when compared to the inversion of the full matrix N.

To obtain S^T, two formulas are used for the diagonal and the above-diagonal elements (Koch 1988, pp. 37):

$$s^T_{ij} = (\,n_{ij} - \sum_{k=1}^{i-1} s^T_{kj}\,)^{1/2} \qquad (3\text{-}43)$$

$$s^T_{ij} = (\,n_{ij} - \sum_{k=1}^{i-1} s^T_{ki}\;s^T_{kj})^{1/2} \qquad (3\text{-}44)$$

for i = 1, 2, ….. m and j = 1, 2, …. M

The Cholesky algorithm can also be used to decorrelate a set of correlated observations to allow for easy accumulation of the normal equations (Milbert 1985). This can be done by factorizing the cofactor matrix of the observations, $\sum$, as:

$$\sigma_o^2 \sum = R\,R^T \qquad (3\text{-}45)$$

so that

$$\sigma_o^2 \sum^{-1} = P = (\,R^{-1}\,)^T\,R^{-1} \qquad (3\text{-}46)$$

Instead of the normal equations (3-3), the model becomes:

$$N'\,X = C' \qquad (3\text{-}47)$$

where

$$N' = A'^T A' \qquad (3\text{-}48)$$
$$C' = A'^T Y' \qquad (3\text{-}49)$$
$$A' = R^{-1} A \qquad (3\text{-}50)$$
$$Y' = R^{-1} Y \qquad (3\text{-}51)$$

Therefore, the least-squares solution will be:

$$X^{\wedge} = [\,(A')^T A'\,]^{-1} (A')^T Y' \qquad (3\text{-}52)$$

in which the transformed observations, Y', have an identity cofactor matrix because of

$$
\begin{aligned}
E\{\varepsilon' \ \varepsilon'^T\} &= E\{ (R^{-1}\varepsilon)\ (R^{-1}\varepsilon^T) \} \\
&= R^{-1}\ E\{\varepsilon\,\varepsilon^T\}\,(R^{-1})^T \\
&= R^{-1}\ \sum\ (R^{-1})^T = R^{-1}\ (R\,R^T)\,\sigma_o^2\,(R^{-1})^T \\
&= (R^{-1}R)\,\sigma_o^2\,(R^T)^{-1} = \sigma_o^2\, I
\end{aligned}
$$

Another advantage of the Cholesky factorization algorithm is that it reveals the defects in the normal matrix. The number of the generated zeros on the diagonal represents the number of defects. This property is important in data snooping since deleting an erroneous observation might create a new observation defect in the network.

CHAPTER IV

DETECTING INCONSITENT AND NO-CHECK OBSERVATIONS

4.1 Observation Defects in GPS Networks

For the analysis of geodetic networks, in general, there is a distinction of two types of defects: datum defects and observation defects (Delikaraoglou 1985). The datum defects have been addressed in section 3.2. This section of the thesis is devoted to the second type of defects. The number of the observation defects is not always known before the adjustment process. Moreover, in some cases this number may increase during the adjustment. Figure 3 shows an example of some deficiencies in a triangulation network.

Observation defects can be thought of as a lack of observations or deficiencies in an observation campaign. In other words, these defects arise when the number and geometry of observations are inadequate to estimate uniquely the network parameters. An example of this situation is point E in figure 3. Suppose that there were two observations connecting point E to any other neighboring points, and these observations being conflicted and therefore flagged as erroneous observations when performing data snooping. Deleting these erroneous observations will disconnect point E from the network, with the result that new defect (singularities) will be encountered when inverting the normal equation matrix. In other situations, the observation defects result when some of the network points are situated in a special or poor-geometry configuration (for a certain type of measurements). For instance, point C in the triangulation network in figure 3 is a defected point since it is nearly on the line between points A and G. Therefore, such observation defects could be detected by viewing a sketch of the network with expert eyes.

The most crucial type of observation defects (which was the main target of this study) is the "no-check". For example, the line from point A to point B in figure 3 is connected to the network only through point A, and therefore there is independent check for the observed vector components of this line. The threat of this type of observations lies in two features: (1) no blunder detection algorithm can be applied on no-check observations, and (2) the uncertainties of the distant point (point B in figure 3) cannot be judged correctly even though this point might have small standard deviations. This is because blunders in the no-check observation will adversely affect the adjusted coordinates of this point, but not the statistics.

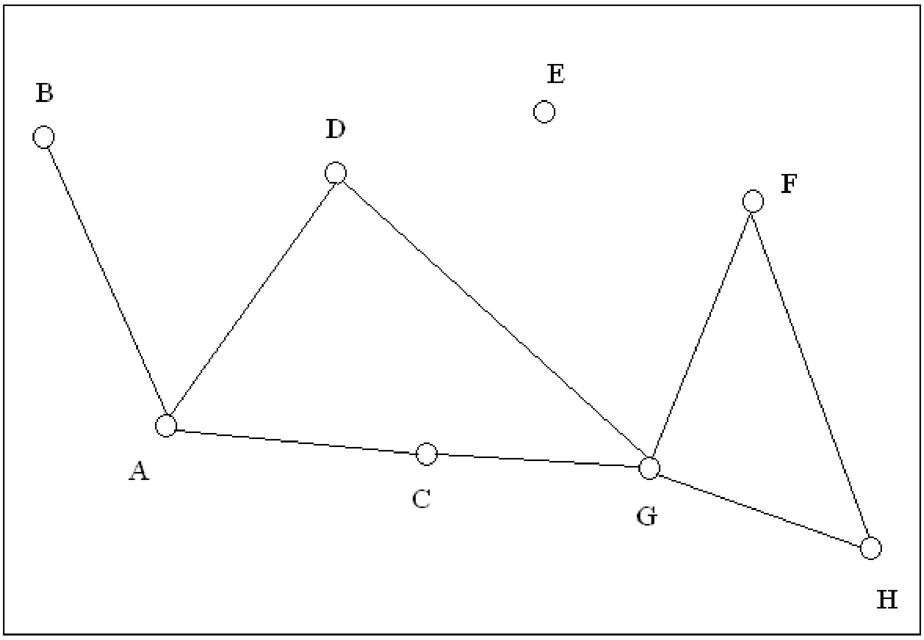


Figure 3: Example of Observation Defects

To overcome the observation defects, after detecting them, the observation plan must be modified by introducing new measurements, or deleting some measurements and parameters from the adjustment. For example, a new vector from point B to any point in the network (including A) needs to be observed to control this weak part of the network. This task is known as the third order design (Schmitt 1985). It is known that in case of adding or deleting observations, the sequential adjustment may be an appropriate and effective tool for performing such a function, particularly in large network.

Because the hazard of the no-check observations, zero residuals, in this case, are not necessarily a good indication about the quality of the adjustment. Also, small standard deviations cannot be interpreted as a measure of the reliability of the estimated parameters. The results of the least-squares adjustment need to be examined carefully to reveal any deficiencies in the observation campaign and to detect no-check observations.

4.2 Redundancy Number and Its Implementations

For measuring the quality of an observation, there are two measures. One is precision (the standard deviation is set up as a measure of precision). The other is reliability. A redundancy number (known also as redundancy contribution) is used to judge the reliability of the adjustment of individual observations. A redundancy number of the i-th observation is the diagonal element of the matrix product ($Q_{\varepsilon}^{\wedge}$ P) of the cofactor matrix of residual ($Q_{\varepsilon}^{\wedge}$) and the weight matrix (P).

The discussion in this section refers to the Gauss-Markoff model with full rank, which is discussed in section 3.1. However, the same argument and the corresponding equations are valid for the extended Gauss-Markoff model with pseudo observations, which is reviewed in section 3.2.

The residual vector ($\varepsilon^{\wedge}$) can be written as the difference between the observations (Y) and the adjusted observations ($Y^{\wedge}$):

$$\varepsilon^{\wedge} = Y - Y^{\wedge} \tag{4-1}$$

Since the cofactor matrix of the adjusted observations follows from equation (3-7), the cofactor matrix of the residuals is:

$$\begin{aligned} Q_{\varepsilon}^{\wedge} &= Q_Y - Q_Y^{\wedge} \\ &= P^{-1} - A\, N^{-1} A^T \end{aligned} \tag{4-2}$$

which, when multiplying by the weight matrix P, yields a redundancy matrix:

$$\begin{aligned} Q_{\varepsilon}^{\wedge} P &= (P^{-1} - A\, N^{-1} A^T) P \\ &= I - A\, N^{-1} A^T P \\ &= I - Q_Y^{\wedge} P \end{aligned} \tag{4-3}$$

The matrix ($A\ N^{-1}\ A^T\ P$) is, among other characteristics discussed by Pope (1976), an idempotent matrix because of

$$\begin{aligned} (A\, N^{-1} A^T P)(A\, N^{-1} A^T P) &= A\, N^{-1} (A^T P\, A) N^{-1} A^T P \\ &= A\, (N^{-1} N) N^{-1} A^T P \\ &= A\, N^{-1} A^T P \end{aligned} \tag{4-4}$$

which, in turn, means that the redundancy matrix is also idempotent:

$$(Q_{\hat{\varepsilon}} P)(Q_{\hat{\varepsilon}} P) = Q_{\hat{\varepsilon}} P \qquad (4\text{-}5)$$

It is known that the trace of an idempotent matrix equals its rank (Koch 1988, pp. 58). So:

$$\begin{aligned} \text{trace}\,(Q_{\hat{\varepsilon}} P) &= \text{trace}\,(I - A N^{-1} A^T P) \\ &= \text{trace}\,(I) - \text{trace}\,(A N^{-1} A^T P) \\ &= n - \text{trace}\,(N^{-1} A^T P) \\ &= n - \text{rank}\,(N) \\ &= n - \text{rank}\,(A) \\ &= n - m \end{aligned} \qquad (4\text{-}6)$$

(for the extended Gauss-Markoff model, the right side of equation 4-6 will be $n + d - m$).

Since (n-m) is the degree of freedom, or total redundancy, in the Gauss-Markoff model of full rank, it can be seen that:

$$\sum_{i=1}^{n} (r_i) = \text{trace}\,(Q_{\hat{\varepsilon}} P) = n - m \qquad (4\text{-}7)$$

where r_i is the diagonal element of ($Q_{\hat{\varepsilon}} P$), which is called the redundancy number and indicates the contribution of the i-th observation to the overall degree of freedom.

Under the assumption that there is no correlation between the observations, i.e., P is a diagonal matrix, it can be written that:

$$r_i = q_i\, p_i \qquad (4\text{-}8)$$

where q_i and p_i are i-th diagonal element of $Q_{\hat{\varepsilon}}$ and P respectively.

From equation (4-2), and observing $Q_{\hat{\varepsilon}}$ as being non-negative definite matrix, it is concluded that:

$$O \le q_i \le 1/p_i \qquad (4\text{-}9)$$

which, when multiplying by p_i, yields

$$O \le r_i \le 1 \qquad (4\text{-}10)$$

If follows from equation (4-1) that

$$Q_{\hat{Y}} = Q_Y - Q_{\hat{\varepsilon}} \qquad (4\text{-}11)$$

Concerning equations (4-10) and (4-11), many investigators (e.g. Caspary 1987 and Leick 1990) provide some interpretations for redundancy number in different cases as follows:

(i) If the redundancy number is close to zero, that means that the variance of the adjusted observation is close to the variance of the observation itself since equation (4-11) will be $Q_{\hat{Y}} \approx Q_Y$. This implies that the increase in precision of this adjusted observation is low.

(ii) If the redundancy number is zero, the corresponding i-th observation is not checked by any other observation since, in this case, equation (4-3) turns to be

$$r_i = (Q_{\hat{\varepsilon}} P)_i = 1 - [Q_{\hat{Y}i} P_i] = 0$$
$$= 1 - [P_{Yi} / P_{\hat{Y}i}] = 0$$

i.e.,

$$P_{\hat{Y}i} = P_{Yi} \qquad (4\text{-}12)$$

Also, some parameters (e.g. the distant end point of a no-check observation) cannot be computed without this uncontrolled observation. Therefore, any undetected gross error in this observation is directly transferred into the estimated parameters.

(iii) On the other hand, if the redundancy number is close to one, i.e., $P_{Yi} / P_{\hat{Y}i}$ almost equals zero, it indicates that the observations are adjusted with high precision.

(iv) Consequently, if r_i is one, that means that the i-th adjusted observation is perfectly checked by the model since equation (4-3) becomes

$$r_i = (Q_{\hat{Y}} P)_i = 1 - (Q_{\hat{Y}} P)_i = 1$$
$$= 1 - [P_{Yi} / P_{\hat{Y}i}] = 1$$

i.e.,

$$P_{\hat{Y}i} = \infty \qquad (4\text{-}13)$$

So, in this case any gross error will be revealed in the residual of this observation with the result that it will not have any effect at all on the estimation of the unknown parameters (Kavouras 1982). Examples of typical values of redundancy numbers in geodetic networks are given in table 2 (Caspary 1987):

Table 2: Typical redundancy numbers in geodetic networks

Network Type	Values of Redundancy Contribution
Traverse net	0.1 – 0.2
Trialateration net	0.3 – 0.6
Combined net	0.5 – 0.8
Leveling net	0.2 – 0.5

Accordingly, redundancy numbers that are close to, or equal, zero can be thought of as an indication of poor geometry of some parts in a network, and a measure of low reliability of subsets of the observations.

4.3 Antenna Setup Error Sources

Generally speaking, reliability of geodetic operations requires, among other considerations, self-checking measurement procedures that provide checks for gross and systematic errors. The usual method of ensuring that the observations are valid is to repeat the observations under different circumstances. This replication, or redundancy, besides validating the observations, puts some limits on those contributions to errors that arise from setup and other systematic effects (Morgan et al. 1986).

The U.S National Geodetic Reference System specifications require 10 part-per-million (1:100,000) minimum geometric accuracy standard for first-order control surveys to meet mapping, land information, property, and engineering requirements (FGCC 1984). It is known that baselines can be measured routinely using the GPS with uncertainties of better than 10 ppm. Assuming that, with the recent advances in the GPS relative positioning, the measurement errors and biases have been eliminated by differencing or proper modelling, the errors associated with the antenna setup become more significant and need to be considered.

The error sources in setting up the antenna relative to the station mark are (Minkel 1989): collimation error; error in setting the tripod; and antenna height measurement error. Collimation error results from inaccurate plumbing over the ground station. Most tribraches have a 2-minute bull's-eye bubble that is used for levelling and plumbing. Moreover, a 2-miute error translated to a horizontal error of 0.001 m (1 mm) at an antenna height of 1.5 m (Minkel 1989). Another method for plumbing is to use a plum bob, which under calm weather conditions, may be accurate to a millimeter or so. Therefore, a plumb bob could be a simple, quick, and good check on the optical plummet. Tripod setting might occur and cause significant errors in both collimation and antenna height measurements. Precautions should be considered when setting the tripod in loose soil.

Probably, the most important error source is that due to inaccuracies in the height determination of the antenna. Normally, a height is determined by taping the distance from the plumb point of the station to a reference point on the antenna. The distance from the reference point to the phase center of the antenna is known and given by the receiver's manufacturer. From these two distances, the antenna height (from the phase center to the plumb point of the station) is computed. Therefore, there are two possible sources of error in the antenna height determination. One error may be in measuring or recording the distance from the antenna reference point to the plumb point. The second error might be in using the incorrect edge-to-center distance. The later error may occur in surveys with different types of antennas, where an incorrect distance may be used for the antenna.

Some precautionary guidelines are presented by the FGCC (1989) for GPS field procedures in order to minimize the antenna setup errors. These instructions could be summarized as follows:

(a) The antenna height should be measured in both feet and metric units.
(b) The antenna height is preferred to be measured before and after each survey session.
(c) Checks should be performed for collimation and levelling before and after each occupation.
(d) The plumb bob, or any other independent plumb point check, should be used.
(e) Frequently, the optical plummets should be checked and adjusted if required.

(f) If an antenna is moved during an observing session, the set of observations for that session may not be acceptable.

(g) For all survey orders and classes, the antenna must be stably located over the mark for the duration of the observing session within allowable setup error which is determined from the equation:

$$k = 0.1\, p\, d\, (\beta) \qquad (4\text{-}14)$$

where,

k = the repeatable setup error in (cm) for any component (horizontal or vertical) at the 95% confidence level.
p = minimum geometric standard in parts-per-million (ppm).
d = distance between any two stations of a survey (km).
β = 0.05 = critical region factor for the 95% confidence level (β = 1 – 0.95 = 0.05).

Some examples of the allowable setup errors are given in table 3 (FGCC 1989):

Table 3: Allowable Setup Errors (k) in Centimeters at 95% level of Confidence
(with k_{min} = 0.3 cm and k_{max} = 10 cm)

Survey Class	p (ppm)	d (km)				
		0.01	0.1	1.0	10	100
AA	0.01	0.3	0.3	0.3	0.3	0.3
A	0.10	0.3	0.3	0.3	0.3	0.3
B	1.00	0.3	0.3	0.3	0.3	0.3
1	10.0	0.3	0.3	0.3	0.5	5.0
2-I	20.0	0.3	0.3	0.3	1.0	10.0
2-II	50.0	0.3	0.3	0.3	2.5	(10)
3-I	100.0	0.3	0.3	0.5	5.0	(10)

Moreover, committing an undetectable setup error is still possible with the result that the least-squares results might be significantly influenced. A method is required to obtain some measure about the sensitivity of the estimated parameters to such undetectable errors.

4.4 A Proposed Method for Detecting Inconsistent Observations Associated with Incorrect Antenna Placements

The proposed procedure is based on analyzing the least-squares adjustment results to reveal a measure of the "sensitivity" of detecting any existent setup errors. It should be emphasized that the suggested method is not intended to compute the magnitude of such setup errors, if any. The main purpose, here, is to obtain "an indication of controllability" of the setup errors and, therefore, reveal information about the reliability of the estimated parameters.

We start again with equation (3-8) for the residuals vector:

$$\varepsilon^{\wedge} = Y - A\,X^{\wedge} = (\,I - A\;N^{-1}\;A^{T}\,P\,)\,Y$$

Instead of using the model

$$Y - \varepsilon = A\,X$$

For the observation equations, we will use an extended model:

$$Y - \varepsilon = A\,X + B\,\tau \qquad (4\text{-}15)$$

which is often used in the hypothesis testing for outliers (Kok 1984, Koch 1988). τ is an t x 1 vector which contains the setup errors assumed to be present in the observations, and B is an n x t coefficient matrix corresponding to the derivatives of the observations with respect to those assumed errors such that rank (A, B) $\leq$ n.

The symbol "t" refers to number of possible setup errors. In the network mode of GPS surveying, a number of receivers are deployed to a number of stations to collect data simultaneously in the so-called "session". Some stations are occupied more than once, in different sessions, in order to obtain redundancy in the GPS networks. This leads to a number of total setup operations greater than the number of stations, with the result that the number of possible setup errors, t, is greater than the number of the stations, m.

Substituting equation (4-15) in equation (3-8) yields the following:

$$\varepsilon^{\wedge} = [\,I - A\;N^{-1}\;A^{T}\,P\,]\;[\,A\,X + B\,\tau\,] \qquad (4\text{-}16)$$

Differentiating (4-16) with respect to the setup error vector τ yields:

$$\delta\varepsilon^{\wedge} / \delta\tau = [\, I - A\ N^{-1}\ A^{T}\ P\,]\ B \qquad (4\text{-}17)$$

where $(\delta\varepsilon^{\wedge}/\delta\tau)$ is an n x t matrix that describes the sensitivity of the residuals, $\varepsilon^{\wedge}$, to detect effects of setup errors, τ.

In order to interpret the results of equation (4-17), two different situations are discussed:

4.4.1 Detecting Uncontrolled Setup Errors:

The worst situation occurs when

$$\delta\varepsilon^{\wedge} / \delta\tau = 0 \qquad (4\text{-}18)$$

which happens if and only if the matrix B belongs to the null space of the matrix $[\, I - A\ N^{-1}\ A^{T}\ P\,]$, i.e., the range space of the design matrix A. Therefore, the matrix B can be expressed as a linear combination of A:

$$B = A\ G \qquad (4\text{-}19)$$

for some matrix G (i.e., rank [A,B] = rank A).

It is known that the matrix $[\, I - A\ N^{-1}\ A^{T}\ P\,]^{T}$ is orthogonal to the design matrix A since

$$[\, I - A\ N^{-1}\ A^{T}\ P\,]\ A = 0 \qquad (4\text{-}20)$$

Therefore, an additional vector $B\ \tau$ with

$$B\ \tau \in R(A) \qquad (4\text{-}21)$$

Does not change the residuals, if they are computed via (3-8).

A geometrical interpretation of this result is shown in figure 4. In this case, the setup errors will not be revealed in the residuals (and, hence, do not affect the adjusted observations), but are directly transferred to the estimation of the parameters. Accordingly, it can be said that if $\delta\varepsilon^{\wedge}/\delta\tau$ equals zero, the setup errors are "uncontrolled". This situation is the case of no-check observations.

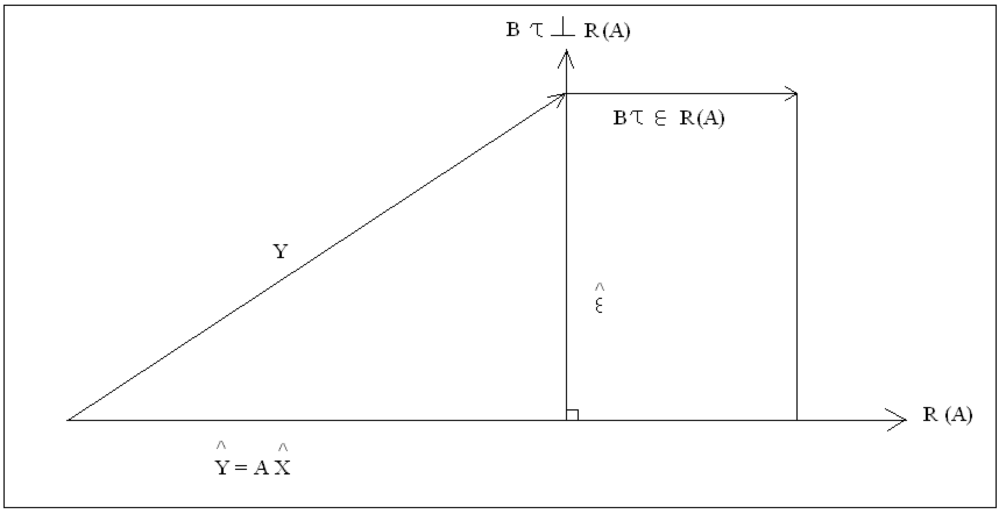


Figure 4: Influence of Setup Errors on the Residuals

4.4.2 Detecting Controlled Setup Errors:

The best situation is encountered when

$$A^T P B = 0 \tag{4-22}$$

which yields (from equation 4-17):

$$\delta\varepsilon^{\wedge} / \delta\tau = B \tag{4-23}$$

Integrating (4-23) gives

$$\varepsilon^{\wedge} = B \tau + \varepsilon \tag{4-24}$$

Equation (4-24) could be interpreted as the estimated residuals, $\varepsilon^{\wedge}$, and describes effects of both the observational error, ε , and the setup errors, τ. Therefore, the adjusted observations are determined with high precision which results in an accurate estimation of the parameters. In this case, it can be said that the setup error are "controlled".

A measure that is easy to interpret, in any situation, is developed based on the computed n x t matrix $(\delta\varepsilon^{\wedge}/\delta\tau)$. It can be realized from equation (4-17) that each column of this matrix corresponds to one of the setup error. Therefore, it could be written that

$$(\delta\varepsilon^{\wedge}/\delta\tau)_i = [\, I - A\ N^{-1}\ A^T\ P \,]\ b_i \tag{4-25}$$

for i = 1, 2, …. t

A weighted L2-norm of this column (or vector) is

$$\| (\delta\varepsilon^{\wedge}/\delta\tau)_i \|^2_P = (\delta\varepsilon^{\wedge}/\delta\tau)_i^T \quad P \ (\delta\varepsilon^{\wedge}/\delta\tau)_I \tag{4-26}$$

which, after some manipulation, can be expressed as

$$\| (\delta\varepsilon^{\wedge}/\delta\tau)_i \|^2_P = b_i^T [P - P A N^{-1} A^T P] b_i \tag{4-27}$$

A normalized weighted L2-norm of this vector is

$$S_i = b_i^T [P - P A N^{-1} A^T P] b_i \ / \ b_i^T \ P \ b_i \tag{4-28}$$

The maximum value of S_i occurs when A^T P B =0, and therefore $(S_i)_{max}$ = 1. That is corresponding to the case 4.4.2 when the setup errors are 100% revealed in the residuals. On the other hand, S_i equals zero when the setup errors belong to the range space of the design matrix A, and they cannot be detected in the residuals, i.e., no-check observations.

Accordingly, it can be written that

$$0 \le S_i \le 1 \tag{4-29}$$

This indication, S_i , is called a "measure of sensitivity" since it describes the sensitivity of the residuals to detect the effects of setup errors.

4.5 The Use of the Produced Method for Detecting No-Check Observations

A no-check observation is a result of deficiencies in planning the observation campaign. This type of observation defects has to be located to detect weak parts of the network. A no-check observation is characterized by a setup error that lies in the range space of the design matrix A, with the result that these errors will be transferred directly to the estimated parameters. Based on this rule, it can be expected that this error cannot be revealed in the residuals. That was the key used in the proposed method to detect no-check observations.

Depending on how the matrix B was constructed, it can be expressed as a linear combination of the coefficient matrix A; in this case: B (nxt) = A (n,m) L (m,t) for some matrix L. According to the argument of case

(4.4.1), this situation will cause $(\delta\hat{\varepsilon}/\delta\tau)$ to be zero since the setup error vector, $B\ \tau$ belongs to R(A).

The use of the redundancy number may not be the finest tool in recognizing no-check observations. This routine relies on examining the diagonal element of the matrix$[\ I - A\ N^{-1}\ A^T\ P\ ]$. It is known that this matrix is, in general, not a diagonal matrix. Therefore, neglecting the influences of the off-diagonal elements might produce misleading interpretation.

The proposed procedure has some merits, over the redundancy number concept, in terms of detecting no-check observations. The method carries the test on the obtained $(\delta\hat{\varepsilon}/\delta\tau)$ matrix, which is the product of multiplying $[\ I - A\ N^{-1}\ A^T\ P\ ]$ by B. That means that we have included some other information, throughout the matrix B, about possible setup errors that might be present in the observations. Also, it is known that a product of two matrices can contain some zero elements even if one of the matrices has non-zero diagonal elements. A third advantage is that the matrix $(\delta\hat{\varepsilon}/\delta\tau)$ reveals the influences of the off-diagonal elements of the original matrix $[\ I - A\ N^{-1}\ A^T\ P\ ]$. Therefore, the suggested method identifies no-check observations more effectively.

The main advantage of the proposed method is its simplicity which enables it to be included in any least-squares adjustment procedure without additional computations. The method is directly based on the obtained least-squares results and is used efficiently for both the original two objectives: (1) obtaining a sensitivity measure about the controllability of setup errors; and (2) detecting no-check observations in a GPS network.

4.6 A Practical Experiment with a Small GPS Network

A small GPS network was used to test the developed program and the proposed method of detecting inconsistent and no-check observations. The network, in figure 5, consists of 23 stations connected by 36 baselines. Station 1 has previously known accurate coordinates, in the GRS80 datum, and therefore was held fixed to overcome the expected three datum defects. The estimated Cartesian coordinates of the stations are presented in table 4. The adjusted baseline components along with their residuals are provided in table 5.

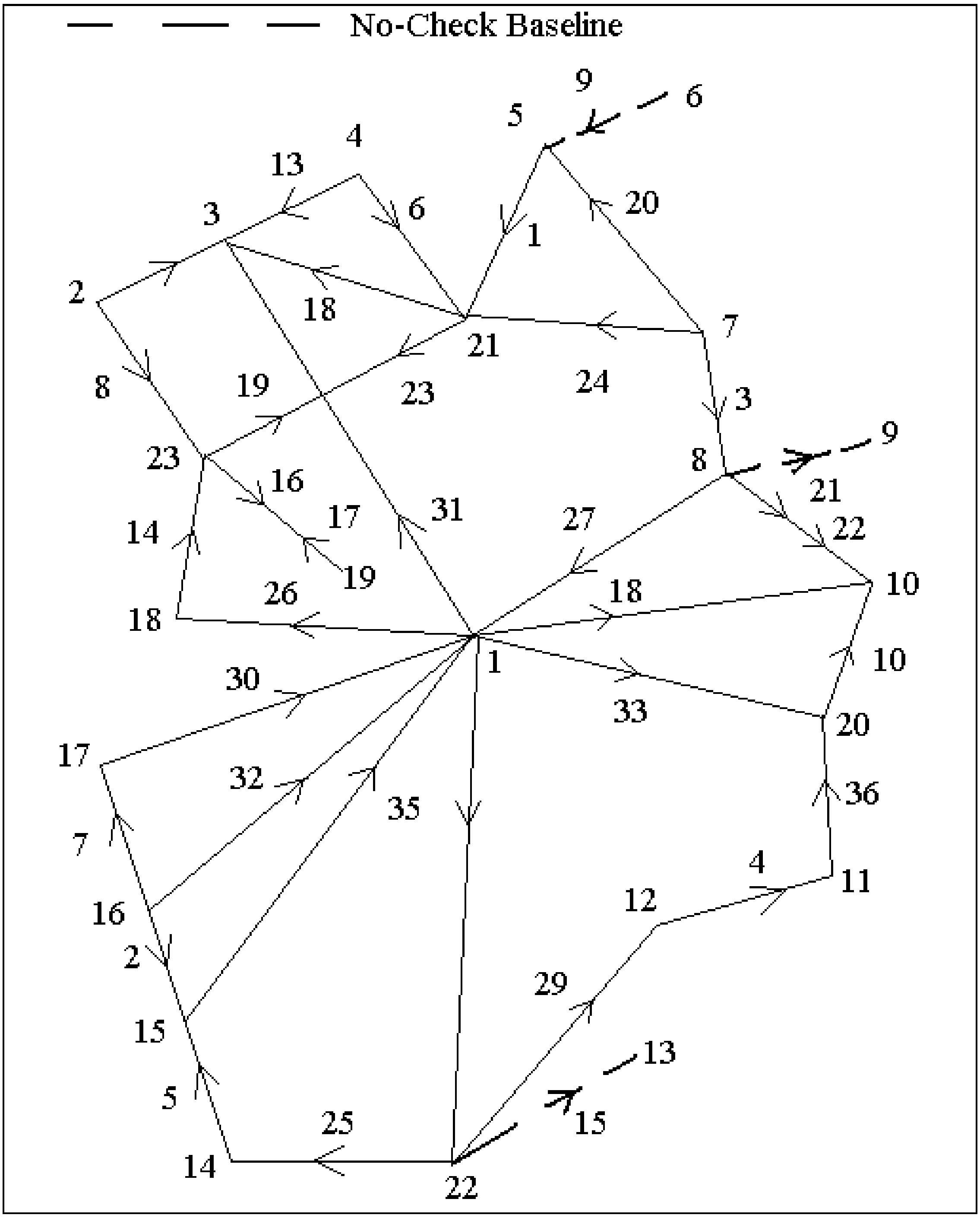


Figure 5: A Sketch of the Tested GPS Network

Table 4: The Estimated Cartesian Coordinates (m) of the GPS Network Stations

Station	X	(stdv)	Y	(stdv)	Z	(stdv)
1	593898.888	(0.000)	-4856214.546	(0.000)	4078710.706	(0.000)
2	592228.445	(0.081)	-4857180.590	(0.139)	4077844.926	(0.116)
3	592177.300	(0.078)	-4856760.593	(0.133)	4078352.338	(0.097)
4	592154.053	(0.085)	-4856289.716	(0.145)	4078915.013	(0.110)
5	592143.654	(0.076)	-4855982.308	(0.143)	4079276.772	(0.110)
6	592078.258	(0.089)	-4855599.001	(0.199)	4079741.541	(0.152)
7	592784.445	(0.071)	-4855427.648	(0.122)	4079821.118	(0.088)
8	593319.278	(0.056)	-4855416.043	(0.102)	4079738.310	(0.076)
9	593354.851	(0.084)	-4854952.029	(0.148)	4080287.962	(0.110)
10	593996.999	(0.044)	-4854862.455	(0.091)	4080291.250	(0.072)
11	595126.249	(0.060)	-4855004.190	(0.192)	4079989.698	(0.121)
12	595200.236	(0.061)	-4855346.527	(0.188)	4079573.785	(0.124)
13	595660.216	(0.076)	-4855788.863	(0.204)	4078986.747	(0.158)
14	595703.643	(0.060)	-4857185.141	(0.182)	4077307.170	(0.134)
15	595157.857	(0.045)	-4857243.109	(0.157)	4077326.103	(0.107)
16	594632.457	(0.041)	-4857273.274	(0.137)	4077342.816	(0.082)
17	594053.416	(0.064)	-4857338.677	(0.122)	4077351.903	(0.113)
18	593329.609	(0.083)	-4857089.844	(0.115)	4077767.661	(0.097)
19	593452.569	(0.086)	-4856581.524	(0.153)	4078345.751	(0.137)
20	594576.862	(0.041)	-4855007.774	(0.124)	4080036.405	(0.078)
21	592709.211	(0.074)	-4856232.800	(0.127)	4078884.680	(0.092)
22	595768.849	(0.064)	-4856306.567	(0.177)	4078337.358	(0.134)
23	592759.323	(0.076)	-4856986.598	(0.131)	4077988.316	(0.099)

Table 5: The Adjusted Baseline Components (m) of the GPS Network

No.	From	To	dX	(V_{dX})	dY	(V_{dY})	dZ	(V_{dZ})
1	5	21	565.567	(0.058)	-250.491	(0.015)	-392.092	(-0.114)
2	16	15	525.401	(0.058)	39.165	(-0.207)	-16.712	(0.118)
3	8	7	-534.832	(0.041)	-11.605	(-0.059)	82.808	(0.032)
4	12	11	-73.987	(0.033)	342.338	(-0.062)	415.913	(0.031)
5	14	15	-545.786	(-0.057)	-48.968	(0.014)	18.933	(-0.029)
6	4	21	555.158	(-0.006)	56.917	(-0.004)	-30.334	(0.004)
7	16	17	-579.041	(-0.007)	-65.403	(0.028)	9.087	(-0.019)
8	2	23	530.878	(-0.001)	193.992	(-0.020)	143.390	(0.015)
9	6	5	65.387	(0.000)	-383.307	(0.000)	-464.769	(0.000)
10	20	10	-579.863	(0.032)	145.319	(-0.023)	254.846	(0.026)
11	2	3	- 51.145	(-0.000)	419.996	(0.007)	507.411	(-0.005)
12	8	9	35.573	(0.000)	464.014	(0.000)	549.652	(0.000)
13	4	3	23.247	(0.006)	-470.877	(-0.006)	-562.676	(0.003)
14	18	23	-570.286	(-0.019)	103.246	(0.085)	220.655	(-0.043)
15	22	13	-108.633	(0.000)	517.704	(0.000)	649.389	(0.000)
16	23	19	693.246	(0.000)	405.073	(0.002)	357.435	(-0.001)
17	19	23	-693.246	(0.000)	-405.073	(0.004)	-357.435	(-0.001)
18	21	3	-531.911	(0.019)	-527.794	(-0.035)	-532.342	(0.022)
19	23	21	-50.112	(-0.008)	753.798	(-0.015)	896.364	(-0.001)
20	7	5	-640.801	(0.045)	-554.661	(-0.019)	-544.346	(0.007)
21	8	10	677.721	(-0.037)	553.588	(0.061)	552.941	(-0.037)
22	8	10	677.721	(-0.035)	553.588	(0.055)	552.941	(-0.034)
23	21	23	50.112	(0.035)	-753.798	(-0.062)	-896.364	(0.024)
24	7	21	-75.234	(-0.145)	-805.152	(0.165)	-936.438	(-0.077)
25	22	14	-65.206	(-0.066)	-878.574	(0.025)	-1030.188	(-0.058)
26	1	18	-569.279	(-0.083)	-875.298	(0.088)	-943.045	(-0.048)
27	8	1	579.610	(-0.005)	-798.503	(0.006)	-1027.604	(0.005)
28	1	22	1869.962	(0.119)	-92.021	(0.443)	-373.348	(-0.354)
29	22	12	-568.613	(0.363)	960.040	(0.339)	1236.427	(-0.182)
30	17	1	-154.528	(-0.038)	1124.131	(0.050)	1358.803	(-0.035)
31	1	3	-1721.588	(-0.073)	-546.048	(0.089)	-358.368	(-0.051)
32	16	1	-733.569	(-0.027)	1058.728	(0.055)	1367.890	(-0.044)
33	1	20	677.975	(-0.059)	1206.771	(0.057)	1352.699	(-0.036)
34	1	10	`98.111	(-0.027)	1352.091	(0.052)	1580.545	(-0.019)
35	15	1	-1258.970	(-0.083)	1019.563	(0.231)	1384.602	(-0.154)
36	11	20	-549.386	(0.139)	-3.584	(0.312)	46.706	(0.171)

The matrix B, which contains the derivatives of observations with respect to the setup errors as appeared in equation 4-17, has n x t size, where n is the number of observations and t is the number of setup errors. It is inconvenient to construct B with this huge size. For example, the small network has been observed in 19 sessions with 54 different station occupations. That means that n = 36 x 3 = 108, and t = 54 x 3 = 162, which makes B require a computer space approximately 35 kilobytes. To overcome this problem, the test 4-17 is performed in a slightly different way. The strategy of performing the test is based on the idea that the occurrence of setup errors in a station will affect those baselines that pass by that station and are observed in the same session. With this concept, the test is carried out session by session and baseline by baseline. For each baseline, each of the end points is checked separately where n is now the number of baselines connected to this station in the current session. Consequently, the number of possible setup errors, t, is always 3. This procedure reduce the required size of matrices and, hence, facilitates checking.

A no-check observation, for example on baseline 9 in figure 5, will be the only baseline passing by one of its end points (station 6). In this case, the matrix B of that point will be a 3 x 3 matrix and is a linear combination of the corresponding part of the design matrix A. Therefore, equation (4-19) holds which validates the equality in equation (4-18). Consequently, this baseline is flagged to be a no-check observation. The complete results of the performed test are presented in table 6.

Table 6: Results of Detecting No-Check Observations in the GPS Network

Session No.	Baseline No.	From	(status)	To	(status)	Final Result
1	5	14	(O.K.)	15	(O.K.)	Good
1	25	22	(O.K.)	14	(O.K.)	Good
2	4	12	(O.K.)	11	(O.K.)	Good
3	15	22	(O.K.)	13	(N.C.)	No-Check
3	29	22	(O.K.)	12	(O.K.)	Good
4	16	23	(O.K.)	19	(O.K.)	Good
4	17	19	(O.K.)	23	(O.K.)	Good
4	19	23	(O.K.)	21	(O.K.)	Good
5	14	18	(O.K.)	23	(O.K.)	Good
6	13	4	(O.K.)	3	(O.K.)	Good
6	31	1	(O.K.)	3	(O.K.)	Good
7	28	1	(O.K.)	22	(O.K.)	Good
7	35	15	(O.K.)	1	(O.K.)	Good
8	2	16	(O.K.)	15	(O.K.)	Good
8	32	16	(O.K.)	1	(O.K.)	Good
9	7	16	(O.K.)	17	(O.K.)	Good
10	26	1	(O.K.)	18	(O.K.)	Good
10	30	17	(O.K.)	1	(O.K.)	Good
11	18	21	(O.K.)	3	(O.K.)	Good
11	23	21	(O.K.)	23	(O.K.)	Good
12	9	6	(N.C.)	5	(O.K.)	No-Check
12	20	7	(O.K.)	5	(O.K.)	Good
13	1	5	(O.K.)	21	(O.K.)	Good
13	6	4	(O.K.)	21	(O.K.)	Good
14	8	2	(O.K.)	23	(O.K.)	Good
14	11	2	(O.K.)	3	(O.K.)	Good
15	3	8	(O.K.)	7	(O.K.)	Good
15	24	7	(O.K.)	21	(O.K.)	Good
16	12	8	(O.K.)	9	(N.C.)	No-Check
16	22	8	(O.K.)	10	(O.K.)	Good
17	21	8	(O.K.)	10	(O.K.)	Good
17	27	8	(O.K.)	1	(O.K.)	Good
18	10	20	(O.K.)	10	(O.K.)	Good
18	34	1	(O.K.)	10	(O.K.)	Good
19	33	1	(O.K.)	20	(O.K.)	Good
19	36	11	(O.K.)	20	(O.K.)	Good

N.C. = No-Check Station

CHAPTER V

SUMMARY AND COINCLUSIONS

With the high accuracy of GPS measurements, post-adjustment analysis of the obtained lease-squares results become a must. It has been a matter of principle in every geodetic problem that not only the estimated solution but also a measure of its quality is provided. The quality of a network could be, generally, made up of three factors: Economy; Precision; and Reliability. The network reliability, which describes the ability of redundant observations to check model errors, is the major area of this study.

Due to lack of observations or weak geometry of a network, some observation and configuration defects exist. Among these defects, a no-check observation is of special interest because it can dramatically decrease the quality of the results. No-check observations may also be produced during the adjustment process itself in case of some observations being rejected when applying a testing procedure for outlier detection. Therefore, the detection of no-check observations in a GPS network cannot be based only on experience of survoyers when reviewing a sketch of the network. A more rigorous and effective test is needed to reveal these harmful situations. A careful analysis of the least-squares adjustment results, mainly the computed residuals, provides a basic tool for developing an efficient test to detect no-check observations.

Errors associated with antenna setup, relative to the station mark, contribute a significant error source in GPS measurements. Probably the largest error in antenna setup is because of inaccuracies and blunders in the antenna height measurements. This error may be due to an error in measuring the distance from the station mark to a reference point on the antenna edge, or an error in using the incorrect known distance from the reference point to the phase center of the antenna. Some precautionary instructions (reviewed in section 4.3) are recommended as field checks to detect such setup errors. However, these guidelines cannot assure that all possible errors are discovered in the field.

Based on analyzing the least-squares adjustment results, a proposed algorithm was constructed to detect no-check observations and, in the same time, to give a measure of sensitivity, which describes the controllability of setup errors. The performed analysis shows that the setup errors are not revealed in the residuals in case of no-check observations. Instead, these errors are completely transferred to the estimated parameters even though those parameters might have small standard deviations. This result highlights the importance of detecting no-check observations in GPS networks as far as the reliability is concerned.

The proposed test should be implemented as a post-adjustment analysis procedure in any GPS network. It was designed such that its integration in a least-squares routine is straightforward without additional sophisticated computations or large computer memory requirements. The result of the developed program are in table 6, which it is easily understood and interpreted by survoyers. The test is also recommended to be repeated again if some baselines are to be removed after being flagged as erroneous observations. This step is suggested because changing the network geometry might create new no-check observations which were previously checked by the rejected baselines. Therefore, the testing algorithm may be invoked more than once during the adjustment process.

Upon detecting no-check observations in a GPS network, the observations campaign should be modified in order to overcome these observation deficiencies. It is highly recommended that new baselines be observed to provide some redundant occupations of the no-check stations and, therefore, produce some checks for those uncontrolled observations.

The estimated parameters in a GPS network should not be the only important output of a least-squares adjustment process. Post-adjustment analysis is a helpful tool that reveals valuable information about the observations and the network geometry. Small standard deviations, as a measure of precision, may not be the only indication for a "good" GPS network. The ability of the network redundant observations to detect model errors is the second measure of quality control of GPS network. Detecting observation defects and producing strong sensitivity measures could be thought of as indicators for reliable GPS networks.

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APPENDIX A

NANI: A program for Network Adjustment with No-Check Identification

```
      PROGRAM NANI
c
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c
c     THE OHIO STATE UNIVERSITY – COLUMBUS, OHIO, USA
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c
c
c     This program adjusts a GPS network and identifies
c     no-check observations
c
c
c     Required Parameters:
c        nobs = No of observations = 3 * No. of baselines
c        npar = No. of parameters = 3 * No. of network stations
c        nses = No. of sessions in GPS campaign
c        maxses = Maximum No. of baselines observed in one session
c
c
      PARAMETERS (nobs =108, npar =69, nses = 19, maxses = 6)
c
c
      implicit double precision (a-h, o-z)
      double precision n1, m1
      integer*2 obs, obsn, ses, observ, jfrom, jto, session
c
c
      dimension p(3,3), b(3,3), a(3,3), coeff(3,npar), bpy(3), y(3), ising(5)
      dimension atpa(npar,npar), atpy(npar), x(npar), bp(3,3), bpa(3,3)
      dimension s(npar,npar), dummy(nobs), atpai(npar,npar), fixed(3)
      dimension v(nobs), v1(3) , pris(maxses*3,3)
      dimension bb(maxses*3,3), anat(maxses*3, maxses*3)

      dimension obs(maxses), iif(maxses), iit(maxses), aa(maxses*3,npar)
```

```
      dimension ap(maxses*3,maxses*3), kobs(maxses*3)
      dimension kfrom(maxses), kto(maxses), pp(3,3)
      dimension ap2(maxses*3,maxses*3), anatp(maxses*3,maxses*3)
      dimension temp1(3,3), temp2(3,3)
c
c
$large: atpa, s, atpai, atpy, v, dummy
c
c
c
      open (1, file = 'network.inp')
      open (2, file = 'netcov.inp')
      open (3, file = 'results.out')
      open (4, file ='fixed.inp')
c
c
c
      do 3 i = 1, npar
            atpy(i) = 0.d0
            do 3 j = 1, npar
                  atpa (i,j) = 0.d0
                  s (i,j) = 0.d0
3     continue
      nstat = npar / 3
      nbase = nobs / 3
      write (*,*) ' Please Wait'
      do 70 k1 = 1, nbase
            jk = 2*k1+k1-2
            do 8 i = 1, 3
                  dummy (i) = 0.d0
                  bpy (i) = 0.d0
            do 6 j = 1, 3
                  a(i,j) = 0.d0
                  b(i,j) = 0.d0
                  p(i,j) = 0.d0
                  bp(i,j) = 0.d0
                  bpa(i,j) = 0.d0
6           continue
            do 7 k = 1, npar
                  coeff(i,k) = 0.d0
7           continue
8           continue
c
```

```
c
        read(1,*,end=71) line, ifrom, ito, session, dx, dy, dz
        y(1)=0.d0
        y(2)=0.d0
        y(3)=0.d0
        do 10 i = 1, 3
                read (2,*) (p(i,j), j = 1,i)
10      continue
        do 20 i2 = 1, 2
        do 20 j2 = i2+1, 3
                p(i2, j2) = p(j2,i2)
20      continue
        call ainv(p,dummy,3)
        j1 = 2*ifrom+ifrom-2
        j2 = 2*ito+ito-2
        do 25 i = 1, 3
                coeff(i,j1+i-1)= -1.do
                coeff(i,j2+i-1)= -1.do
25      continue
        l1 = 1
        do 60 k2 = 1, npar
                l4 = 2*k2+k2-2
                do 30 i = 1,3
                do 30 j = 1,3
                        b(i,j) = coeff(i,j+l1-1)
30      continue
        l1 = l1 + 3
        call ab (b, p, bp, 3, 3, 3)
        call ab (bp, y, bpy, 3, 3, 1)
        atpy(l4) = atpy(l4)+bpy(1)
        atpy(l4+1) = atpy(l4+1)+bpy(2)
        atpy(l4+2) = atpy(l4+2)+bpy(3)
        l2 = 1
        do 50 k3 = 1, nstat
                do 35 i = 1, 3
                do 35 j = 1, 3
                        a(i,j) = coeff(i,j+l2-1)
35      continue
        call ab (bp, a, bpa, 3, 3, 3)
        l3 = 1
        do 40 i = 1, 3
        do 40 j = 1, 3
                atpa(i+l4-1,j+l2-1)=atpa(i+l4-1,j+l2-1)+bpa(i,j)
```

```
40      continue
        l2 = l2+3
50      continue
60      continue
70      continue
c
c
71      read(4,*) icode
        If(icode.eq.1) then
                read (4,*) ae
                read (4,*) finv
                read (4,*) nfix
                read (4,*) (fixed(i), i=1,3)
                call togeod (ae,finv,fixed(1),fixed(2),fixed(3),dlat,dlon,h)
                call dtodms (dlat,latd,latm,slat)
                call dtodms (dlon,lond, lonm, slon)
        elseif(icode.eq.2) then
                read (4,*) nfix
                read (4,*) ae
                read (4,*) finv
                read (4,*) latd, latm, slat
                read (4,*) lond, lonm, slon
                read (4,*) h
                call dmstod (latd,latm,slat,dlat)
                call dmstod (lond,lonm,slon,dlon)
                call frgeod (ae,finv,fixed(1),fixed(2),fixed(3),dlat,dlon,h)
        else
                write(*,*) 'error in CONVERT: Check the Fixed data file'
                write(3,*) 'error in CONVERT: Check the Fixed data file'
                go to 281
        endif
        locat=2*nfix+nfix-2
        do 101 i=locat,locat+2
101     atpa(i,i)=1d20
        do 102 j = 1,5
102     ising(i)=0
        call chdec(npar,atpa,s,nsing,ising)
        if (nsing.gt.0) then
                write(3,31) nsing
                write(*,31) nsing
                go to 456
        endif
        call ainv(s,dummy,npar)
```

```
      call atra(s,atpai,npar,npar)
      do 829 i=1,npar
      do 829 j=1,npar
         if(abs(atpai(i,j).lt.1d-12) atpai(i,j)=0.d0
829   continue
      call ab(atpai,atpy,x,npar,npar,1)
      do 103 i=1,npar
         if(abs(x).le.1d-12) x(i)=0.d0
103   continue
      l5=1
104   do 105 i=1,3
105   x(i+l5-1)=x(i+l5-1)+fixed(i)
      l5=l5+3
      if(l5.lt.npar) go to 104
      vtpv=0.d0
      l1=1
      rewind 1
      rewind 2
      do 170 k7=1,nbase
         do 108 i=1,3
            y(i)=0.d0
            v1(i)=0.d0
            do 106 j=1,3
106         p(i,j)=0.d0
         do 107 k=1,npar
            coeff(i,k)=0.d0
107      continue
108      continue
      read(1,*,end=171) line,ifrom,ito,session,dx,dy,dz
      y(1)=dx
      y(2)=dy
      y(3)=dz
      do 110 i=1,3
         read(2,*) (p(i,j),j=1,i)
110   continue
      do 120 i2=1,2
      do 120 j2=12+1,3
         p(i2,j2)=p(j2,i2)
120   continue
      call ainv(p,dummy,3)
      j1=2*ifrom+ifrom-2
      j2=2*ito+ito-2
      do 125 i=1,3
```

```
          coeff(i,j1+i-1)=-1.d0
          coeff(i,j2+i-1)=-1.d0
125   continue
      call ab(coeff,x,v1,3,npar,1)
      do 130 i=1,3
130       v1(i)=v1(i)-y(i)
      sumf=0.0
      call atba (v1,p,sumf,dummy,3,1)
      ctpv=vtpv+sumf
      do 135 i=1,3
135   v(l1+i-1)=v1(i)
      l1=l1+3
170   continue
171   asigma=dsqrt(vtpv/(nob-npar+3))
      do 190 i=1,npar
      do 190 j=1,npar
          atpai(i,j)=asigma*asigma*atpai(i,j)
190   continue
      write(3,4) nbase
      write(3,5) nstat
      write(3,84) nnfix
      do 83 i=1,nstat
          l9=2*i+i-2
          stdvx=dsqrt(atpai(l9,l9))
          stdvy=dsqrt(atpai(l9+1,l9+1))
          stdvz=dsqrt(atpai(l9+2,l9+2))
          write(3,97) i,x(l9),stdvx,x(l9+1),stdvy,x(l9+2),stdvz
83    continue
      write(3,96) asigma
      write(3,99)
      rewind 1
      ly=1
      do 280 k8=1,nbase
          l8=2*k8+k8-2
          d0 208 i=1,3
              y(i)=0.d0
              v1(i)=0.d0
208       continue
      read(1,*,end=281) line,ifrom,ito,session,dx,dy,dz
      y(1)=dx
      y(2)=dy
      y(3)=dz
      do 235 i=1,3
```

```
235          v1(i)=y(i)-v(ly+i-1)
      write(3,199) line,ifrom,ito,v1(1),v(l8),v1(2),v(l8+1),v1(3),v(l8+2)
      ly=ly+3
280   continue
281   f=1.d0/finv
      esq=(2.d0-f)*f
      esqone=1.d0-esq
      pi=4.d0*datan(1.d0)
      ro=pi/180.d0
      second=206265.d0
      write(3,284) nfix
      do 380 i=1,nstat
             j9=2*i+i-2
             call togeod(ae,finv,x(j9),x(j9+1),x(j9+2),dlat,dlon,h)
             call dtodms(dlat,latd,latm,slat)
             call dtodms(dlon,lond,lonm,slon)
             do 330 i1=1,3
             do 330 j1=1,3
                    a(i1,j1)=0.d0
                    b(i1,j1)=0.d0
330          continue
      sinp=dsin(dlat*ro)
      cosp=dcos(dlat*ro)
      sin1=dsin(dlon*ro)
      cos1=dcos(dlon*ro)
      w=dsqrt(1.d0-esq*sinp*sinp)
      m1=ae*esqone/w/w/w
      n1=ae/w
      p(1,1)=-sinp*cos1*second/(m1+h)
      p(1,2)=-sinp*sin1*second/(m1+h)
      p(1,3)=cosp*second/(m1+h)
      p(2,1)=-sin1*second/(n1+h)/cosp
      p(2,2)=cos1*second/(n1+h)/cosp
      p(2,3)=0.d0
      p(3,1)=cosp*cos1
      p(3,2)=cosp*sin1
      p(3,3)=sinp
      do 340 i2=1,3
      do 340 j2=1,3
             a(i2,j2)=atpai(i2+j9-1,j2+j9-1)
340   continue
      call abat(p,a,b,dummy,3,3)
      stphi=dsqrt(b(1,1)*30.922d0)
```

```
      stlam=dsqrt(b(2,2)*30.922d0)
      sth=dsqrt(b(3,3))
      write(3,385) I,latd,latm,slat,stphi,lond,lonm,slon,stlam,h,sth
380   continue
456   write(3,457)
      call nocheck(nob,npar,nses,maxses,atpai,bb,anat,pris,obs,iif,
     + iit,aa,ap,kobs,kfrom,kto,pp,ap2,anatp,temp1,temp2,dummy)
c
c
4     format(2x,'No of baselines =',i3,/)
5     format(2x,'No of stations =',i3,/)
31    format(/,5x,'After fixing the datum, ',i2,' singularities have been
     +  encountered !',//,10x,' Please, check your network.',
     +  //,10x,'The program is terminated')
72    format(6(f11.5))
80    format(i3,3(3x,f15.5))
84    format(//,5x,'THE ESTIMATED PARAMETERS in (GRS80)
     +   Cartesian Coordinates:',//,
     +  7x,'(Note that station No.',i3,' has been fixed)',
     +  ///,'Station',3x,'X     (stdev)',9x,'Y     (stdev)',12x,
     +  'Z      (stdev)',/)
96    format(///,5x,'A-POSTERIORI STANDARD DEVIATION OF',
     +  'UNIT WEIGHT σ = ',f10.5)
97    format(i3,3(2x,f14.3,1x,'(',f6.3,')'))
99    format(//,5x,'THE ADJUSTED OBSERVATIONS AND THEIR
     +  RESIDUALS:',///,
     +  'Vector',3x,'From',3x,'To',/,13x,'dX  (vdX)',10x,'dY',
     +  4x,'(vdY)',12x,'dZ    (vdZ)',/)
199   format(i3,4x,i4,3x,i4,/,8x,3(2x,f11.3,1x,'(',f6.3,')'))
284   format(///,5x,'THE ESTIMATED PARAMETRS IN (GRS80)
     +   Geodetic Coordinates:',//,
     +  7x,'(Note that station No.',i3,' has been fixed)',
     +  ///,'Station',3x,'Latitude     (st.dv)',6x,'Longitude     (st.dv)',8x,
     +  'Height      (st.dv)',/
     + 11x,'d:m:sec     (m)',11x,'d:m:sec     (m)',10x,
     +  '   m     (m)',/)
385   format(i2,2x,i4,1x,i2,1x,f8.5,2x,'(',f6.3,')',1x,i4,1x,i2,1x,
     +  f8.5,1x,'(',f6.3,')',3x,f9.3,'(',f6.3,')')
457   format(///,20x,'End of Adjustment Computations')
c
c
c
      END
```

```
      subroutine nocheck(nobs,npar,nses,maxses,atpai,b,anat,pris,obs,
     + ifrom,ito,a,p,kobs,kfrom,kto,pp,p2,anatp,temp1,temp2,summy)
c
c
c     This subroutine for detecting No-Check observations in a GPS
c     network.
c
      implicit real*8 (a-h,o-z)
      integer*2 obs,obsn,ses,observ,jfrom,jto,session
c
      dimesion atpai(npar,npar),b(maxses*3,3)
      dimesion anat(maxses*3,maxses*3),pris(maxses*3,3)
      dimesion obs(maxses),ifrom(maxses),ito(maxses)
      dimesion a(maxses*3,npar),p(maxses*3,maxses*3)
      dimesion kobs(maxses),kfrom(maxses),kto(maxses)
      dimesion pp(3,3),p2(maxses*3,maxses*3)
      dimesion dummy(nobs),anatp(maxses*3,maxses*3)
      dimesion temp1(3,3),temp2(3,3)
c
$large:atpai,dummy
c
      open (1,file='network.inp')
      open (2,file='netcov.inp')
      open (3,file='results.out')
c
      write(3,901)
      rewind 1
      rewind 2
      write(*,*) 'no-check detection'
      write(*,*) '    '
      ses=1
1     do 5 i=1,maxses
      ifrom(i)=0
      kfrom(i)=0
      ito(i)=0
5     kto(i)=0
      do 9 i=1,maxses*3
            do 6 kk=1,3
                  pris(i,kk)=0.d0
6           b(i,kk)=0.d0
      do 7 k=1,npar
7     a(i,k)=0.d0
      do 8 j=1,maxses*3
```

```
            p(i,j)=0.d0
            p2(i,j)=0.d0
8           anat(i,j)=0.d0
9       continue
        write(*,*) 'session # ',ses
        obsn=1
        do 100 i=1,nobs
            read(1,*,end=101) observ,jfrom,jto,session,dx,dy,dz
            do 55 i2=1,3
            do 55 j2=1,3
55          pp(i2,j2)=0.d0
            do 60 i2=1,3
60          read(2,*) (pp(i2,j2),j2=1,i2)
            if(session.ne.ses) go to 100
            obs(obsn)=observ
            ifrom(obsn)=jfrom
            ito(obsn)=jto
            do 65 i2=1,2
            do 65 j2=1+1,3
                pp(i2,j2)=pp(j2,i2)
65          continue
            kk=obsn*2+obsn-2
            do 70 i3=1,3
            do 70 j3=1,3
                p(kk+i3-1,kk+j3-1)=pp(i3,j3)
                p2(kk+i3-1,kk+j3-1)=pp(i3,j3)
70          continue
            obsn=obsn+1
100     continue
101     obsn=obsn-1
        kgomaa=1
1002    i0=1
        iff=1
        kobs(iff)=obs(i0)
        kfrom(iff)=ifrom(i0)
        kto(iff)=ito(i0)
        itest=ifrom(i0)
        do 110 i=2,obsn
            if(ifrom(i).eq.itest) then
                iff=iff+1
                kobs(iff)=obs(i)
                kfrom(iff)=ifrom(i)
                kto(iff)=ito(i)
```

```
            elseif(ito(i).eq.itest) then
                  iff=iff+1
                  kobs(iff)=obs(i)
                  kfrom(iff)=ifrom(i)
                  kto(iff)=ito(i)
            endif
110   continue
      do 130 i=1,iff
            kk=2*i+i-2
            jj=2*kfrom(i)+kfrom(i)-2
            jjj=2*kto(i)+kto(i)-2
            do 120 l=1,3
                  a(l+kk-1,l+jj-1)=-1.d0
                  a(l+kk-1,l+jjj-1)=-1.d0
120         continue
130   continue
      do 150 i=1,iff*3
      do 135 k=1,napar
            dummy(k)=0.d0
            do 135 l=1,npar
135   dummy(k)=dummy(k)+a(i,l)*atpai(l,k)
      do 150 j=i,iff*3
            anat(i,j)=0.d0
            do 140 l=1,npar
140               anat(i,j)=anat(i,j)+dummy(l)*a(j,l)
150   anat(j,i)=anat(i,j)
      if(iff.ne.1) then
            do 170 ii=1,iff-1
                  jm=2*ii+ii-2
                  jm1=2*ii+ii+1
                  do 160 i=1,jm+2
                  do 160 j=jm1,3*iff
                        cc=dsqrt(p(i,j))
                        dd=dsqrt(p(j,j))
                        p(i,j)=0.2d0*cc*dd
                        p(j,i)=p*(i,j)
160               continue
170         continue
      endif
      do 180 i=1,npar
180   dummy(i)=0.d0
      im=iff*3-1
      do 185 k=1,iff*3
```

```
      do 182 j=1,im
         dummy(j)=p(i,j+1)/p(1,1)
182   continue
      dummy(iff*3)=1/p(1,1)
      do 184 l=1,im
         do 183 j=1,im
            p(l,j)=p(l+1,j+1)-p(l+1,1)*dummy(j)
183      continue
         p(l,iff*3)=-p(l+1,1)*dummy(iff*3)
184   continue
      do 185 j=1,iff*3
         p(iff*3,j)=dummy(j)
185   continue
      do 190 l=1,iff*3
      do 190 m=1,iff*3
         anatp(l,m)=0.d0
      do 190 n=1,iff*3
190   anatp(l,m)=anatp(l,m)+anat(l,n)*p(n,m)
      do 196 i=1,iff*3
      do 196 j=1,iff*3
         if(i.eq.j) anatp(i,j)=1.d0-anatp(i,j)
         if(i.ne.j) anatp(i,j)=-anatp(i,j)
196   continue
      do 200 i=1,3
200   b(i,j)=1.d0
      if(iff.ne.1) then
         if=1
         do 230 ji=2,iff
            if(ifrom(ji).eq.itest) then
               if=if+1
               km=2*if+if-2
               b(km,1)=1.d0
               b(km+1,2)=1.d0
               b(km+2,3)=1.d0
            elseif(ito(ji).eq.itest) then
               if=if+1
               kkm=2*if+if-2
               b(kkm,1)=1.d0
               b(kkm+1,2)=1.d0
               b(kkm+2,3)=1.d0
            endif
230      continue
      endif
```

```
          do 240 l=1,iff*3
          do 240 m=1,3
               pris(l,m)=0.d0
          do 240 n=1,iff*3
240       pris(l,m)=pris(l,m)+anatp(l,n)*b(n,m)
          noch=0
          do 250 i=1,iff*3
          do 250 j=1,3
               if(abs(pris(i,j)).lt.1.d-12) pris(i,j)=0.d0
               if(pris(i,j).eq.0.d0) noch=noch+1
250       continue
          itest=ito(i0)
          ifd=1
          do 610 i=2,obsn
               if(ifrom(i).eq.itest) then
               do 610 i=2,obsn
                    if(ifrom(i).eq.itest) then
                         ifd=ifd+1
                         kobs(ifd)=obs(i)
                         kfrom(ifd)=ifrom(i)
                         kto(ifd)=ito(i)
                    elseif(ito(i).eq.itest) then
                         ifd=ifd+1
                         kobs(ifd)=obs(i)
                         kfrom(ifd)=ifrom(i)
                         kto(ifd)=ito(i)
                    endif
610            continue
          do 614 i=1,18
          do 614 j=1,npar
614       a(i,j)=0.d0
          do 630 i=1,ifd
               kk=2*i+i-2
               jj=2*kfrom(i)+kfrom(i)-2
               jjj=2*kto(i)+kto(i)-2
               do 620 l=1,3
                    a(l+kk-1,l+jj-1)=-1.d0
                    a(l+kk-1,l+jj-1)=-1.d0
620            continue
630       continue
          do 634 i=1,18
          do 634 j=1,18
634       anat(i,j)=0.d0
```

```
      do 650 i=1,ifd*3
      do 650 k=1,npar
          dummy(k)=0.d0
          do 635 l=1,npar
635       dummy(k)=dummy(k)+a(i,j)*atpai(l,k)
      do 650 j=i,ifd*3
          anat(i,j)=0.d0
          do 640 l=1,npar
640       anat(i,j)=anat(i,j)+dummy(l)*a(j,l)
650   anat(j,i)=anat(i,j)
      if(ifd.ne.1) then
          do 655 ii=2,ifd
              if(kobs(ii).ne.obs(ii)) then
                do 654 i3=ii+1,obsn
                  if(kobs(ii).eq.obs(i3)) then
                    jj=2*ii+ii-2
                    kk=2*i3+i3-2
                    do 653 i=1,3
                    do 653 j=1,3
                      p2(i+jj-1,j+jj-1)=p2(i+kk-1,j+kk-1)
653                 continue
                  endif
654           continue
              endif
655       continue
      endif
      if(ifd.ne.1) then
          do 670 ii=1,ifd-1
              jm=2*ii+ii-2
              jm1=2*ii+ii+1
              do 660 i=1,jm+2
              do 660 j=jm1,3*ifd
                  cc=dsqrt(p2(i,i))
                  dd=dsqrt(p2(j,j))
                  p2(i,j)=0.2d0*cc*dd
                  p2(j,i)=p2(i,j)
660           continue
670       continue
      endif
      do 680 i=1,npar
680   dummy(i)=0.d0
      im=ifd*3-1
      do 685 k=1,ifd*3
```

```
                do 682 j=1,im
                    dummy(j)=p2(i,j+1)/p2(1,1)
682             continue
                dummy(ifd*3)=1./p2(1,1)
                do 684 l=1,im
                    do 683 j=1,im
                        p2(l,j)=p2(l+1,j+1)-p2(l+1,1)*dummy(j)
683                 continue
                    p2(l,ifd*3)=-p2(l+1,1)*dummy(ifd*3)
684             continue
                do 685 j=1,ifd*3
                    p2(ifd*3,j)=dummy(j)
685             continue
                do 690 l=1,ifd*3
                do 690 m=1,ifd*3
                    anatp(l,m)=0.d0
                do 690 n=1,ifd*3
690             anatp(l,m)=anatp(l,m)+anat(l,n)*p2(n,m)
                do 696 i=1,ifd*3
                do 696 j=1,ifd*3
                    if(i.eq.j) anatp(i,j)=1.d0-anatp(i,j)
                    if(i.ne.j) anatp(i,j)=-anatp(i,j)
696             continue
                do 700 i=1,3
700             b(i,j)=-1.d0
                if(ifd.ne.1) then
                    if=1
                    do 730 ji=2,ifd
                        if(ifrom(ji).eq.itest) then
                            if=if+1
                            km=2*if+if-2
                            b(km,1)=1.d0
                            b(km+1,1)=1.d0
                            b(km+2,1)=1.d0
                        elseif(ito(ji).eq.itest) then
                            if=if+1
                            km=2*if+if-2
                            b(km,1)=-1.d0
                            b(km+1,1)=-1.d0
                            b(km+2,1)=-1.d0
                        endif
730                 continue
                endif
```

```
            do 740 l=1,ifd*3
            do 740 m=1,3
                pris(l,m)=0.d0
            do 740 n=1,ifd*3
740         pris(l,m)=pris(l,m)+anatp(l,n)*b(n,m)
            noch1=0
            do 750 i=1,ifd*3
            do 750 j=1,3
                if(abs(pris(i,j)).lt.1.d-12) pris(i,j)=0.d0
                if(pris(i,j).eq.0.d0) noch1=noch1+1
750         continue
        if(noch.eq.0.and.noch1.eq.0) write(3,905) ses,obs(l),ifrom(l),ito(l)
        if(noch.eq.0.and.noch1.eq.0) write(3,904) ses,obs(l),ifrom(l),ito(l)
        if(noch.eq.0.and.noch1.eq.0) write(3,903) ses,obs(l),ifrom(l),ito(l)
        if(noch.eq.0.and.noch1.eq.0) write(3,902) ses,obs(l),ifrom(l),ito(l)
        kgomaa=kgomaa+1
        if(kgomaa.le.obsn) then
            do 805 i=1,6
                kobs(i)=0
                kfrom(i)=0
805         kto(i)=0
            do 809 i=1,18
                do 806 kk=1,3
                    pris(i,kk)=0.d0
806             b(i,kk)=0.d0
                do 807 k=1,npar
807             a(i,k)=0.d0
                do 808 j=1,18
                    anatp(i,j)=0.d0
808                 anat(i,k)=0.d0
809         continue
                do 810 i=1,npar
810             dummy(i)=0.d0
            if(obsn.ne.1) then
                jj=obs(l)
                kk=ifrom(l)
                ll=ito(l)
                do 820 i=1,obsn-1
                    obs(i)=obs(i+1)
                    ifrom(i)=ifrom(i+1)
                    ito(i)=ito(i+1)
820             continue
                obs(obsn)=jj
```

```
                ifrom(obsn)=kk
                ito(obsn)=ll
         endif
         if(obsn.ne.1) then
                do 830 i=1,npar
830             dummy(i)=0.d0
                im=iff*3-1
                do 835 k=1,iff*3
                       do 832 j=1,im
                              dummy(j)=p(i,j+1)/p(1,1)
832                    continue
                       dummy(iff*3)=1./p(1,1)
                       do 834 l=1,im
                              do 833 j=1,im
                                 p(l,j)=p(l+1,j+1)-p(l+1,1)*dummy(j)
833                           continue
                              p(l,iff*3)=-p(l+1,1)*dummy(iff*3)
834                    continue
                do 835 j=1,iff*3
                       p(iff*3,j)=dummy(j)
835             continue
                do 840 i=1,npar
840             dummy(i)=0.d0
                im=ifd*3-1
                do 845 k=1,ifd*3
                       do 842 j=1,im
                       dummy(j)=p2(i,j+1)/p2(l,l)
842                    continue
                       dummy(ifd*3)=1./p2(l,l)
                       do 844 l=1,im
                       do 843 j=1,im
                         do 843 j=1,im
                           p2(l,j)=p2(l+1,j+1)-p2(l+1,1)*dummy(j)
843                      continue
                         p2(l,iff*3)=-p2(l+1,1)*dummy(iff*3)
844                    continue
                do 845 j=1,iff*3
                       p2(iff*3,j)=dummy(j)
845             continue
                do 860 ii=1,obsn-1
                       jm=2*ii+ii-2
                       jm1=2*ii+ii+1
                       do 855 i=1,jm+2
```

```
                    do 855 j=jm1,3*obsn
                        p(i,j)=0.d0
                        p2(i,j)=0.d0
                        p(j,i)=0.d0
                        p2(j,i)=0.d0
855                 continue
860             continue
                do 870 i=1,3
                do 870 j=1,3
870             temp1(i,j)=p(i,j)
                do 880 ii=1,obsn-1
                jj=2*ii+ii-2
                kk=2*ii+ii+1
                do 872 i=1,3
                do 872 j=1,3
872             p(i+jj-1,j+jj-1)=p(i+kk-1,j+kk-1)
                mm=2*obsn+obsn-2
                do 882 i=1,3
                do 882 j=1,3
882             p(i+mm-1,j+mm-1)=temp1(i,j)
                do 883 i=1,3
                do 883 j=1,3
883             temp2(i,j)=p2(i,j)
                do 890 ii=1,obsn-1
                    jj=2*ii+ii-2
                    kk=2*ii+ii+1
                    do 886 i=1,3
                    do 866 j=1,3
886                 p2(i+jj-1,j+jj-1)=p2(i+kk-1,j+kk-1)
890             continue
                mm=2*obsn+obsn-2
                do 892 i=1,3
                do 892 j=1,3
892             p2(i+mm-1,j+mm-1)=temp2(i,j)
            endif
            go to 1002
        endif
c
c
        ses=ses+1
        rewind 1
        rewind 2
        if(ses.le.nses) go to 1
```

```
        write(3,999)
c
c
901     format(//,12x,'results of detecting no-check observations',///,
     +  'Session  Baseline   From (status) To (status) Final',/,
     + ' No                                             Results',/)
902     format(5x,i2,7x,i2,7x,i2,5x,\(n.c)',4x,i2,2x,'(n.c)',4x,
     +  'No Check')
903     format(5x,i2,7x,i2,7x,i2,5x,\(n.c)',4x,i2,2x,'(o.k)',4x,
     +  'No Check')
904     format(5x,i2,7x,i2,7x,i2,5x,\(n.c)',4x,i2,2x,'(n.c)',4x,
     +  'No Check')
905     format(5x,i2,7x,i2,7x,i2,5x,\(n.c)',4x,i2,2x,'(o.k)',4x,
     +  'Good')
999     format(4x,'-----------------------------------------',///,
     +   'N.C = no-check station')
c
c
        Return
C
C
        END
C
C
        Subroutine chdec(n,a,ut,nsing,ising)
C
C       This subroutine computes the Cholesky factor, ut(n,n) of a matric
C       a(n,n). 'nsing' is the number of singularities may be happened
C       in the factorization, while the vector 'ising' will contain the locations
C       of these singularities.
c
c
        implicit double precision (a-h,o-z)
        dimension a(n,n),ut(n,n),ising(6)
$large:a,ut
        do 1 i=1,n
        do 1 j=1,n
            ut(i,j)=0.d0
1       continue
        nsing=0
        do 2 j=1,n
            ising(i)=0
2       continue
```

```
      ut(1,1)=dsqrt(a(1,1))
      do 9 j=2,n
            ut(i,j)=a(i,j)/ut(i,i)
9     continue
      do 6 i=2,n
            sum1=0.d0
            do 3 k=1,i-1
                  sum1=sum1+ut(k,i)**2
3           continue
            ut(i,i)=dsqrt(a(i,i)-sum1)
            if(ut(i,i).eq.0.d0) then
                  ut(i,i)=10d6
                  nsing=nsing+1
                  ising(nsing)=i
                  do 8 jj=1,i-1
                        ut(jj,i)=0.d0
8                 continue
            endif
            do 5 j=i+1,n
                  sum2=0.d0
                  do 4 k=1,i-1
                        sum2=sum2+ut(k,i)*ut(k,h)
4                 continue
                  ut(i,j)=(a(i,j)-sum2/ut(i,i))
5           continue
6     continue
      do 10 i=1,n
      do 10 j=i+1,n
            ut(j,i)=ut(i,j)
10    ut(i,j)=0.d0
      returne
      end
c
c
      Subroutine togeod(a,finv,x,y,z,dlat,dlon,h)
C
C     Subroutine to calculate geodetic coordinates latitude, longitude,
C     height given Cartesian coordinates x,y,z and reference ellipsoid
C     values: semi-major axis (a) and the inverse of flatiining (finv).
C
      implicit double precision (a-h,o-z)
      double precision n
      data maxit/10/
```

```
      data tolsq/1.d-10/
      pi=4.d0*datan(1.d0)
      rtd=180.d0/pi
      flat=1.d0/finv
      esq=(2.d0-flat)*flat
      oneesq=1.d0-esq
      dlon=datan2(y,x)*rtd
      if(dlon.lt.0.d0) dlon=dlon+360.d0
      psq=x*x+y*y
      p=dsqrt(psq)
      r=dsqrt(psq+z*z)
      sinlat=z/r
      dlat=dasin(sinlat)
      h=r-a*(1.d0-flat*sinlat*sinlat)
      do 190 iter=1,maxit
            sinlat=dsin(dlat)
            coslat=dcos(dlat)
            n=a/dsqrt(1.d0-esq*sinlat*sinlat)
            dp=p-(n+h)*coslat
            dz=z-(n*oneesq+h)*sinlat
            h=h+(sinlat*dz+coslat*dp)
            dlat=dlat+(coslat*dz-sinlat*dp)/(n+h)
            if (dp*dp+dz*dz.lt.tolsq) then
                  dlat=dlat+rtd
                  retune
            endif
100   continue
      write(*,101) maxit
101   format('problem in togeod, did not converge in',i2,
     +  ' iterations')
      dlat=dlat*rtd
      returne
      end
c
c
      Subroutine frgeod(a,finv,x,y,z,dlat,dlon,h)
C
C     Subroutine to calculate Cartesian coordinates x,y,z
C     given geodetic coordinates latitude, longitude, height and
C     reference ellipsoid
C     values: semi-major axis (a) and the inverse of flatiining (finv).
C
c
```

```
         implicit double precision (a-h,o-z)
         double precision n
         pi=4.d0*datan(1.d0)
         ro=pi/180,d0
         flat=1.d0/finv
         esq=(2.d0-flat)*flat
         sinlat=dsin(dlat*ro)
         n=a/dsqrt(1.d0-esq*sinlat*sinlat)
         p=(n+h)*dcos(dlat*ro)
         z=(n*(1.d0-esq)+h)*sinlat
         x=p*dcos(dlon*ro)
         y=p*dsin(dlon*ro)
         retune
         end
c
c
c
         Subroutine dmstod(m,n,s,ang)
C
C        Convert deg,min,sec to degree
C
         Implicit double precision (a-h,o-z)
         If(m.lt.0) ang=-(abs(m)+(n/60.d0)+(s/3600.d0))
         If(m.gt.0) ang=(m+(n/60.d0)+(s/3600.d0))
         Returne
         End
C
C
         Subroutine dtodms(ang,d,m,s)
C        Convert degree to deg,min,sec
C
         Implicit double precision (a-h,o-z)
         If(ang.gt.180.d0) ang=(360.d0-ang)*(-1.d0)
         Id=ang
         Rm=(ang-id)*60.d0
         M1=rm
         M=abs(rm)
         S=(rm-m)*60.d0
         S=abs(s)
         Returne
         End
C
C
```

```
      Subroutine abat(a,b,r,w,m,n)
C     subroutine to form matrix product:
C           r = a . b . at
C     Array dimension: a(m,n), b(n,n), r(m,m)
C     Scratch vector: w(n)
C
      Implicit double precision (a-h,o-z)
      Dimension a(m,n),b(n,n),r(m,m),w(n)
$large:a,b,r,w
C
      Do 15 i=1,m
      Do 15 k=1,n
            W(k)=0
            Do 5 l=1,n
5           w(k)=w(k)+a(I,.l)*b(l,k)
      Do 15 j=1,m
            R(I,j)=0
            Do 10 l=1,n
10          r(I,j)=r(I,j)+w(l)*a(j,l)
15    r(j,i)=r(I,j)
      Returne
      End
C
C
      Subroutine ab(a,b,c,I,j,k)
C     subroutine to form matrix product:
C           c = a . b
C     Array dimension: a(I,j), b(j,k), c(I,k)
C
      Implicit double precision (a-h,o-z)
      Dimension a(I,j),b(j,k),c(I,k)
$large:a,b,c
C
      Do 10 l=1,i
      Do 10 m=1,k
            c(l,m)=0.d0
      Do 10 n=1,j
10          c(l,m)=c(l,m)+a(l,n)*b(n,m)
      Returne
      End
C
C
c
```

```
      Subroutine atrans(a,at,l,m)
C     subroutine to transpose a matrix
C     Array dimension: a(l,m), at(m,l)
C
      Implicit double precision (a-h,o-z)
      Dimension a(l,m),at(m,l)
$large:a,at
C
      Do 20 j=1,m
      Do 20 i=1,l
20          at(j,i)=a(I,j)
      Returne
      End
C
C
      Subroutine ainv(a,b,I)
C     subroutine to inverse a matrix:
C           a = a-inverse
C     Array dimension: a(I,i)
C     scratch vector: b(i)
C
      implicit double precision (a-h,o-z)
      dimension a(i,i),b(i)
$large:a,b
c
      if(i.eq.1) go to 10
      im=i-1
      do 5 k=1,i
            do 2 j=1,im
2           b(j)=a(i,j+1)/a(1,1)
            b(i)=1./a(1,1)
            do 4 l=1,im
                  do 3 j=1,im
3                 a(l,j)=a(l+1,j+1)-a(l+1,1)*b(j)
4           a(l,i)=-a(l+1,1)*b(l)
            do 5 j=1,i
5     a(i,j)=b(j)
      returne
10    a(l,l)=1./a(l,l)
      Returne
      End
C
C
```

```
      Subroutine atra(a,ata,n,m)
C     subroutine to form matrix product:
C           ata = at . a
C     Array dimension: a(n,m), ata(m,m)
C
      Implicit double precision (a-h,o-z)
      Dimension a(n,m),ata(m,m)
$large:a,ata
c
      do 5 i=1,m
      do 5 j=1,m
            ata(i,j)=0.d0
      do 5 k=1,n
5           ata(i,j)=ata(i,j)+a(k,i)*a(k,j)
      Returne
      End
C
      Subroutine atba(a,b,r,w,m,n)
C     subroutine to form matrix product:
C           r = at . b . a
C     Array dimension: a(m,n), b(m,m), r(n,n)
C     scratch vector: w(m)
C
      Implicit double precision (a-h,o-z)
      Dimension a(m,n),b(m,m),r(n,n),w(m)
$large:a,b,r,w
C
      do 15 i=1,n
      do 15 k=1,m
            w(k)=0.d0
            do 5 l=1,m
5           w(k)=w(k)+a(l,i)*b(l,k)
      do 15 j=1,n
            r(i,j)=0
            do 10 k=1,m
10          r(i,j)=r(i,j)+w(k)*a(k,j)
15    r(j,i)=r(i,j)
      returne
      end
c
c
c -------------------------- END --------------------------------------
```